# Trig, identities and equations Answers

1)

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| **(a)** $\sin\left(2x-\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$ | M1 | M1 for dealing with cosec |
| $2x-\frac{\pi}{3}=\frac{\pi}{4},\frac{3\pi}{4}$ | M1 | M1 for a correct order of operations |
| $x=\frac{7\pi}{24},\frac{13\pi}{24}$ | A1, A1 | |
| (0.916, 1.70) | [4] | |
| **(b) (i)** | | |
| $10\cos^2 y+5\sin y\cos y-5\sin^2 y=7$ | M1 | M1 for expansion |
| $10+5\tan y-5\tan^2 y=7\sec^2 y$ | M1 | M1 for division by $\cos^2$ |
| $10+5\tan y-5\tan^2 y=7\left(\tan^2 y+1\right)$ | M1 | M1 for use of correct identity |
| $12\tan^2 y-5\tan y-3=0$ | A1 | |
| Or | [4] | |
| $10-15\sin^2 y+5\sin y\cos y=7$ | | M1 for expansion and use of identity |
| $3\sec^2 y-15\tan^2 y+5\tan y=0$ | | M1 for division by $\cos^2$ |
| $3\left(1+\tan^2 y\right)-15\tan^2 y+5\tan y=0$ | | M1 for use of correct identity |
| Or | | |
| $15\cos^2 y+5\sin y\cos y-5=7$ | | M1 for expansion and use of identity |
| $15+15\tan y-12\sec^2 y=0$ | | M1 for division by $\cos^2$ |
| $15+5\tan y-12\left(1+\tan^2 y\right)=0$ | | M1 for use of correct identity |

2)

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| **(i)** $(\sec^2 x-1)-2\sec x+1=0$ | M1 | M1 for use of correct identity |
| $\sec x(\sec x-2)=0$ | M1 | M1 for solution of quadratic in sec or cos |
| $\cos x=0.5$, $x=60°, 300°$ | A1, A1 | A1 for one correct solution |
| | [4] | |
| Alt scheme: | | |
| $\frac{\sin^2 x}{\cos^2 x}-\frac{2}{\cos x}+1=0$ | | M1 for dealing with tan and sec correctly and for use of correct identity |
| $\sin^2 x-2\cos x+\cos^2 x=0$, | | M1 for solution to obtain $\cos x$ |
| $\cos x=0.5$, $x=60°, 300°$ | | |
| **(ii)** $\tan^2 3y=\frac{1}{5}$, $\tan 3y=(\pm)\frac{1}{\sqrt{5}}$ | M1 | M1 for correctly obtaining in terms of 1 trig ratio and square rooting |
| (or $\sin 3y=(\pm)\frac{1}{\sqrt{6}}$, $\cos 3y=(\pm)\frac{\sqrt{5}}{\sqrt{6}}$) | | |
| $3y=0.42,\ 2.72$, etc. | M1 | M1 for dealing with '3' correctly |
| $y=0.140,\ 0.907,\ 1.19,\ 1.95$ | A1, A1 | A1 for first A1 for others |
| | [4] | |
| **(iii)** $\sin\left(z+\frac{\pi}{4}\right)=\frac{2}{5}$ | M1 | M1 for dealing with '2' and cosec correctly |
| $z+\frac{\pi}{4}=0.4115,\ 2.730,\ 6.695$ | DM1 | DM1 for dealing with $\frac{\pi}{4}$ correctly |
| $z=1.94,\ 5.91$ | A1,A1 | |
| | [4] | |

3)

| | | |
|---|---|---|
| **(i)** $15 + 2\frac{\sin^2\theta}{\cos^2\theta} = \frac{7}{\cos^2\theta}$ | **M1** | **M1** for dividing by $\cos^2\theta$ |
| $15 + 2\tan^2\theta = 7\sec^2\theta$ | **M1** | **M1** for $\frac{1}{\cos^2\theta} = \sec^2\theta$ |
| $15 + 2\tan^2\theta = 7(1 + \tan^2\theta)$ | **M1** | **M1** for $\sec^2\theta = 1 + \tan^2\theta$ |
| leading to $\tan^2\theta = \frac{8}{5}$ | **A1** | **A1** for rearrangement to get required result |
| or | | |
| $15\cos^2\theta + 2\sin^2\theta = 7(\cos^2\theta + \sin^2\theta)$ | **[M1]** | **M1** for use of identity |
| $8\cos^2\theta = 5\sin^2\theta$ | **[M1]** | **M1** for simplification |
| leading to $\tan^2\theta = \frac{8}{5}$ | **[M1]** **[A1]** | **M1** for use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ |
| **(ii)** $\tan\theta = \pm\sqrt{\frac{8}{5}}$ | **M1** | **M1** for attempt to solve |
| leading to θ = 0.902, 2.24 | **A1, A1** | |
| (also, $\sin\theta = \pm\sqrt{\frac{8}{13}}$, $\cos\theta = \pm\sqrt{\frac{5}{13}}$ ) | **[M1]** **[7]** | **M1** for attempt to solve |

4)

| | | |
|---|---|---|
| $\frac{\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$ | **B1** | **B1** for $\cot A = \frac{\cos A}{\sin A}$ |
| $= \frac{\cos A + \cos^2 A + \sin^2 A}{\sin A(1+\cos A)}$ | **M1** | **M1** for obtaining as a single fraction |
| $= \frac{(1+\cos A)}{\sin A(1+\cos A)}$ | **M1** | **M1** for use of $\cos^2 \mathrm{A} + \sin^2 A = 1$ |
| $= \frac{1}{\sin A}$ = cosecA | **A1** | **A1** for correct simplification – answer given. |
| Alternate solution: | | |
| $\cot A + \frac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$ | **[M1]** | **M1** for multiplying by (1 – cos $A$) |
| $= \cot A + \frac{\sin A(1-\cos A)}{\sin^2 A)}$ | **[M1]** | **M1** for use of $\cos^2 A + \sin^2 A = 1$ anywhere |
| $= \cot A + \frac{1-\cos A}{\sin A}$ | **[M1]** | **M1** for cancelling sin $A$ |
| $= \cot A - \cot A + \frac{1}{\sin A}$ leading to cosec$A$ | **[A1]** **[4]** | **A1** for subtraction and simplification |

5)