

Answer the following questions in the spaces provided.

1. Consider the arithmetic sequence 2, 5, 8, 11, .....

(a) Find  $u_{101}$ .

(3)

(b) Find the value of  $n$  so that  $u_n = 152$ .

(3)

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(Total 6 marks)

2. Let  $f(x) = \log_a x, x > 0$ .

(a) Write down the value of

(i)  $\log_a a$ ;

(ii)  $\log_a 1$ ;

(iii)  $\log_a (a^4)$ .

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(Total 3 marks)

3. Given that  $p = \log_a 5$ ,  $q = \log_a 2$ , express the following in terms of  $p$  and/or  $q$ .

(a)  $\log_a 10$

(b)  $\log_a 8$

(c)  $\log_a 2.5$

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(Total 6 marks)

4. (a) Given that  $(2^x)^2 + (2^x) - 12$  can be written as  $(2^x + a)(2^x + b)$ , where  $a, b \in \mathbb{R}$ , find the value of  $a$  and of  $b$ .

(b) Hence find the **exact** solution of the equation  $(2^x)^2 + (2^x) - 12 = 0$ , and explain why there is only one solution.

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(Total 6 marks)

5. Let  $\ln a = p$ ,  $\ln b = q$ . Write the following expressions in terms of  $p$  and  $q$ .

(a)  $\ln a^3 b$

(b)  $\ln \left( \frac{\sqrt{a}}{b} \right)$

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(Total 6 marks)

6. Find the **exact** value of  $x$  in each of the following equations.

(a)  $5^{x+1} = 625$

(b)  $\log_a (3x + 5) = 2$

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(Total 6 marks)

7. Expand  $(2x - 1)^4$  using the Binomial Expansion.

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(Total 6 marks)

Answer the following questions on the separate paper provided. You may use a calculator for this section.

8. (a) Write down the first three terms of the sequence  $u_n = 3n$ , for  $n \geq 1$ . (1)

(b) Find

(i)  $\sum_{n=1}^{20} 3n$ ;

(ii)  $\sum_{n=21}^{100} 3n$ .

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(5)

(Total 6 marks)

9. An investment, originally worth \$1250 grows at the rate of 12% per year, compounded annually. Find:
- (a) The value of the investment after 5 years.
- (b) The number of years that must pass before the investment is worth more than \$10 000

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(Total 6 marks)

10. Consider the expansion of the expression  $(x^3 - 3x)^6$ .

(a) Write down the number of terms in this expansion.

(b) Find the term in  $x^{12}$ .

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(Total 6 marks)

11. (a) Consider the geometric sequence  $-3, 6, -12, 24, \dots$

(i) Write down the common ratio.

(ii) Find the 15<sup>th</sup> term.

Consider the sequence  $x - 3, x + 1, 2x + 8, \dots$

(3)

(b) When  $x = 5$ , the sequence is geometric.

(i) Write down the first three terms.

(ii) Find the common ratio.

(2)

(c) Find the other value of  $x$  for which the sequence is geometric.

(4)

(d) For this value of  $x$ , find

(i) the common ratio;

(ii) the sum of the infinite sequence.

(3)

(Total 12 marks)

[illegible]