

Normal Distribution and Binomial Distribution Quiz Answers

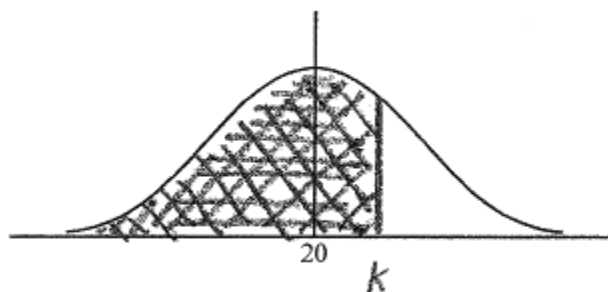
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1. (a) $\sigma = 3$ (A1)
evidence of attempt to find $P(X \leq 24.5)$ (M1)

e.g. $z = 1.5, \frac{24.5 - 20}{3}$

$P(X \leq 24.5) = 0.933$ A1 N3 3

- (b) (i)



A1A1 N2

Note: Award A1 with shading that clearly extends to right of the mean, A1 for any correct label, either k, area or their value of k

- (ii) $z = 1.03(64338)$ (A1)
attempt to set up an equation (M1)

e.g. $\frac{k - 20}{3} = 1.0364, \frac{k - 20}{3} = 0.85$

$k = 23.1$ A1 N3 5

2. (a) symmetry of normal curve (M1)
e.g. $P(X < 25) = 0.5$
 $P(X > 27) = 0.2$ A1 N2 2

(b) **METHOD 1**

finding standardized value (A1)

e.g. $\frac{27-25}{\sigma}$

evidence of complement (M1)

e.g. $1-p$, $P(X < 27)$, 0.8

finding *z*-score (A1)

e.g. $z = 0.84\dots$

attempt to set up equation involving the standardized value M1

e.g. $0.84 = \frac{27-25}{\sigma}$, $0.84 = \frac{X-\mu}{\sigma}$

$\sigma = 2.38$ A1 N3 5

METHOD 2

set up using normal CDF function and probability (M1)

e.g. $P(25 < X < 27) = 0.3$, $P(X < 27) = 0.8$

correct equation A2

e.g. $P(25 < X < 27) = 0.3$, $P(X > 27) = 0.2$

attempt to solve the equation using GDC (M1)

e.g. solver, graph, trial and error (more than two trials must be shown)

$\sigma = 2.38$ A1 N3 5

[7]

3. (a) evidence of appropriate approach (M1)

e.g. $1 - 0.85$, diagram showing values in a normal curve

$P(w \geq 82) = 0.15$ A1 N2

(b) (i) $z = -1.64$ A1 N1

(ii) evidence of appropriate approach (M1)
e.g. $-1.64 = \frac{x - \mu}{\sigma}, \frac{68 - 76.6}{\sigma}$
 correct substitution A1
e.g. $-1.64 = \frac{68 - 76.6}{\sigma}$
 $\sigma = 5.23$ A1 N1

(c) (i) $68.8 \leq \text{weight} \leq 84.4$ A1A1A1 N3
Note: Award A1 for 68.8, A1 for 84.4, A1 for giving answer as an interval.

(ii) evidence of appropriate approach (M1)
e.g. $P(-1.5 \leq z \leq 1.5), P(68.76 < y < 84.44)$
 $P(\text{qualify}) = 0.866$ A1 N2

(d) recognizing conditional probability (M1)
e.g. $P(A | B) = \frac{P(A \cap B)}{P(B)}$
 $P(\text{woman and qualify}) = 0.25 \times 0.7$ (A1)
 $P(\text{woman} | \text{qualify}) = \frac{0.25 \times 0.7}{0.866}$ A1
 $P(\text{woman} | \text{qualify}) = 0.202$ A1 N3

[15]

4. (a) correct substitution into formula for $E(X)$ (A1)
e.g. 0.05×240
 $E(X) = 12$ A1 N2 2

(b) evidence of recognizing binomial probability (may be seen in part (a)) (M1)
e.g. $\binom{240}{15} (0.05)^{15} (0.95)^{225}, X \sim B(240, 0.05)$
 $P(X = 15) = 0.0733$ A1 N2 2

(c) $P(X \leq 9) = 0.236$ (A1)
 evidence of valid approach (M1)
e.g. using complement, summing probabilities
 $P(X \geq 10) = 0.764$ A1 N3 3
[7]

5. (a) evidence of recognizing binomial probability (may be seen in (b) or (c)) (M1)
e.g. probability = $\binom{7}{4}(0.9)^4(0.1)^3$, $X \sim B(7, 0.9)$, complementary probabilities
 probability = 0.0230 A1 N2

(b) correct expression A1A1 N2
e.g. $\binom{7}{4}p^4(1-p)^3$, $35p^4(1-p)^3$

Note: Award A1 for binomial coefficient $\left(\text{accept} \binom{7}{3}\right)$,
 A1 for $p^4(1-p)^3$.

(c) evidence of attempting to solve **their** equation (M1)
e.g. $\binom{7}{4}p^4(1-p)^3 = 0.15$, sketch
 $p = 0.356, 0.770$ A1A1 N3
[7]

6. (a) 36 outcomes (seen anywhere, even in denominator) (A1)
 valid approach of listing ways to get sum of 5, showing at least two pairs (M1)
e.g. (1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2), lattice diagram
 $P(\text{prize}) = \frac{4}{36} \left(= \frac{1}{9} \right)$ A1 N3

(b) recognizing binomial probability (M1)
e.g. $B\left(8, \frac{1}{9}\right)$, binomial pdf, $\binom{8}{3}\left(\frac{1}{9}\right)^3\left(\frac{8}{9}\right)^5$
 $P(3 \text{ prizes}) = 0.0426$ A1 N2 [5]

7. (a) (i) valid approach (M1)
e.g. $np, 5 \times \frac{1}{5}$
 $E(X) = 1$ A1 N2

(ii) evidence of appropriate approach involving binomial (M1)
e.g. $X \sim B\left(5, \frac{1}{5}\right)$
 recognizing that Mark needs to answer 3 **or more** questions correctly (A1)
e.g. $P(X \geq 3)$
 valid approach M1
e.g. $1 - P(X \leq 2), P(X = 3) + P(X = 4) + P(X = 5)$
 $P(\text{pass}) = 0.0579$ A1 N3

(b) (i) evidence of summing probabilities to 1 (M1)
e.g. $0.67 + 0.05 + (a + 2b) + \dots + 0.04 = 1$
 some simplification that clearly leads to required answer
e.g. $0.76 + 4a + 2b = 1$ A1
 $4a + 2b = 0.24$ AG N0

(ii) correct substitution into the formula for expected value (A1)
e.g. $0(0.67) + 1(0.05) + \dots + 5(0.04)$
 some simplification (A1)
e.g. $0.05 + 2a + 4b + \dots + 5(0.04) = 1$
 correct equation A1
e.g. $13a + 5b = 0.75$
 evidence of solving (M1)
 $a = 0.05, b = 0.02$ A1A1 N4

(c) attempt to find probability Bill passes
e.g. $P(Y \geq 3)$

(M1)

correct value 0.19

A1

Bill (is more likely to pass)

A1 N0

[17]