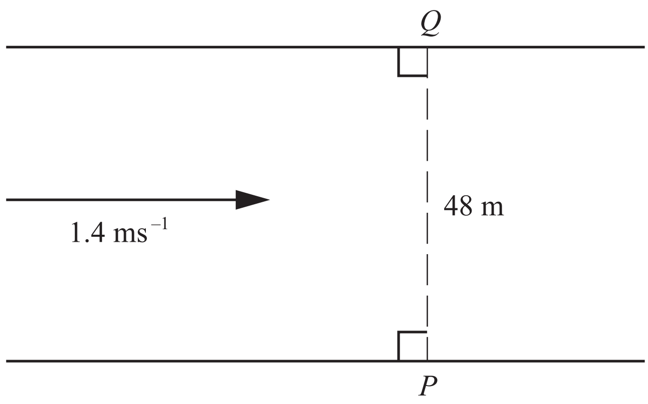
Vectors and Velocity

1)



The diagram shows a river with parallel banks. The river is 48 m wide and is flowing with a speed of $1.4\,\text{ms}^{-1}$. A boat travels in a straight line from a point P on one bank to a point Q which is on the other bank directly opposite P. Given that the boat takes 10 seconds to cross the river, find

(i) the speed of the boat in still water, [4]

(ii) the angle to the bank at which the boat should be steered. [2]

2) An aircraft, whose speed in still air is $350\,\text{kmh}^{-1}$, flies in a straight line from A to B, a distance of 480 km. There is a wind of $50\,\text{kmh}^{-1}$ blowing from the north. The pilot sets a course of 130°.

(i) Calculate the time taken to fly from A to B. [5]

(ii) Calculate the bearing of B from A. [3]

3) A coastguard station receives a distress call from a ship which is travelling at $15\,\text{km}\,\text{h}^{-1}$ on a bearing of 150°. A lifeboat leaves the coastguard station at 1500 hours; at this time the ship is at a distance of 30 km on a bearing of 270°. The lifeboat travels in a straight line at constant speed and reaches the ship at 1540 hours.

(i) Find the speed of the lifeboat. [5]

(ii) Find the bearing on which the lifeboat travelled. [3]

4) In this question, $\begin{pmatrix}1\\0\end{pmatrix}$ is a unit vector due east and $\begin{pmatrix}0\\1\end{pmatrix}$ is a unit vector due north.

A lighthouse has position vector $\begin{pmatrix}27\\48\end{pmatrix}$ km relative to an origin O. A boat moves in such a way that its position vector is given by $\begin{pmatrix}4+8t\\12+6t\end{pmatrix}$ km, where t is the time, in hours, after 1200.

(i) Show that at 1400 the boat is 25 km from the lighthouse. [4]

(ii) Find the length of time for which the boat is less than 25 km from the lighthouse. [4]

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5) At 10 00 hours, a ship P leaves a point A with position vector $(-4\mathbf{i} + 8\mathbf{j})$ km relative to an origin O, where $\mathbf{i}$ is a unit vector due East and $\mathbf{j}$ is a unit vector due North. The ship sails north-east with a speed of $10\sqrt{2}$ km h^{-1}. Find

(i) the velocity vector of P, [2]

(ii) the position vector of P at 12 00 hours. [2]

At 12 00 hours, a second ship Q leaves a point B with position vector $(19\mathbf{i} + 34\mathbf{j})$ km travelling with velocity vector $(8\mathbf{i} + 6\mathbf{j})$ km h^{-1}.

(iii) Find the velocity of P relative to Q. [2]

(iv) Hence, or otherwise, find the time at which P and Q meet and the position vector of the point where this happens. [3]

6) In this question $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a unit vector due east and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is a unit vector due north. At 12 00 a coastguard, at point O, observes a ship with position vector $\begin{pmatrix} 16 \\ 12 \end{pmatrix}$ km relative to O. The ship is moving at a steady speed of 10 kmh^{-1} on a bearing of 330°.

(i) Find the value of p such that $\begin{pmatrix} -5 \\ p \end{pmatrix}$ kmh^{-1} represents the velocity of the ship. [2]

(ii) Write down, in terms of t, the position vector of the ship, relative to O, t hours after 12 00. [2]

(iii) Find the time when the ship is due north of O. [2]

(iv) Find the distance of the ship from O at this time. [2]

7) **EITHER**

At 12 00 hours, a ship has position vector $(54\mathbf{i} + 16\mathbf{j})$ km relative to a lighthouse, where $\mathbf{i}$ is a unit vector due East and $\mathbf{j}$ is a unit vector due North. The ship is travelling with a speed of $20\,\text{km h}^{-1}$ in the direction $3\mathbf{i} + 4\mathbf{j}$.

(i) Show that the position vector of the ship at 15 00 hours is $(90\mathbf{i} + 64\mathbf{j})$ km. [2]

(ii) Find the position vector of the ship t hours after 12 00 hours. [2]

A speedboat leaves the lighthouse at 14 00 hours and travels in a straight line to intercept the ship. Given that the speedboat intercepts the ship at 16 00 hours, find

(iii) the speed of the speedboat, [3]

(iv) the velocity of the speedboat relative to the ship, [1]

(v) the angle the direction of the speedboat makes with North. [2]

8)

A plane, whose speed in still air is $420\,\text{km h}^{-1}$, travels directly from A to B, a distance of 1000 km. The bearing of B from A is 230° and there is a wind of $80\,\text{km h}^{-1}$ from the east.

(i) Find the bearing on which the plane was steered. [4]

(ii) Find the time taken for the journey. [4]

9) A pilot flies his plane directly from a point A to a point B, a distance of 450 km. The bearing of B from A is 030°. A wind of $80\,\text{km h}^{-1}$ is blowing from the east. Given that the plane can travel at $320\,\text{km h}^{-1}$ in still air, find

(i) the bearing on which the plane must be steered, [4]

(ii) the time taken to fly from A to B. [4]