Year 12 EOY Paper 1

1

(a) attempt to find d *(M1)*

e.g. $\frac{u_3 - u_1}{2}$, $8 = 2 + 2d$

$d = 3$ ***A1*** ***N2***

[2 marks]

(b) correct substitution *(A1)*

e.g. $u_{20} = 2 + (20-1)3$, $u_{20} = 3 \times 20 - 1$

$u_{20} = 59$ ***A1*** ***N2***

[2 marks]

(c) correct substitution *(A1)*

e.g. $S_{20} = \frac{20}{2}(2+59)$, $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$

$S_{20} = 610$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

2

(a) attempt to form composite *(M1)*

e.g. $g(7-2x)$, $7 - 2x + 3$

$(g \circ f)(x) = 10 - 2x$ ***A1*** ***N2***

[2 marks]

(b) $g^{-1}(x) = x - 3$ ***A1*** ***N1***

[1 mark]

(c) **METHOD 1**

valid approach *(M1)*

e.g. $g^{-1}(5), 2$, $f(5)$

$f(2) = 3$ ***A1*** ***N2***

[2 marks]

METHOD 2

attempt to form composite of f and g^{-1} *(M1)*

e.g. $(f \circ g^{-1})(x) = 7 - 2(x-3)$, $13 - 2x$

$(f \circ g^{-1})(5) = 3$ ***A1*** ***N2***

[2 marks]

Total [5 marks]

3

(a) finding $f'(x)=\frac{1}{2}x$ ***A1***

attempt to find $f'(4)$ ***(M1)***

correct value $f'(4)=2$ ***A1***

correct equation in any form ***A1*** ***N2***

e.g. $y-6=2(x-4)$, $y=2x-2$

[4 marks]

4

(a) **METHOD 1**

evidence of choosing $\sin^2\theta+\cos^2\theta=1$ ***(M1)***

correct working ***(A1)***

e.g. $\cos^2\theta=\frac{9}{13}$, $\cos\theta=\pm\frac{3}{\sqrt{13}}$, $\cos\theta=\sqrt{\frac{9}{13}}$

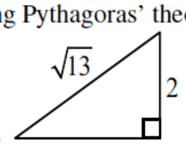
$\cos\theta=-\frac{3}{\sqrt{13}}$ ***A1*** ***N2***

Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

[3 marks]

METHOD 2

approach involving Pythagoras' theorem ***(M1)***

e.g. $2^2+x^2=13$, 

finding third side equals 3 ***(A1)***

$\cos\theta=-\frac{3}{\sqrt{13}}$ ***A1*** ***N2***

Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

[3 marks]

(b) correct substitution into $\sin 2\theta$ (seen anywhere) ***(A1)***

e.g. $2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)$

correct substitution into $\cos 2\theta$ (seen anywhere) ***(A1)***

e.g. $\left(-\frac{3}{\sqrt{13}}\right)^2-\left(\frac{2}{\sqrt{13}}\right)^2$, $2\left(-\frac{3}{\sqrt{13}}\right)^2-1$, $1-2\left(\frac{2}{\sqrt{13}}\right)^2$

valid attempt to find $\tan 2\theta$ ***(M1)***

e.g. $\dfrac{2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)}{\left(-\frac{3}{\sqrt{13}}\right)^2-\left(\frac{2}{\sqrt{13}}\right)^2}$, $\dfrac{2\left(-\frac{2}{3}\right)}{1-\left(-\frac{2}{3}\right)^2}$

correct working ***A1***

e.g. $\dfrac{\frac{(2)(2)(-3)}{13}}{\frac{9}{13}-\frac{4}{13}}$, $\dfrac{-\frac{12}{\left(\sqrt{13}\right)^2}}{\frac{18}{13}-1}$, $\dfrac{-\frac{12}{13}}{\frac{5}{13}}$

$\tan 2\theta = -\frac{12}{5}$ ***A1*** ***N4***

> **Note:** If students find answers for $\cos\theta$ which are not in the range [-1, 1], award full ***FT*** in (b) for correct ***FT*** working shown.

