Vectors

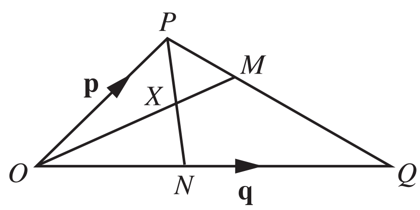
1) Relative to an origin O, points A and B have position vectors $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} 29 \\ -13 \end{pmatrix}$ respectively.

(i) Find a unit vector parallel to $\overrightarrow{AB}$. [3]

The points A, B and C lie on a straight line such that $2\overrightarrow{AC} = 3\overrightarrow{AB}$.

(ii) Find the position vector of the point C. [4]

2)



In the diagram $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$, $\overrightarrow{PM} = \frac{1}{3}\overrightarrow{PQ}$ and $\overrightarrow{ON} = \frac{2}{5}\overrightarrow{OQ}$.

(i) Given that $\overrightarrow{OX} = m\overrightarrow{OM}$, express $\overrightarrow{OX}$ in terms of m, $\mathbf{p}$ and $\mathbf{q}$. [2]

(ii) Given that $\overrightarrow{PX} = n\overrightarrow{PN}$, express $\overrightarrow{OX}$ in terms of n, $\mathbf{p}$ and $\mathbf{q}$. [3]

(iii) Hence evaluate m and n. [2]

3) The vector $\overrightarrow{OP}$ has a magnitude of 10 units and is parallel to the vector $3\mathbf{i} - 4\mathbf{j}$. The vector $\overrightarrow{OQ}$ has a magnitude of 15 units and is parallel to the vector $4\mathbf{i} + 3\mathbf{j}$.

(i) Express $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [3]

(ii) Given that the magnitude of $\overrightarrow{PQ}$ is $\lambda\sqrt{13}$, find the value of λ. [3]

4) Given that $\mathbf{a} = 5\mathbf{i} - 12\mathbf{j}$ and that $\mathbf{b} = p\mathbf{i} + \mathbf{j}$, find

(i) the unit vector in the direction of $\mathbf{a}$, [2]

(ii) the values of the constants p and q such that $q\mathbf{a} + \mathbf{b} = 19\mathbf{i} - 23\mathbf{j}$. [3]

Vectors

5) Relative to an origin O, the position vectors of points A and B are $\begin{pmatrix} 7 \\ 24 \end{pmatrix}$ and $\begin{pmatrix} 10 \\ 20 \end{pmatrix}$ respectively. Find

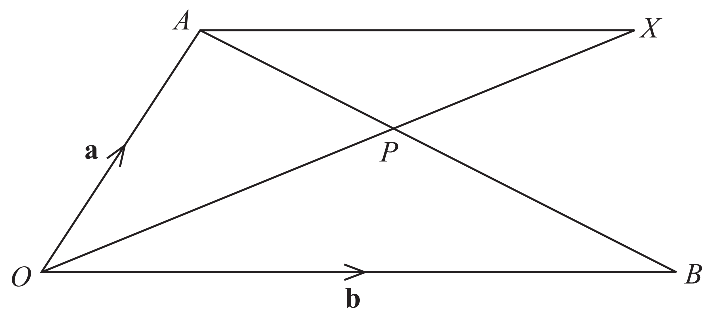
(i) the length of $\overrightarrow{OA}$, [2]

(ii) the length of $\overrightarrow{AB}$. [2]

Given that ABC is a straight line and that the length of $\overrightarrow{AC}$ is equal to the length of $\overrightarrow{OA}$, find

(iii) the position vector of the point C. [3]

6)

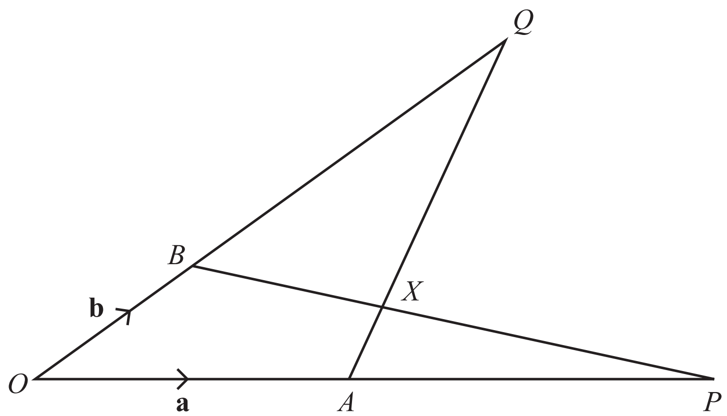


In the diagram $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$.

