Vectors 3 Answers

1) (a) (i) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= -6\boldsymbol{i} - 2\boldsymbol{j}$ *(A1)(A1)* *(N2)*

(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$

$= -2\boldsymbol{i} + 0\boldsymbol{j}\ (= -2\boldsymbol{i})$ *(A1)(A1)* *(N2)*

[4 marks]

(b) $\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB}$

$= -3\boldsymbol{i} + 3\boldsymbol{j}$ *(A1)*

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= -9\boldsymbol{i} - 7\boldsymbol{j}$ *(A1)*

Let θ be the angle between $\overrightarrow{BD}$ and $\overrightarrow{AC}$

$\cos\theta = \left(\dfrac{(-3\boldsymbol{i} + 3\boldsymbol{j}) \bullet (-9\boldsymbol{i} - 7\boldsymbol{j})}{|(-3\boldsymbol{i} + 3\boldsymbol{j})||-9\boldsymbol{i} - 7\boldsymbol{j}|}\right)$ *(M1)*

numerator = + 27 – 21 (= 6) *(A1)*

denominator $= \sqrt{18}\sqrt{130}\ \left(= \sqrt{2340}\right)$ *(A1)*

therefore, $\cos\theta = \dfrac{6}{\sqrt{2340}}$

$\theta = 82.9^\circ\ (= 1.45 \text{ rad})$ *(A1)* *(N3)*

[6 marks]

(c) $\boldsymbol{r} = \boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{t}(2\boldsymbol{i} + 7\boldsymbol{j})\quad \left(= (1 + 2t)\boldsymbol{i} + (-3 + 7\boldsymbol{t})\boldsymbol{j}\right)$ *(A1)* *(N1)*

[1 mark]

(d) **EITHER**

$4\boldsymbol{i} + 2\boldsymbol{j} + s(\boldsymbol{i} + 4\boldsymbol{j}) = \boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{t}(2\boldsymbol{i} + 7\boldsymbol{j})$ (may be implied) *(M1)*

$\left.\begin{array}{r} 4 + s = 1 + 2t \\ 2 + 4s = -3 + 7t \end{array}\right\}$ *(A1)*

$t = 7$ and/or $s = 11$ *(A1)*

Position vector of P is $15\boldsymbol{i} + 46\boldsymbol{j}$ *(A1)* *(N2)*

OR

$7x - 2y = 13$ or equivalent *(A1)*

$4x - y = 14$ or equivalent *(A1)*

$x = 15$, $y = 46$ *(A1)*

Position vector of P is $15\boldsymbol{i} + 46\boldsymbol{j}$ *(A1)* *(N2)*

[4 marks]

Total [15 marks]

Vectors 3 Answers

2) (a) (i) Evidence of subtracting all three components in the correct order **M1**

e.g. $\vec{AB} = \vec{OB} - \vec{OA} = (4\boldsymbol{i} - 5\boldsymbol{j} + 21\boldsymbol{k}) - (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k})$

$= 2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k}$ **AG** **N0**

(ii) $\left|\vec{AB}\right| = \sqrt{2^2 + (-8)^2 + 20^2} \quad \left(= \sqrt{468} = 6\sqrt{13} = 2\sqrt{117} = 21.6\right)$ **(A1)**

$\boldsymbol{u} = \frac{1}{\sqrt{468}}(2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k})$ **A1** **N2**

$\left(= \frac{2}{\sqrt{468}}\boldsymbol{i} - \frac{8}{\sqrt{468}}\boldsymbol{j} + \frac{20}{\sqrt{468}}\boldsymbol{k},\ 0.0925\boldsymbol{i} - 0370\boldsymbol{j} + 0.925\boldsymbol{k},\ etc.\right)$

(iii) If the scalar product is zero, the vectors are perpendicular. **R1**

> **Note:** Award ***R1*** for stating the relationship between the scalar product and perpendicularity, seen anywhere in the solution.

Finding an appropriate scalar product $\left(u \bullet \vec{OA},\ \text{or}\ \vec{AB} \bullet \vec{OA}\right)$ **M1**

e.g. $u \bullet \vec{OA} = \left(\frac{2}{\sqrt{468}}\right) \times 2 + \left(\frac{-8}{\sqrt{468}}\right) \times 3 + \left(\frac{20}{\sqrt{468}}\right) \times 1 \quad \left(= \frac{4 - 24 + 20}{\sqrt{468}}\right)$

$\vec{AB} \bullet \vec{OA} = 2 \times 2 + (-8) \times 3 + 20 \times 1$

$\boldsymbol{u} \bullet \vec{OA} = 0$ or $\vec{AB} \bullet \vec{OA} = 0$ **A1** **N0**

