Vectors 2

1) The line L passes through $\mathrm{A}(0, 3)$ and $\mathrm{B}(1, 0)$. The origin is at O. The point $\mathrm{R}(x, 3-3x)$ is on L, and (OR) is perpendicular to L.

(a) Write down the vectors $\vec{\mathrm{AB}}$ and $\vec{\mathrm{OR}}$.

(b) Use the scalar product to find the coordinates of R.

2) Consider the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 4\boldsymbol{i} + \boldsymbol{j} - p\boldsymbol{k}$.

(a) Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ find the value of p.

(b) Given that $q|\boldsymbol{u}| = 14$, find the value of q.

3) A particle is moving with a constant velocity along line L. Its initial position is $\mathrm{A}(6, -2, 10)$. After one second the particle has moved to $\mathrm{B}(9, -6, 15)$.

(a) (i) Find the velocity vector, $\vec{\mathrm{AB}}$.

(ii) Find the speed of the particle. *[4 marks]*

(b) Write down an equation of the line L. *[2 marks]*

4) Consider the points $\mathrm{A}(1, 5, 4)$, $\mathrm{B}(3, 1, 2)$ and $\mathrm{D}(3, k, 2)$, with (AD) perpendicular to (AB).

(a) Find

(i) $\vec{\mathrm{AB}}$;

(ii) $\vec{\mathrm{AD}}$, giving your answer in terms of k. *[3 marks]*

(b) Show that $k = 7$. *[3 marks]*

The point C is such that $\vec{\mathrm{BC}} = \frac{1}{2}\vec{\mathrm{AD}}$.

(c) Find the position vector of C. *[4 marks]*

(d) Find $\cos \mathrm{A\hat{B}C}$. *[3 marks]*

5) The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{OR} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

(a) Find

(i) $\overrightarrow{PQ}$;

(ii) $\overrightarrow{PR}$. *[3 marks]*

(b) Show that $\cos R\hat{P}Q = \frac{1}{2}$. *[7 marks]*

(c) (i) Find $\sin R\hat{P}Q$.

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$. *[6 marks]*