

SL - Binomial Questions Answers

0 min
0 marks

1. (a) evidence of attempt to find $P(X \leq 475)$ (M1)
e.g. $P(Z \leq 1.25)$
 $P(X \leq 475) = 0.894$ A1 N2
- (b) evidence of using the complement (M1)
e.g. $0.73, 1 - p$
 $z = 0.6128$ (A1)
 setting up equation (M1)
e.g. $\frac{a - 450}{20} = 0.6128$
 $a = 462$ A1 N3
- [6]**
2. $A \sim N(46, 10^2)$ $B \sim N(\mu, 12^2)$
- (a) $P(A > 60) = 0.0808$ A2 N2
- (b) correct approach (A1)
e.g. $P\left(Z < \frac{60 - \mu}{12}\right) = 0.85$, sketch
 $\frac{60 - \mu}{12} = 1.036\dots$ (A1)
 $\mu = 47.6$ A1 N2
- (c) (i) route A A1 N1

(ii) **METHOD 1**

$$P(A < 60) = 1 - 0.0808 = 0.9192$$

valid reason

e.g. probability of A getting there on time is greater than probability of B

$$0.9192 > 0.85$$

A1
R1

N2

METHOD 2

$$P(B > 60) = 1 - 0.85 = 0.15$$

valid reason

e.g. probability of A getting there late is less than probability of B

$$0.0808 < 0.15$$

A1
R1

N2

- (d) (i) let X be the number of days when the van arrives before 07:00

$$P(X = 5) = (0.85)^5 \\ = 0.444$$

(A1)
A1 N2

(ii) **METHOD 1**

evidence of adding correct probabilities

$$\text{e.g. } P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

correct values $0.1382 + 0.3915 + 0.4437$

$$P(X \geq 3) = 0.973$$

(M1)

(A1)

A1 N3

METHOD 2

evidence of using the complement

$$\text{e.g. } P(X \geq 3) = 1 - P(X \leq 2), 1 - p$$

correct values $1 - 0.02661$

$$P(X \geq 3) = 0.973$$

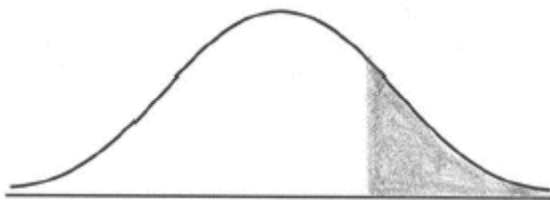
(M1)

(A1)

A1 N3

[13]

3. (a)



A1A1 N2

Note: Award A1 for vertical line to right of mean, A1 for shading to right of **their** vertical line.

(b) evidence of recognizing symmetry (M1)
e.g. 105 is one standard deviation above the mean so d is one standard deviation below the mean, shading the corresponding part,
 $105 - 100 = 100 - d$
 $d = 95$ A1 N2

(c) evidence of using complement (M1)
e.g. $1 - 0.32$, $1 - p$
 $P(d < X < 105) = 0.68$ A1 N2

[6]

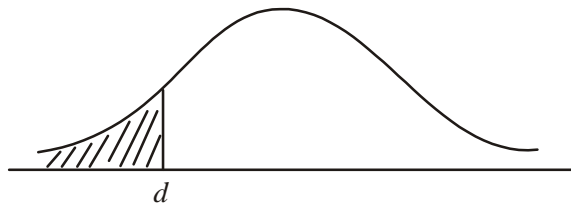
4. $X \sim N(7, 0.5^2)$

(a) (i) $z = 2$ (M1)
 $P(X < 8) = P(Z < 2) = 0.977$ A1 N2

(ii) evidence of appropriate approach (M1)
e.g. symmetry, $z = -2$
 $P(6 < X < 8) = 0.954$ (tables 0.955) A1 N2

Note: Award M1A1(AP) if candidates refer to 2 standard deviations from the mean, leading to 0.95.

(b) (i)



A1A1 N2

Note: Award A1 for d to the left of the mean, A1 for area to the left of d shaded.

(ii) $z = -1.645$ (A1)

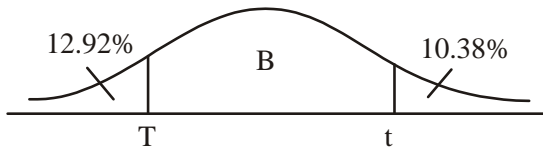
$$\frac{d-7}{0.5} = -1.645 \quad (M1)$$

$d = 6.18$ A1 N3

- (c) $Y \sim N(\mu, 0.5^2)$
 $P(Y < 5) = 0.2$ (M1)
 $z = -0.84162\dots$ A1
 $\frac{5-\mu}{0.5} = -0.8416$ (M1)
 $\mu = 5.42$ A1 N3

[13]

5. (a)



A1A1 N2

Notes: Award A1 for three re.g.ions, (may be shown by lines or shading) A1 for clear labelling of two re.g.ions (may be shown by percentages or cate.g.ories).

r and t need not be labelled, but if they are, they may be interchanged.