(i) Given that $\overrightarrow{OX} = \mu\overrightarrow{OP}$, where μ is a constant, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Given also that $\overrightarrow{AX} = \lambda\overrightarrow{OB}$, where λ is a constant, use a vector method to find the value of μ and of λ. [5]

7)



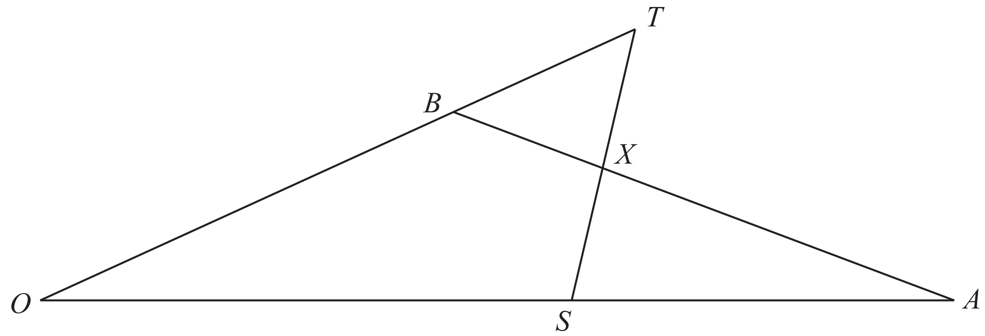
In the diagram $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OQ} = 3\mathbf{b}$.

(i) Given that $\overrightarrow{AX} = \mu\overrightarrow{AQ}$, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Given that $\overrightarrow{BX} = \lambda\overrightarrow{BP}$, express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [3]

(iii) Hence find the value of μ and of λ. [3]

8)



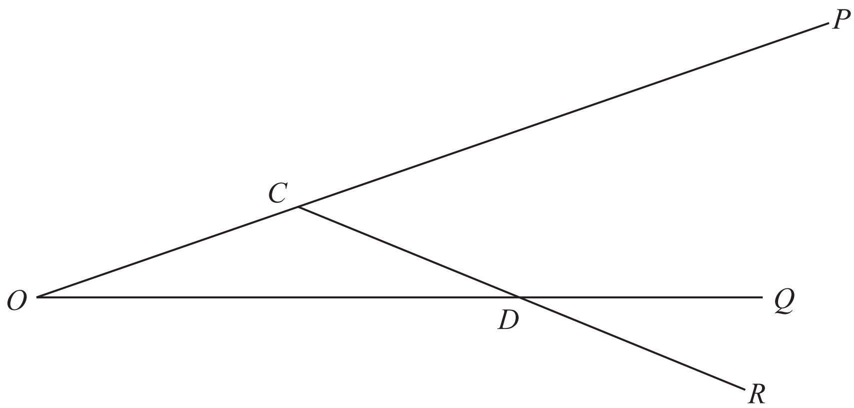
In the diagram above $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OS} = \frac{3}{5}\overrightarrow{OA}$ and $\overrightarrow{OT} = \frac{7}{5}\overrightarrow{OB}$.

(i) Given that $\overrightarrow{AX} = \mu\overrightarrow{AB}$, where μ is a constant, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) Given that $\overrightarrow{SX} = \lambda\overrightarrow{ST}$, where λ is a constant, express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [4]

(iii) Hence evaluate μ and λ. [4]

9)



In the diagram above $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$. The points P and Q lie on OC and OD produced respectively, so that $OC : CP = 1 : 2$ and $OD : DQ = 2 : 1$. The line CD is extended to R so that $CD = DR$.

(i) Find, in terms of $\mathbf{c}$ and/or $\mathbf{d}$, the vectors $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and $\overrightarrow{OR}$. [5]

(ii) Show that the points P, Q and R are collinear and find the ratio $PQ : QR$. [5]