[5 marks]

Total [8 marks]

5

(a) valid approach ***(M1)***

e.g. b^2-4ac, $\Delta = 0$, $(-4k)^2-4(2k)(1)$

correct equation ***A1***

e.g. $(-4k)^2-4(2k)(1)=0$, $16k^2=8k$, $2k^2-k=0$

correct manipulation ***A1***

e.g. $8k(2k-1)$, $\frac{8\pm\sqrt{64}}{32}$

$k=\frac{1}{2}$ ***A2*** ***N3***

[5 marks]

(b) recognizing vertex is on the x-axis ***M1***

e.g. $(1, 0)$, sketch of parabola opening upward from the x-axis

$p \geq 0$ ***A1*** ***N1***

[2 marks]

Total [7 marks]

6

(a) correct derivatives **applied** in quotient rule *(A1)A1A1*

$1,\ -4x+5$

Note: Award *(A1)* for 1, *A1* for $-4x$ and *A1* for 5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule *A1*

e.g. $\dfrac{1\times(-2x^2+5x-2)-x(-4x+5)}{(-2x^2+5x-2)^2}$, $\dfrac{-2x^2+5x-2-x.-4x+5}{(-2x^2+5x-2)^2}$

correct working *(A1)*

e.g. $\dfrac{-2x^2+5x-2-(-4x^2+5x)}{(-2x^2+5x-2)^2}$

expression clearly leading to the answer *A1*

e.g. $\dfrac{-2x^2+5x-2+4x^2-5x}{(-2x^2+5x-2)^2}$

$f'(x)=\dfrac{2x^2-2}{(-2x^2+5x-2)^2}$ *AG* *N0*

[6 marks]

(b) evidence of attempting to solve $f'(x)=0$ *(M1)*

e.g. $2x^2-2=0$

evidence of correct working *A1*

e.g. $x^2=1,\ \dfrac{\pm\sqrt{16}}{4},\ 2(x-1)(x+1)$

correct solution to quadratic *(A1)*

e.g. $x=\pm 1$

correct x-coordinate $x=-1$ $\left(\text{may be seen in coordinate form } \left(-1, \dfrac{1}{9}\right)\right)$ *A1* *N2*

attempt to substitute -1 into f (do not accept any other value) *(M1)*

e.g. $f(-1)=\dfrac{-1}{-2\times(-1)^2+5\times(-1)-2}$

correct working

e.g. $\dfrac{-1}{-2-5-2}$ *A1*

correct y-coordinate $y=\dfrac{1}{9}$ $\left(\text{may be seen in coordinate form } \left(-1, \dfrac{1}{9}\right)\right)$ *A1* *N2*

[7 marks]

(c) recognizing values between max and min *(R1)*

$\frac{1}{9} < k < 1$ ***A2*** ***N3***

[3 marks]

Total [16 marks]

7

(a) (i) 100 (metres) ***A1*** ***N1***

(ii) 50 (metres) ***A1*** ***N1***

[2 marks]

(b) (i) identifying symmetry with $h(2) = 9.5$ ***(M1)***

subtraction ***A1***

e.g. $100 - h(2)$, $100 - 9.5$

$h(8) = 90.5$ ***AG*** ***N0***

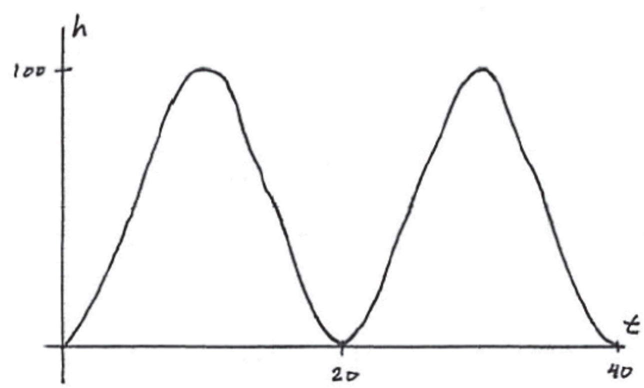
(ii) recognizing period ***(M1)***

e.g. $h(21) = h(1)$

$h(21) = 2.4$ ***A1*** ***N2***

[4 marks]

(c)



A1A1A1 ***N3***

Note: Award ***A1*** for end points $(0, 0)$ **and** $(40, 0)$, ***A1*** for range $0 \le h \le 100$, ***A1*** for approximately correct sinusoidal shape, with two cycles.

[3 marks]

(d) evidence of a quotient involving 20, 2π or 360° to find b ***(M1)***

e.g. $\frac{2\pi}{b} = 20$, $b = \frac{360}{20}$

$b = \frac{2\pi}{20}$ $\left(= \frac{\pi}{10}\right)$ (accept $b = 18$ if working in degrees) ***A1*** ***N2***

$a = -50$, $c = 50$ ***A2A1*** ***N3***

[5 marks]

Total [14 marks]