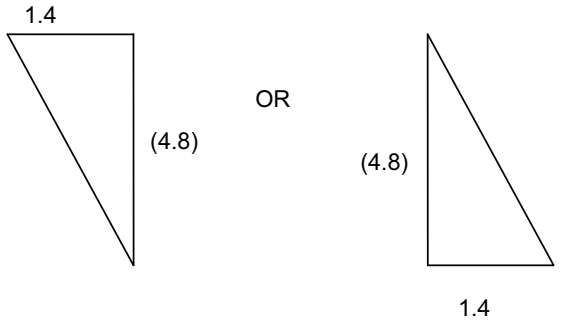
Vectors and Velocity Answers

1)

(i) speed of travel = 4.8 or distance downstream = 14 B1



draw right angle triangle with 1.4 and (4.8) at 90° B1

$\sqrt{1.4^2 + (4.8)^2}$ M1

5 A1

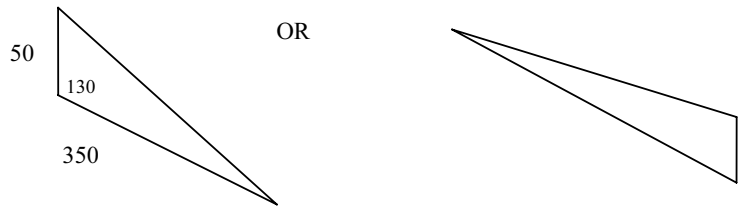
(ii) $\tan^{-1}\dfrac{(4.8)}{1.4}$ oe M1

73.7 or 1.29 radians A1

[6]

2)

9 **(i)**



B1

$V^2 = 50^2 + 350^2 - 2 \times 50 \times 350 \cos 130$ M1

$V = 384$ A1

$T = \dfrac{480}{V}$ M1

1.25 hours A1

(ii) $\dfrac{\sin\alpha}{350} = \dfrac{\sin 130}{V}$ or $\dfrac{\sin\beta}{50} = \dfrac{\sin 130}{V}$ M1

$\alpha = 44.3$ or $\beta = 5.72$ A1

135.7 or 136 A1√

[8]

OR

(i) Diagram B1

$X = 350 \sin 50 (= 268)$

$Y = 50 + 350 \cos 50 (= 275)$

$V^2 = X^2 + Y^2$

Finds components and uses Pythagoras M1

$V = 384$ A1

$T = \dfrac{480}{V}$ M1

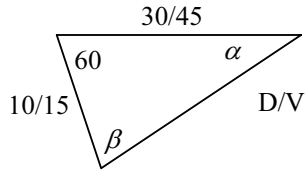
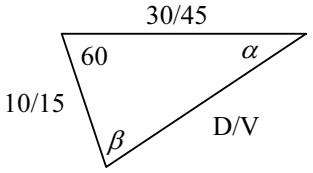
75 mins or 1.25 hours A1

(ii) $\tan\alpha = \dfrac{Y}{X}$ M1

$\alpha = 44.3$ A1

135.7 or 136 A1√

3)

9	**(i)**	Either  	B1
		10 or 45 found	B1
		Uses cosine rule	M1
		$D^2 = 10^2 + 30^2 - 2 \times 10 \times 30 \times \cos 60$ or $V^2 = 15^2 + 45^2 - 2 \times 15 \times 45 \times \cos 60$	A1
		39.7 or 39.8 or $15\sqrt{7}$	A1
	(ii)	$\dfrac{\sin\alpha}{10/15} = \dfrac{\sin 60}{D/V}$ (or $\dfrac{\sin\beta}{30} = \dfrac{\sin 60}{D}$ **and use** β)	M1
		$\alpha = 19.1$ or $\beta = 101$	A1
		251	A1√ **[8]**
9	**(i)**	Or  	B1
		10	B1
		$D\sin\alpha = 10\sin 60$ and $D\cos\alpha = 25$ or $V\sin\alpha = 15\sin 60$ and $V\cos\alpha = 37.5$	B1
		Solve equations	M1
		$V = 39.7$ or 39.8	A1
	(ii)	$\tan\alpha = \dfrac{10\sin 60}{25}$	M1
		$\alpha = 19.1$	A1
		251	A1√ **[8]**

4)

(i)	$\overrightarrow{OP} = \begin{pmatrix} 20 \\ 24 \end{pmatrix}$	B1
	$\overrightarrow{PL} = \begin{pmatrix} 7 \\ 24 \end{pmatrix}$ or $\overrightarrow{LP} = \begin{pmatrix} -7 \\ -24 \end{pmatrix}$	B1
	$\sqrt{7^2 + 24^2} = 25$	M1A1
	$\overrightarrow{PL} = \begin{pmatrix} 23 - 8t \\ 36 - 6t \end{pmatrix}$ oe	B1
(ii)	$(23 - 8t)^2 + (36 - 6t)^2 = 25^2$	M1
	Solve 3 term quadratic $\left[100\left(t^2 - 8t + 12\right) = 0\right]$	M1
	$6 - 2 = 4$ hours	A1 [8]

5)