(b) **METHOD 1**

- $P(X < r) = 0.1292$ (A1)
 $r = 6.56$ A1 N2
 $1 - 0.1038 (= 0.8962)$ (may be seen later) A1
 $P(X < t) = 0.8962$ (A1)
 $t = 7.16$ A1 N2

METHOD 2

- finding z -values $-1.130\dots, 1.260\dots$ A1A1
evidence of setting up one standardized equation (M1)
e.g. $\frac{r-6.84}{0.25} = -1.13K$, $t = 1.260 \times 0.25 + 6.84$
 $r = 6.56, t = 7.16$ A1A1 N2N2

[7]

6. (a) evidence of approach (M1)
e.g. finding 0.84..., using $\frac{23.7-21}{\sigma}$
- correct working (A1)
e.g. $0.84... = \frac{23.7-21}{\sigma}$, graph
- $\sigma = 3.21$ A1 N2
- (b) (i) evidence of attempting to find $P(X < 25.4)$ (M1)
e.g. using $z = 1.37$
 $P(X < 25.4) = 0.915$ A1 N2
- (ii) evidence of recognizing symmetry (M1)
e.g. $b = 21 - 4.4$, using $z = -1.37$
 $b = 16.6$ A1 N2
- [7]**

7. METHOD 1

- (a) $\sigma = 10$ (A1)
 $1.12 \times 10 = 11.2$ A1
 $11.2 + 100$ (M1)
 $x = 111.2$ A1 N2
- (b) $100 - 11.2$ (M1)
 $= 88.8$ A1 N2
- [6]**

METHOD 2

- (a) $\sigma = 10$ (A1)
Evidence of using standardisation formula (M1)
 $\frac{x-100}{10} = 1.12$ A1
 $x = 111.2$ A1 N2
- (b) $\frac{100-x}{10} = 1.12$ A1
 $x = 88.8$ A1 N2
- [6]**

8. (a) Evidence of using the complement *e.g.* $1 - 0.06$ (M1)
 $p = 0.94$ A1 N2
- (b) For evidence of using symmetry (M1)
 Distance from the mean is 7 (A1)
e.g. diagram, $D = \text{mean} - 7$
 $D = 10$ A1 N2
- (c) $P(17 < H < 24) = 0.5 - 0.06$ (M1)
 $= 0.44$ A1
 $E(\text{trees}) = 200 \times 0.44$ (M1)
 $= 88$ A1 N2
- [9]**

9. $X \sim N(\mu, \sigma^2)$
 $P(X > 90) = 0.15$ **and** $P(X < 40) = 0.12$ (M1)
 Finding standardized values 1.036, -1.175 A1A1
 Setting up the equations $1.036 = \frac{90 - \mu}{\sigma}$, $-1.175 = \frac{40 - \mu}{\sigma}$ (M1)
 $\mu = 66.6$, $\sigma = 22.6$ A1A1N2N2
- [6]**

10. (a) $P(H < 153) = 0.705 \Rightarrow z = 0.538(836\dots)$ (A1)
 Standardizing $\frac{153 - \mu}{5}$ (A1)
 Setting up **their** equation $0.5388\dots = \frac{153 - \mu}{5}$ M1
 $\mu = 150.30K$
 $= 150$ (to 3sf) A1 N3
- (b) $Z = \frac{153 - \mu}{5} = 1.138\dots$ (accept 1.14 from $\mu = 150.3$, or 1.2
 from $\mu = 150$) (A1)
 $P(Z > 1.138) = 0.128$ (accept 0.127 from $z = 1.14$, or 0.115
 from $z = 1.2$) A1 N2
- [6]**

11. (a) 0.0668

A2 N2

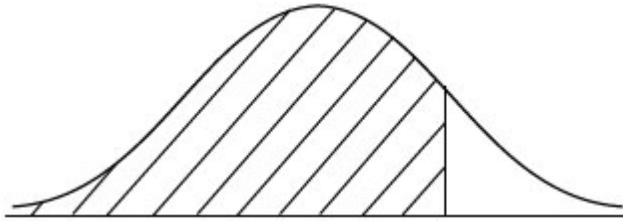
(b) Using the standardized value 1.645

(A1)

$k = 26.1$ kg

A1 N2

(c)



A1A1 N2

Note: Award A1 for vertical line to right of the mean, A1 for shading to left of **their** vertical line.

[6]

12.

