

Vectors 2 Answers

1) (a) $\vec{AB} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\vec{OR} = \begin{pmatrix} x \\ 3-3x \end{pmatrix}$ *A1A1 N2*

(b) $\vec{AB} \cdot \vec{OR} = x - 3(3-3x)$ *A1*

$\vec{AB} \cdot \vec{OR} = 0 \quad (10x - 9 = 0)$ *M1*

R is $\left(\frac{9}{10}, \frac{3}{10} \right)$ *A1A1 N2*

2) (a) $\mathbf{u} \cdot \mathbf{v} = 8 + 3 + p$ *(A1)*

For equating scalar product equal to zero *(M1)*

$8 + 3 + p = 0$

$p = -11$ *A1 N3*

(b) $|\mathbf{u}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}, 3.74$ *(M1)*

$q\sqrt{14} = 14$ *A1*

$q = \sqrt{14} \quad (= 3.74)$ *A1 N2*

3) (a) (i) evidence of approach *(M1)*

e.g. $\vec{AO} + \vec{OB}, \mathbf{B} - \mathbf{A}, \begin{pmatrix} 9-6 \\ -6+2 \\ 15-10 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$ (accept $(3, -4, 5)$) *A1 N2*

(ii) evidence of finding the magnitude of the velocity vector *M1*

e.g. speed $= \sqrt{3^2 + 4^2 + 5^2}$

speed $= \sqrt{50} \quad (= 5\sqrt{2})$ *A1 N1*

(b) correct **equation** (accept Cartesian and parametric forms) *A2 N2*

e.g. $\mathbf{r} = \begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$

[6 marks]

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- 4) (a) (i) evidence of combining vectors *(M1)*
 e.g. $\vec{AB} = \vec{OB} - \vec{OA}$ (or $\vec{AD} = \vec{AO} + \vec{OD}$ in part (ii))

$$\vec{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$$
 A1 *N2*
- (ii) $\vec{AD} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$ *A1* *N1*
- [3 marks]*
- (b) evidence of using perpendicularity \Rightarrow scalar product = 0 *(M1)*
 e.g. $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0$
 $4 - 4(k-5) + 4 = 0$ *A1*
 $-4k + 28 = 0$ (accept any correct equation clearly leading to $k = 7$) *A1*
 $k = 7$ *AG* *N0*
- [3 marks]*
- (c) $\vec{AD} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ *(A1)*
 $\vec{BC} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ *A1*
 evidence of correct approach *(M1)*
 e.g. $\vec{OC} = \vec{OB} + \vec{BC}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$
 $\vec{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ *A1* *N3*
- [4 marks]*
- (d) **METHOD 1**
 choosing appropriate vectors, \vec{BA} , \vec{BC} *(A1)*
 finding the scalar product *M1*
 e.g. $-2(1) + 4(1) + 2(-1)$, $2(1) + (-4)(1) + (-2)(-1)$
 $\cos A\hat{B}C = 0$ *A1* *N1*
- METHOD 2**
 \vec{BC} parallel to \vec{AD} (may show this on a diagram with points labelled) *R1*
 $\vec{BC} \perp \vec{AB}$ (may show this on a diagram with points labelled) *R1*
 $\hat{A}BC = 90^\circ$
 $\cos A\hat{B}C = 0$ *A1* *N1*
- [3 marks]*

Total [13 marks]

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- 5) (a) (i) evidence of approach **(M1)**
 e.g. $\vec{PQ} = \vec{PO} + \vec{OQ}$, $Q - P$
 $\vec{PQ} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ **A1** **N2**
- (ii) $\vec{PR} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ **A1** **N1**

[3 marks]

- (b) **METHOD 1**
- choosing correct vectors \vec{PQ} and \vec{PR} **(AI)(AI)**
 finding $\vec{PQ} \cdot \vec{PR}$, $|\vec{PQ}|$, $|\vec{PR}|$ **(AI) (AI)(AI)**
- $$\vec{PQ} \cdot \vec{PR} = -2 + 4 + 4 (= 6)$$
- $$|\vec{PQ}| = \sqrt{(-1)^2 + 2^2 + 1^2} (= \sqrt{6}), |\vec{PR}| = \sqrt{2^2 + 2^2 + 4^2} (= \sqrt{24})$$
- substituting into formula for angle between two vectors **M1**
- e.g. $\cos R\hat{P}Q = \frac{6}{\sqrt{6} \times \sqrt{24}}$
- simplifying to expression clearly leading to $\frac{1}{2}$ **A1**
- e.g. $\frac{6}{\sqrt{6} \times 2\sqrt{6}}, \frac{6}{\sqrt{144}}, \frac{6}{12}$
- $\cos R\hat{P}Q = \frac{1}{2}$ **AG** **N0**

METHOD 2

- evidence of choosing cosine rule (seen anywhere) **(M1)**
- $\vec{QR} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$ **A1**
- $$|\vec{QR}| = \sqrt{18}, |\vec{PQ}| = \sqrt{6} \text{ and } |\vec{PR}| = \sqrt{24}$$
- (AI)(AI)(AI)**
- $$\cos R\hat{P}Q = \frac{(\sqrt{6})^2 + (\sqrt{24})^2 - (\sqrt{18})^2}{2\sqrt{6} \times \sqrt{24}}$$
- A1**
- $$\cos R\hat{P}Q = \frac{6 + 24 - 18}{24} \left(= \frac{12}{24} \right)$$
- A1**
- $$\cos R\hat{P}Q = \frac{1}{2}$$
- AG** **N0**

[7 marks]

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(c) (i) **METHOD 1**

evidence of appropriate approach **(M1)**

e.g. using $\sin^2 \hat{R}PQ + \cos^2 \hat{R}PQ = 1$, diagram

substituting correctly **(A1)**

$$e.g. \sin \hat{R}PQ = \sqrt{1 - \left(\frac{1}{2}\right)^2}$$

$$\sin \hat{R}PQ = \sqrt{\frac{3}{4}} \quad \left(= \frac{\sqrt{3}}{2} \right) \quad \begin{matrix} \textbf{A1} & & \textbf{N3} \end{matrix}$$

METHOD 2

since $\cos \hat{P} = \frac{1}{2}$, $\hat{P} = 60^\circ$ **(A1)**

evidence of approach

e.g. drawing a right triangle, finding the missing side **(A1)**

$$\sin \hat{P} = \frac{\sqrt{3}}{2} \quad \begin{matrix} \textbf{A1} & & \textbf{N3} \end{matrix}$$

(ii) evidence of appropriate approach **(M1)**

e.g. attempt to substitute into $\frac{1}{2}ab \sin C$

correct substitution

$$e.g. \text{area} = \frac{1}{2} \sqrt{6} \times \sqrt{24} \times \frac{\sqrt{3}}{2} \quad \begin{matrix} \textbf{A1} \\ \textbf{A1} \end{matrix}$$

$$\text{area} = 3\sqrt{3} \quad \begin{matrix} \textbf{N2} \\ \textbf{[6 marks]} \end{matrix}$$

Total [16 marks]