

Vectors 4 Answers

1) (a) (i) Evidence of approach

$$e.g. \vec{JQ} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}, \vec{JQ} = \vec{JO} + \vec{OQ}$$
M1

$$\vec{JO} = \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$$
AG
No

$$(ii) \quad \vec{MK} = \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$$
A1
N1

[2 marks]

$$(b) \quad (i) \quad \mathbf{r} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} \text{ or } \mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + t \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$$
A2
N2

Note: Award **A1** if “ $\mathbf{r} =$ ” is missing.

$$(ii) \quad \text{Evidence of choosing correct vectors} \quad \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}, \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix} \quad (\mathbf{A1})(\mathbf{A1})$$

Evidence of calculating magnitudes **(A1)(A1)**

$$e.g. \sqrt{(-6)^2 + 7^2 + 10^2} = \sqrt{185} \quad \sqrt{6^2 + (-7)^2 + 10^2} = \sqrt{185}$$

$$\begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix} = -36 - 49 + 100 \quad (=15) \quad (\text{accept } -15) \quad (\mathbf{A1})$$

For evidence of substitution into the correct formula **M1**

$$e.g. \cos\theta = \frac{15}{\sqrt{185}\sqrt{185}} \quad \left(= \frac{15}{185} = 0.0811 \right) \quad \left(\text{accept } \frac{-15}{\sqrt{185}\sqrt{185}} \right)$$

$$\theta = 1.49 \text{ (radians), } 85.3^\circ \quad \begin{matrix} \mathbf{A1} & \mathbf{N4} \end{matrix}$$

[9 marks]

(c) **METHOD 1**

Geometric approach **(M1)**

Valid reasoning **A2**

$$e.g. \text{ diagonals bisect each other, } \vec{OD} = \vec{OM} + \frac{1}{2} \vec{MK}$$

Calculation of mid point **(A1)**

$$e.g. \left(\frac{6+0}{2}, \frac{0+7}{2}, \frac{0+10}{2} \right)$$

$$\vec{OD} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix} \quad (\text{accept } (3, 3.5, 5)) \quad \begin{matrix} \mathbf{A1} & \mathbf{N3} \end{matrix}$$

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METHOD 2

Correct approach

(M1)

$$\text{e.g. } \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$$

Two correct equations

A1

$$\text{e.g. } 6 - 6t = 6s, \quad 7t = 7 - 7s, \quad 10t = 10s$$

Attempt to solve

(M1)

One correct parameter

$$s = 0.5 \quad t = 0.5$$

A1

$$\vec{\text{OD}} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix} \quad (\text{accept } (3, 3.5, 5))$$

A1

N3

METHOD 3

Correct approach

(M1)

$$\text{e.g. } \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} + t \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$$

Two correct equations

A1

$$\text{e.g. } -6t = 6s, \quad 7 + 7t = 7 - 7s, \quad 10 + 10t = 10s$$

Attempt to solve

(M1)

One correct parameter

$$s = 0.5 \quad t = -0.5$$

A1

$$\vec{\text{OD}} = \begin{pmatrix} 3 \\ 3.5 \\ 5 \end{pmatrix} \quad (\text{accept } (3, 3.5, 5))$$

A1

N3

[5 marks]

Total [16 marks]

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- 2) $p\mathbf{w} = p\mathbf{i} + 2p\mathbf{j} - 3p\mathbf{k}$ (seen anywhere) **(A1)**
attempt to find $\mathbf{v} + p\mathbf{w}$ **(M1)**
e.g. $3\mathbf{i} + 4\mathbf{j} + \mathbf{k} + p(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$
- collecting terms $(3 + p)\mathbf{i} + (4 + 2p)\mathbf{j} + (1 - 3p)\mathbf{k}$ **A1**
- attempt to find the dot product **(M1)**
e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p)$
- setting **their** dot product equal to 0 **(M1)**
e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p) = 0$
- simplifying **A1**
e.g. $3 + p + 8 + 4p - 3 + 9p = 0, 14p + 8 = 0$
- $p = -0.571 \quad \left(-\frac{8}{14}\right)$ **A1** **N3**
- [7 marks]**
- 3) correct substitutions for $\mathbf{v} \cdot \mathbf{w}; |\mathbf{v}|; |\mathbf{w}|$ **(A1)(A1)(A1)**
e.g. $2k + (-3) \times (-2) + 6 \times 4, 2k + 30; \sqrt{2^2 + (-3)^2 + 6^2}, \sqrt{49}; \sqrt{k^2 + (-2)^2 + 4^2}, \sqrt{k^2 + 20}$
- evidence of substituting into the formula for scalar product **(M1)**
e.g. $\frac{2k + 30}{7 \times \sqrt{k^2 + 20}}$
- correct substitution **A1**
e.g. $\cos \frac{\pi}{3} = \frac{2k + 30}{7 \times \sqrt{k^2 + 20}}$
- $k = 18.8$ **A2** **N5**
- [7 marks]**

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- 4) (a) evidence of correct approach **A1**
e.g. $\vec{PQ} = \vec{OQ} - \vec{OP}$, $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$
 $\vec{PQ} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ **AG** **No**
[1 mark]
- (b) (i) correct description **R1** **N1**
e.g. reference to $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$ being the position vector of a point on the line,
a vector to the line, a point on the line.
- (ii) **any** correct expression in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ **A2** **N2**
where \mathbf{a} is $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$, and \mathbf{b} is a scalar multiple of $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$
e.g. $\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} 3+2s \\ -3-4s \\ 8+6s \end{pmatrix}$
[3 marks]
- (c) **one** correct equation **(A1)**
e.g. $3+s=-1$, $-3-2s=5$
- $s = -4$ **A1**
 $p = -4$ **A1** **N2**
[3 marks]
- (d) **one** correct equation **A1**
e.g. $-3+t=-1$, $9-2t=5$
- $t = 2$ **A1**
substituting $t = 2$ **A1**
e.g. $2+2q=-4$, $2q=-6$
 $q = -3$ **AG** **No**
[3 marks]

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(e) choosing correct direction vectors $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$ **(AI)(AI)**

finding correct scalar product and magnitudes **(AI)(AI)(AI)**

scalar product $(1)(1) + (-2)(-2) + (-3)(3)$ ($= -4$)

$$\text{magnitudes } \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}, \sqrt{1^2 + (-2)^2 + (-3)^2} = \sqrt{14}$$

evidence of substituting into scalar product

M1

$$\text{e.g. } \cos \theta = \frac{-4}{3.741... \times 3.741...}$$

$$\theta = 1.86 \text{ radians (or } 107^\circ\text{)}$$

A1

N4

[7 marks]

Total [17 marks]