[6 marks]

(b) (i) **EITHER**

$S\left(\frac{2+4}{2}, \frac{3-5}{2}, \frac{1+21}{2}\right)$ ***(M1)(A1)***

Therefore, $\vec{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$ (accept (3, −1, 11)) **A1** **N3**

OR

$\vec{OS} = \vec{OA} + \frac{1}{2}\vec{AB}$ ***(M1)***

$= (2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}) + \frac{1}{2}(2\boldsymbol{i} - 8\boldsymbol{j} + 20\boldsymbol{k})$ ***(A1)***

$\vec{OS} = 3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}$ **A1** **N3**

(ii) $L_1 : \boldsymbol{r} = (3\boldsymbol{i} - \boldsymbol{j} + 11\boldsymbol{k}) + t(2\boldsymbol{i} + 3\boldsymbol{j} + 1\boldsymbol{k})$ **A1** **N1**

[4 marks]

continued ...

(c) Using direction vectors $(e.g.\ 2\boldsymbol{i}+3\boldsymbol{j}+1\boldsymbol{k}$ and $-2\boldsymbol{i}+5\boldsymbol{j}-3\boldsymbol{k})$ ***(M1)***
Valid explanation of why L_1 is not parallel to L_2 ***R1*** ***N2***

e.g. Direction vectors are not scalar multiples of each other.
Angle between the direction vectors is not zero or 180.
Finding the angle

$\boldsymbol{d_1} \bullet \boldsymbol{d_2} \neq |\boldsymbol{d_1}||\boldsymbol{d_2}|$.

Note: Award ***R0*** for "direction vectors are not equal".

[2 marks]

(d) Setting up any **two** of the three equations ***(M1)***
For each correct equation ***A1A1***
e.g. $3+2t=5-2s$, $-1+3t=10+5s$, $11+t=10-3s$

Attempt to solve these equations ***(M1)***
Finding **one** correct parameter $(s=-1,\ t=2)$ ***(A1)***

P has position vector $7\boldsymbol{i}+5\boldsymbol{j}+13\boldsymbol{k}$ ***A2*** ***N4***

Note: Award ***(M1)A2*** if the same parameter is used for both lines in the initial correct equations.
Award no further marks.

[7 marks]

Total [19 marks]

Vectors 3 Answers

3)

(a)	(i)	$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$	*(M1)*	
		$= \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$	*A1*	*N2*
	(ii)	$\boldsymbol{r} = \overrightarrow{OP} + s\,\overrightarrow{PQ}$	*(M1)*	
		$= -5\boldsymbol{i} + 11\boldsymbol{j} - 8\boldsymbol{k} + s(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$	*A1*	
		$= (-5+s)\boldsymbol{i} + (11-2s)\boldsymbol{j} + (-8+3s)\boldsymbol{k}$	*AG*	*N0*

[4 marks]

(b) If $(2,\ y_1,\ z_1)$ lies on L_1 then $-5+s=2$ *(M1)*

$s = 7$ *A1*

$y_1 = -3,\ z_1 = 13$ *A1A1* *N3*

[4 marks]

(c) Evidence of correct approach *(M1)*

e.g. $(-5+s)\boldsymbol{i} + (11-2s)\boldsymbol{j} + (-8+3s)\boldsymbol{k} = 2\boldsymbol{i} + 9\boldsymbol{j} + 13\boldsymbol{k} + t(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$

At least two correct equations *A1A1*

e.g. $-5+s = 2+t,\ 11-2s = 9+2t,\ -8+3s = 13+3t$

Attempting to solve **their** equations *(M1)*

One correct parameter ($s=4,\ t=-3$) *A1*

$\overrightarrow{OT} = -\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ *A2* *N4*

[7 marks]

(d) Direction vector for L_1 is $\boldsymbol{d}_1 = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ *(A1)*

> **Note:** Award ***A1FT*** for their vector from (a)(i).

Direction vector for L_2 is $\boldsymbol{d}_2 = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ *(A1)*

$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = 6,\ |\boldsymbol{d}_1| = \sqrt{14},\ |\boldsymbol{d}_2| = \sqrt{14}$ *(A1)(A1)(A1)*

$\cos\theta = \dfrac{6}{\sqrt{14}\sqrt{14}} \quad \left(= \dfrac{6}{14} = \dfrac{3}{7}\right)$ *A1*

$\theta = 64.6^\circ \quad (= 1.13 \text{ radians})$ *A1* *N4*

> **Note:** Award marks as per the markscheme if their (correct) direction vectors give $d_1 \bullet d_2 = -6$, leading to $\theta = 115^\circ \quad (= 2.01 \text{ radians})$.

[7 marks]

Total [22 marks]