9 **(i)** $10\sqrt{2}\left(\frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}\right)=10\mathbf{i}+10\mathbf{j}$	M1 A1 [2]	M1 for attempt at a correct direction vector A1 all correct
(ii) $(-4\mathbf{i}+8\mathbf{j})+(20\mathbf{i}+20\mathbf{j})=16\mathbf{i}+28\mathbf{j}$	M1 A1 [2]	M1 for valid attempt A1 all correct
(iii) $(10\mathbf{i}+10\mathbf{j})-(8\mathbf{i}+6\mathbf{j})=2\mathbf{i}+4\mathbf{j}$	M1 A1 [2]	M1 for attempt at vector difference A1 condone negative
(iv) displacement of $(19\mathbf{i}+34\mathbf{j})-(16\mathbf{i}+28\mathbf{j})=3\mathbf{i}+6\mathbf{j}$	M1	M1 for displacement and attempt to obtain time
time =1330 hours (accept 1.5 hours)	A1	A1 for correct time
at $31\mathbf{i}+43\mathbf{j}$	A1 [3]	A1 for correct position vector
Alternative scheme: $(19\mathbf{i}+34\mathbf{j})+(8\mathbf{i}+6\mathbf{j})t=$ $(16\mathbf{i}+28\mathbf{j})+(10\mathbf{i}+10\mathbf{j})t$ or equivalent		M1 for attempt to equate like vectors A marks as above

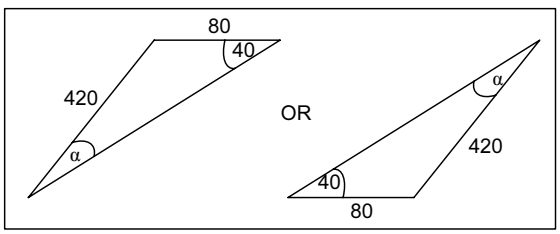
6)

(i) 10sin60 or 10cos30 or 5tan60 or $\sqrt{10^2-5^2}$	M1
$5\sqrt{3}$ or 8.66	A1 [2]
(ii) $\left(\frac{16-5t}{12+8.66t}\right)$ o.e.	M1A1 [2]
(iii) Equate x component to 0 1512 (when $t = 3.2$)	M1 A1 [2]
(iv) Substitute t into y component 39.7 km	M1 A1 [2]

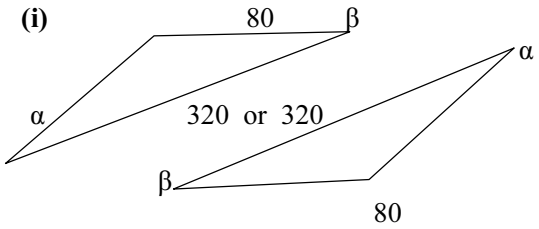
7)

EITHER **(i)** velocity = 12**i** + 16**j** position = (54**i** + 16**j**) + (36**i** + 48**j**) = 90**i** + 64**j** ANSWER GIVEN	**M1** **A1**	**M1** for (3 × their velocity (must in numeric vector form)) + (54**i** + 16**j**)
(ii) (54**i** + 16**j**) + (12*t***i** + 16*t***j**)	**M1, A1**	**M1** for position vector + (their numeric velocity vector × time)
(iii) At 16 00, ship has 'travelled' (102**i** + 80**j**)	**B1**	**B1** for (102**i** + 80**j**)
boat needs to do this in 2 hours so velocity of boat (51**i** + 40**j**) speed $\sqrt{51^2+40^2}$ = 64.8	**M1** **A1**	**M1** for attempt at velocity of boat and speed
(iv) (51**i** + 40**j**) – (12**i** + 16**j**) = 39**i** +24**j**	**B1**	**B1**, allow unsimplified but must be correct
(v) $\tan\alpha = \frac{51}{40}$ angle = 51.9	**M1** **A1** **[10]**	**M1** for use of tan and their velocity vector

8)

(i) 	B1	Correct triangle
$\frac{\sin\alpha}{80} = \frac{\sin 40}{420}$ α = 7.03 or 7 Bearing 223 (230 – α)	M1 A1 A1√ [4]	Use of sine or cosine rule in any triangle with some of 80,420, their *v* and an angle.
(ii) $\frac{v}{\sin their 133} = \frac{420}{\sin 40}$	M1	Use of sine or cosine rule in any triangle with 80 or 420 or both.
v = 478	A1	
Use time $\frac{1000}{v}$	M1	*v* calculated from a triangle
2.09 hours or 2 hours 5minutes	A1 [4]	Units required

9)

5	**(i)** 	B1	B1 for correct triangle Could be implied by subsequent working.
	$\frac{320}{\sin 120°} = \frac{80}{\sin \alpha}$	M1	M1 for complete method (sine rule and/or cosine rule) to find α or β
	α = 12.5° (or β = 47.5°)	A1	A1 for α (or β)
	Bearing = 042.5° or 043°	A1 [4]	A1 for bearing
	(ii) $\frac{v_r}{\sin 47.5°} = \frac{320}{\sin 120°}$, $v_r = 272.4$	M1	M1 for use of complete method (sine rule and/or cosine rule) to find v_r or x
	or $\frac{x}{\sin 120°} = \frac{450}{\sin 47.5°}$	A1	For either v = 272 or x = 529
	Time = $\frac{450}{272.4}$ or $\frac{528.6}{320}$	DM1	DM1 for $\frac{450}{\text{their velocity}}$ or their $\frac{x}{320}$
	= 1.65	A1 [4]	