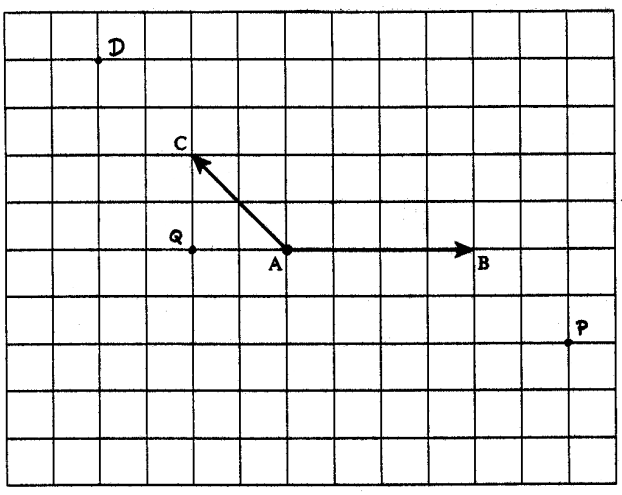
Vectors 1 Answers

1)



(a) Point D (or vector $\vec{AD}$) ***(C2)***

(b) Point P (or vector $\vec{AP}$) ***(C2)***

(c) Point Q (or vector $\vec{AQ}$) ***(C2)***

OR

Scalar projection

$$\vec{AC} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \qquad \vec{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\text{Projection} = \frac{\vec{AC} \cdot \vec{AB}}{\left|\vec{AB}\right|} = \frac{\begin{pmatrix} -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \end{pmatrix}}{4} \qquad \textit{(M1)}$$

$$= -2 \qquad \textit{(A1)} \qquad \textit{(C2)}$$

[6 marks]

2) **METHOD 1**

At point of intersection:

$5 + 3\lambda = -2 + 4t$ ***(M1)***

$1 - 2\lambda = 2 + t$ ***(M1)***

Attempting to solve the linear system ***(M1)***

$\lambda = -1$ (or $t = 1$) ***(A1)***

$\vec{OP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ***(A1)(A1)*** ***(C6)***

METHOD 2

(changing to Cartesian coordinates)

$2x + 3y = 13,\ x - 4y = -10$ ***(M1)(A1)(A1)***

Attempt to solve the system ***(M1)***

$\vec{OP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ***(A1)(A1)*** ***(C6)***

Note: Award ***(C5)*** for the point P(2 , 3).

3 B, or $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ ***C3***

D, or $\boldsymbol{r} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ ***C3***

Note:	Award ***C4*** for B, D and one incorrect, ***C3*** for one correct and nothing else, ***C1*** for one correct and one incorrect, ***C0*** for anything else.

4) Direction vectors are $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j}$. ***(A2)***

$\boldsymbol{a} \cdot \boldsymbol{b} = (1 + 3)$ ***(A1)***

$|\boldsymbol{a}| = \sqrt{10}, |\boldsymbol{b}| = \sqrt{2}$ ***(A1)***

$\cos\theta = \dfrac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\left(= \dfrac{4}{\sqrt{10}\sqrt{2}}\right)$ ***(M1)***

$\cos\theta = \dfrac{4}{\sqrt{20}}$ ***(A1)*** ***(C6)***

5) **METHOD 1**

Using $\boldsymbol{a} \bullet \boldsymbol{b} = ab\cos\theta$ (may be implied) ***(M1)***

$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| \left|\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right| \cos\theta$ ***(A1)***

Correct value of scalar product $\begin{pmatrix} 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 1 \end{pmatrix} = (3 \times -2) + (4 \times 1) = -2$ ***(A1)***

Correct magnitudes $\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| = \sqrt{25}\,(=5), \left|\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right| = \sqrt{5}$ ***(A1)(A1)***

$\cos\theta = \dfrac{-2}{\sqrt{125}}$ ***(A1)*** ***(C6)***

METHOD 2

$\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| = \sqrt{25}$ ***(A1)***

$\left|\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right| = \sqrt{5}$ ***(A1)***

$\left|\begin{pmatrix} 5 \\ 3 \end{pmatrix}\right| = \sqrt{34}$ ***(A1)***

Using cosine rule ***(M1)***

$34 = 25 + 5 - 25\sqrt{5}\cos\theta$ ***(A1)***

$\cos\theta = -\dfrac{2}{\sqrt{125}}$ ***(A1)*** ***(C6)***

6) (a) $\sqrt{16+9} = \sqrt{25} = 5$ ***(M1)(A1)*** ***(C2)***

(b) $\begin{pmatrix} -2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ (so B is $(6, 7)$) ***(M1)(A1)*** ***(C2)***

(c) $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ (not unique) ***(A2)*** ***(C2)***

Note: Award ***(A1)*** if "$\boldsymbol{r} =$" is omitted, i.e. not an equation.

7) (a) $\vec{PQ} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ ***A1A1*** ***N2***

(b) Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ ***A2A1A1*** ***N4***