# Vectors 3 **calculator**

1) *[Maximum mark: 15]*

Points A, B, and C have position vectors $4\boldsymbol{i}+2\boldsymbol{j}$, $\boldsymbol{i}-3\boldsymbol{j}$ and $-5\boldsymbol{i}-5\boldsymbol{j}$. Let D be a point on the $x$-axis such that ABCD forms a parallelogram.

(a) (i) Find $\overrightarrow{BC}$.

(ii) Find the position vector of D. *[4 marks]*

(b) Find the angle between $\overrightarrow{BD}$ and $\overrightarrow{AC}$. *[6 marks]*

The line $L_1$ passes through A and is parallel to $\boldsymbol{i}+4\boldsymbol{j}$. The line $L_2$ passes through B and is parallel to $2\boldsymbol{i}+7\boldsymbol{j}$. A vector equation of $L_1$ is $\boldsymbol{r}=(4\boldsymbol{i}+2\boldsymbol{j})+s(\boldsymbol{i}+4\boldsymbol{j})$.

(c) Write down a vector equation of $L_2$ in the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{q}$. *[1 mark]*

(d) The lines $L_1$ and $L_2$ intersect at the point P. Find the position vector of P. *[4 marks]*

2) The position vector of point A is $2\boldsymbol{i}+3\boldsymbol{j}+\boldsymbol{k}$ and the position vector of point B is $4\boldsymbol{i}-5\boldsymbol{j}+21\boldsymbol{k}$.

(a) (i) Show that $\overrightarrow{AB}=2\boldsymbol{i}-8\boldsymbol{j}+20\boldsymbol{k}$.

(ii) Find the unit vector $\boldsymbol{u}$ in the direction of $\overrightarrow{AB}$.

(iii) Show that $\boldsymbol{u}$ is perpendicular to $\overrightarrow{OA}$. *[6 marks]*

Let S be the midpoint of [AB]. The line $L_1$ passes through S and is parallel to $\overrightarrow{OA}$.

(b) (i) Find the position vector of S.

(ii) Write down the equation of $L_1$. *[4 marks]*

The line $L_2$ has equation $\boldsymbol{r}=(5\boldsymbol{i}+10\boldsymbol{j}+10\boldsymbol{k})+s(-2\boldsymbol{i}+5\boldsymbol{j}-3\boldsymbol{k})$.

(c) Explain why $L_1$ and $L_2$ are not parallel. *[2 marks]*

(d) The lines $L_1$ and $L_2$ intersect at the point P. Find the position vector of P. *[7 marks]*

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3) Points P and Q have position vectors $-5\boldsymbol{i}+11\boldsymbol{j}-8\boldsymbol{k}$ and $-4\boldsymbol{i}+9\boldsymbol{j}-5\boldsymbol{k}$ respectively, and both lie on a line $L_1$.

(a) (i) Find $\overrightarrow{\mathrm{PQ}}$.

(ii) Hence show that the equation of $L_1$ can be written as

$$\boldsymbol{r}=(-5+s)\boldsymbol{i}+(11-2s)\boldsymbol{j}+(-8+3s)\boldsymbol{k}.$$ *[4 marks]*

The point $\mathrm{R}(2,\ y_1,\ z_1)$ also lies on $L_1$.

(b) Find the value of $y_1$ and of $z_1$. *[4 marks]*

The line $L_2$ has equation $\boldsymbol{r}=2\boldsymbol{i}+9\boldsymbol{j}+13\boldsymbol{k}+t(\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k})$.

(c) The lines $L_1$ and $L_2$ intersect at a point T. Find the position vector of T. *[7 marks]*

(d) Calculate the angle between the lines $L_1$ and $L_2$. *[7 marks]*