Trig Identities and Equations Non Calc Answers

1) (a) attempt to substitute $1-2\sin^2\theta$ for $\cos 2\theta$ ***(M1)***

correct substitution ***A1***

e.g. $4-(1-2\sin^2\theta)+5\sin\theta$

$4-\cos 2\theta+5\sin\theta = 2\sin^2\theta+5\sin\theta+3$ ***AG*** ***N0***

(b) evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working ***A1***

e.g. $(2\sin\theta+3)(\sin\theta+1)$, $(2x+3)(x+1)=0$, $\sin x = \frac{-5\pm\sqrt{1}}{4}$

correct solution $\sin\theta=-1$ $\left(\text{do not penalise for including } \sin\theta = -\frac{3}{2}\right)$ ***(A1)***

$\theta = \frac{3\pi}{2}$ ***A2*** ***N3***

[7 marks]

2) (a) $\tan\theta = \frac{3}{4}$ $\left(\text{do not accept } \frac{3}{4}x\right)$ ***A1*** ***N1***

(b) (i) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ ***(A1)(A1)***

correct substitution ***A1***

e.g. $\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$ ***A1*** ***N3***

(ii) correct substitution ***A1***

e.g. $\cos 2\theta = 1-2\left(\frac{3}{5}\right)^2$, $\left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$ ***A1*** ***N1***

[7 marks]

3) evidence of substituting for $\cos 2x$ ***(M1)***

evidence of substituting into $\sin^2 x+\cos^2 x = 1$ ***(M1)***

correct equation in terms of $\cos x$ (seen anywhere) ***A1***

e.g. $2\cos^2 x-1-3\cos x-3=1$, $2\cos^2 x-3\cos x-5=0$

evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

appropriate working ***A1***

e.g. $(2\cos x-5)(\cos x+1)=0$, $(2x-5)(x+1)$, $\cos x = \frac{3\pm\sqrt{49}}{4}$

correct solutions to the equation

e.g $\cos x = \frac{5}{2}$, $\cos x=-1$, $x=\frac{5}{2}$, $x=-1$ ***(A1)***

$x=\pi$ ***A1*** ***N4***

[7 marks]

4)

(a) changing $\tan x$ into $\frac{\sin x}{\cos x}$ *A1*

$e.g.$ $\sin^3 x + \cos^3 x \frac{\sin x}{\cos x}$

simplifying *A1*

$e.g$ $\sin x(\sin^2 x + \cos^2 x)$, $\sin^3 x + \sin x - \sin^3 x$

$f(x) = \sin x$ *AG* *N0*

(b) recognizing $f(2x) = \sin 2x$, seen anywhere *(A1)*

evidence of using double angle identity $\sin(2x) = 2\sin x \cos x$, seen anywhere *(M1)*

evidence of using Pythagoras with $\sin x = \frac{2}{3}$ *M1*

$e.g.$ sketch of right triangle, $\sin^2 x + \cos^2 x = 1$

$\cos x = -\frac{\sqrt{5}}{3}$ $\left(\text{accept } \frac{\sqrt{5}}{3}\right)$ *(A1)*

$f(2x) = 2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$ *A1*

$f(2x) = -\frac{4\sqrt{5}}{9}$ *AG* *N0*

[7 marks]

5)

(a) (i) $\sin 140^\circ = p$ *A1* *N1*

(ii) $\cos 70^\circ = -q$ *A1* *N1*

(b) **METHOD 1**

evidence of using $\sin^2\theta + \cos^2\theta = 1$ *(M1)*

$e.g.$ diagram, $\sqrt{1-p^2}$ (seen anywhere)

$\cos 140^\circ = \pm\sqrt{1-p^2}$ *(A1)*

$\cos 140^\circ = -\sqrt{1-p^2}$ *A1* *N2*

METHOD 2

evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ *(M1)*

$\cos 140^\circ = 2\cos^2 70 - 1$ *(A1)*

$\cos 140^\circ = 2(-q)^2 - 1$ $(= 2q^2 - 1)$ *A1* *N2*

(c) **METHOD 1**

$\tan 140^\circ = \frac{\sin 140^\circ}{\cos 140^\circ} = -\frac{p}{\sqrt{1-p^2}}$ *A1* *N1*

METHOD 2

$\tan 140^\circ = \frac{p}{2q^2 - 1}$ *A1* *N1*

[6 marks]

6) (a) evidence of choosing the formula $\cos 2A = 2\cos^2 A - 1$ ***(M1)***

> **Note:** If they choose another correct formula, do not award the ***M1*** unless there is evidence of finding $\sin^2 A = 1 - \frac{1}{9}$.

correct substitution ***A1***

e.g. $\cos 2A = \left(\frac{1}{3}\right)^2 - \frac{8}{9}$, $\cos 2A = 2 \times \left(\frac{1}{3}\right)^2 - 1$

$\cos 2A = -\frac{7}{9}$ ***A1*** ***N2***

(b) **METHOD 1**

evidence of using $\sin^2 B + \cos^2 B = 1$ ***(M1)***

e.g. $\left(\frac{2}{3}\right)^2 + \cos^2 B = 1$, $\sqrt{\frac{5}{9}}$ (seen anywhere),

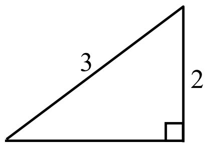
$\cos B = \pm\sqrt{\frac{5}{9}}$ $\left(= \pm\frac{\sqrt{5}}{3}\right)$ ***(A1)***

$\cos B = -\sqrt{\frac{5}{9}}$ $\left(= -\frac{\sqrt{5}}{3}\right)$ ***A1*** ***N2***

METHOD 2

diagram ***M1***

e.g.



for finding third side equals $\sqrt{5}$ ***(A1)***

$\cos B = -\frac{\sqrt{5}}{3}$ ***A1*** ***N2***

[6 marks]

7)

> **Note**: Throughout this question, do **not** accept methods which involve finding θ.

(a) Evidence of correct approach ***A1***

e.g. $\sin\theta = \frac{\text{BC}}{\text{AB}}$, $\text{BC} = \sqrt{3^2 - 2^2} = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$ ***AG*** ***N0***

(b) Evidence of using $\sin 2\theta = 2\sin\theta\cos\theta$ ***(M1)***

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$ ***A1***

$= \frac{4\sqrt{5}}{9}$ ***AG*** ***N0***

(c) Evidence of using an appropriate formula for $\cos 2\theta$ ***M1***

e.g. $\frac{4}{9} - \frac{5}{9}$, $2 \times \frac{4}{9} - 1$, $1 - 2 \times \frac{5}{9}$, $\sqrt{\left(1 - \frac{80}{81}\right)}$

$\cos 2\theta = -\frac{1}{9}$ ***A2*** ***N2***

8)

(a) Evidence of choosing the double angle formula **(M1)**

$f(x) = 15\sin(6x)$ **A1** **N2**

(b) Evidence of substituting for $f(x)$ **(M1)**

e.g. $15\sin 6x = 0$, $\sin 3x = 0$ **and** $\cos 3x = 0$

$6x = 0,\ \pi,\ 2\pi$

$x = 0,\ \frac{\pi}{6},\ \frac{\pi}{3}$ **A1A1A1** **N4**