Binomial Distribution

85 min
85 marks

1.   A multiple choice test consists of ten questions. Each question has five answers.
     Only one of the answers is correct. For each question, Jose randomly chooses one of the five
     answers.

     (a)   Find the expected number of questions Jose answers correctly.
     (1)

     (b)   Find the probability that Jose answers exactly three questions correctly.
     (2)

     (c)   Find the probability that Jose answers more than three questions correctly.
     (3)
     (Total 6 marks)

2. A factory makes switches. The probability that a switch is defective is 0.04.
The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample.

(2)

(b) Find the probability that there are exactly six defective switches in the sample.

(2)

(c) Find the probability that there is at least one defective switch in the sample.

(3)
(Total 7 marks)

3. The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads.

(2)

(b) Find the probability of obtaining **at least** two heads.

(3)
(Total 5 marks)

4. The probability of obtaining heads on a biased coin is $\dfrac{1}{3}$.

(a) Sammy tosses the coin three times. Find the probability of getting

(i) three heads;

(ii) two heads and one tail.

(5)

(b)   Amir plays a game in which he tosses the coin 12 times.

   (i)   Find the expected number of heads.

   (ii)   Amir wins $ 10 for each head obtained, and loses $ 6 for each tail.
           Find his expected winnings.

(5)
(Total 10 marks)

5.   A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random
sample of 100 calculators is tested.

(a)   Write down the expected number of faulty calculators in the sample.

(1)

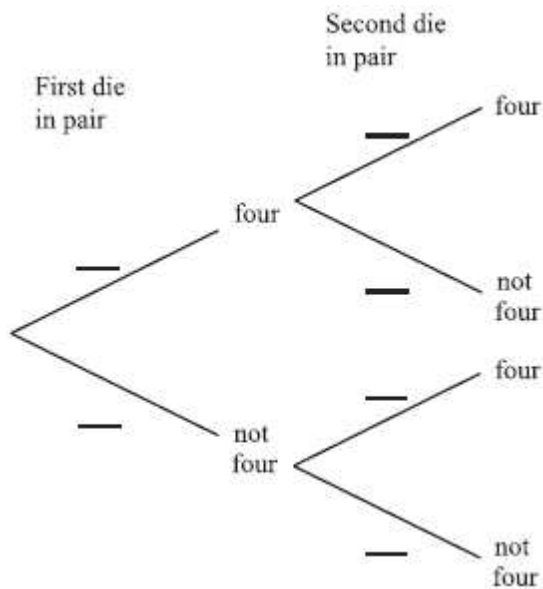(b)   Find the probability that three calculators are faulty.

(2)

(c)   Find the probability that more than one calculator is faulty.

(3)
(Total 6 marks)

6. A pair of fair dice is thrown.

(a) Copy and complete the tree diagram below, which shows the possible outcomes.

(3)

Let E be the event that **exactly** one four occurs when the pair of dice is thrown.

(b) Calculate P(E).

(3)

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs **exactly** three times in the five throws.

(3)

(d) Calculate the probability that event E occurs **at least** three times in the five throws.
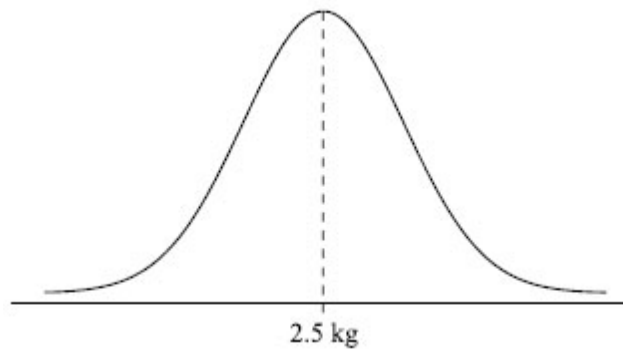
(3)
(Total 12 marks)

7. The weights of chickens for sale in a shop are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

(a) A chicken is chosen at random.

   (i) Find the probability that it weighs less than 2 kg.

   (ii) Find the probability that it weighs more than 2.8 kg.

   (iii) Copy the diagram below. Shade the areas that represent the probabilities from parts (i) and (ii).

2.5 kg

   (iv) **Hence** show that the probability that it weighs between 2 kg and 2.8 kg is 0.7936 (to four significant figures).

(7)

(b) A customer buys 10 chickens.

   (i) Find the probability that all 10 chickens weigh between 2 kg and 2.8 kg.

   (ii) Find the probability that at least 7 of the chickens weigh between 2 kg and 2.8 kg.

(6)
(Total 13 marks)

8. A fair coin is tossed five times. Calculate the probability of obtaining

(a) exactly three heads;

(b) at least one head.

(Total 6 marks)

9. A factory makes calculators. Over a long period, 2% of them are found to be faulty. A random sample of 100 calculators is tested.

    (a)   Write down the expected number of faulty calculators in the sample.

    (b)   Find the probability that three calculators are faulty.

    (c)   Find the probability that more than one calculator is faulty.

        (Total 6 marks)

10. A box contains 35 red discs and 5 black discs. A disc is selected at random and its colour noted. The disc is then replaced in the box.

    (a)   In eight such selections, what is the probability that a black disc is selected

        (i)   exactly once?

        (3)

        (ii)   at least once?

        (3)

    (b)   The process of selecting and replacing is carried out 400 times.

        What is the expected number of black discs that would be drawn?

        (2)
        (Total 8 marks)

11. A fair coin is tossed eight times. Calculate

    (a)   the probability of obtaining exactly 4 heads;

        (2)

    (b)   the probability of obtaining exactly 3 heads;

        (1)

    (c)   the probability of obtaining 3, 4 or 5 heads.

        (3)
        (Total 6 marks)