Trig, identities and equations

1) **(a)** Solve $\operatorname{cosec}\left(2x - \frac{\pi}{3}\right) = \sqrt{2}$ for $0 < x < \pi$ radians. [4]

(b) **(i)** Given that $5(\cos y + \sin y)(2\cos y - \sin y) = 7$, show that $12\tan^2 y - 5\tan y - 3 = 0$. [4]

(ii) Hence solve $5(\cos y + \sin y)(2\cos y - \sin y) = 7$ for $0° < x < 180°$. [3]

2) **(i)** Solve $\tan^2 x - 2\sec x + 1 = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(ii) Solve $\cos^2 3y = 5\sin^2 3y$ for $0 \leqslant y \leqslant 2$ radians. [4]

(iii) Solve $2\operatorname{cosec}\left(z + \frac{\pi}{4}\right) = 5$ for $0 \leqslant z \leqslant 6$ radians. [4]

3) **(i)** Given that $15\cos^2\theta + 2\sin^2\theta = 7$, show that $\tan^2\theta = \frac{8}{5}$. [4]

(ii) Solve $15\cos^2\theta + 2\sin^2\theta = 7$ for $0 \leqslant \theta \leqslant \pi$ radians. [3]

4) Show that $\cot A + \dfrac{\sin A}{1 + \cos A} = \operatorname{cosec} A$. [4]