Note: Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.

$$W \sim N(2.5, 0.3^2)$$

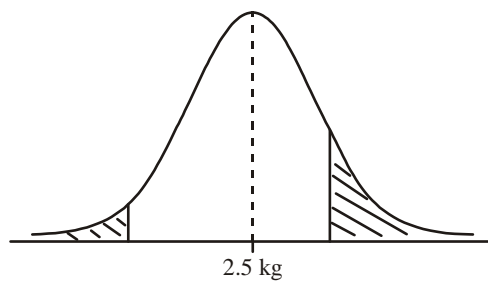
(a) (i) $z = -1.67$ (accept 1.67) (A1)

$P(W < 2) = 0.0478$ (accept answers between 0.0475 and 0.0485) A1 N2

(ii) $z = 1$ (A1)

$P(W > 2.8) = 0.159$ A1 N2

(iii)



A1A1 N2

Note: Award A1 for a vertical line to left of mean and shading to left, A1 for vertical line to right of mean and shading to right.

(iv) Evidence of appropriate calculation M1

eg $1 - (0.047790 + 0.15866)$, $0.8413 - 0.0478$

$P = 0.7936$ AG N0

Note: The final value may vary depending on what level of accuracy is used.

Accept their value in subsequent parts.

(b)	(i)	$X \sim B(10, 0.7935\dots)$		
		Evidence of calculation		M1
		<i>eg</i> $P(X = 10) = (0.7935\dots)^{10}$		
		$P(X = 10) = 0.0990$ (3 sf)		A1 N1
	(ii)	METHOD 1		
		Recognizing $X \sim B(10, 0.7935\dots)$ (may be seen in (i))		(M1)
		$P(X \leq 6) = 0.1325\dots$ (or $P(X = 1) + \dots + P(X = 6)$)		(A1)
		evidence of using the complement		(M1)
		<i>eg</i> $P(X \geq 7) = 1 - P(X \leq 6)$, $P(X \geq 7) = 1 - P(X < 7)$		
		$P(X \geq 7) = 0.867$		A1 N3
		METHOD 2		
		Recognizing $X \sim B(10, 0.7935\dots)$ (may be seen in (i))		(M1)
		For adding terms from $P(X = 7)$ to $P(X = 10)$		(M1)
		$P(X \geq 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030$		(A1)
		$= 0.867$		A1 N3

[13]

13.	(a)	$z = \frac{180 - 160}{20} = 1$		(A1)
		$\phi(1) = 0.8413$		(A1)
		$P(\text{height} > 180) = 1 - 0.8413$		
		$= 0.159$		A1 N3
	(b)	$z = -1.1800$		(A1)
		Setting up equation $-1.18 = \frac{d - 160}{20}$		(M1)
		$d = 136$		A1 N3

[6]

14.

Notes: Accept any suitable notation, as long as the candidate's intentions are clear.

The following symbols will be used in the markscheme.

Girls' height $G \sim N(155, 10^2)$, boys' height $B \sim N(160, 12^2)$

Height H , Female F , Male M .

(a) $P(G > 170) = 1 - P(G < 170)$ (A1)

$$P(G > 170) = P\left(Z < \frac{170-155}{10}\right) \quad (\text{A1})$$

$$P(G > 170) = 1 - \Phi(1.5) = 1 - 0.9332$$

$$= 0.0668 \quad \text{A1} \quad \text{N3}$$

(b) $z = -1.2816$ (A1)

Correct calculation (eg $x = 155 + -1.282 \times 10$) (A1)

$$x = 142 \quad \text{A1} \quad \text{N3}$$

(c) Calculating one variable (A1)

eg $P(B < r) = 0.95, z = 1.6449$

$$r = 160 + 1.645(12) = 179.74$$

$$= 180 \quad \text{A1} \quad \text{N2}$$

Any valid calculation for the second variable, including use of symmetry (A1)

eg $P(B < q) = 0.05, z = -1.6449$

$$q = 160 - 1.645(12) = 140.26$$

$$= 140 \quad \text{A1} \quad \text{N2}$$

Note: Symbols are not required in parts (d) and (e).

(d) $P(M \cap (B > 170)) = 0.4 \times 0.2020, P(F \cap (G > 170)) =$
 0.6×0.0668 (A1)(A1)

$$P(H > 170) = 0.0808 + 0.04008 \quad \text{A1}$$

$$= 0.12088 = 0.121 \text{ (3 sf)} \quad \text{A1} \quad \text{N2}$$

(e) $P(F | H > 170) = \frac{P(F \cap (H > 170))}{P(H > 170)}$ (M1)

$$= \frac{0.60 \times 0.0668}{0.121} \left(= \frac{0.0401}{0.121} \text{ or } \frac{0.04008}{0.1208} \right)$$

A1

$$= 0.332$$

A1 N1

[17]

15. $X \sim N(\mu, \sigma^2)$, $P(X < 3) = 0.2$, $P(X > 8) = 0.1$

$P(X < 8) = 0.9$ (M1)

Attempt to set up equations (M1)

$$\frac{3 - \mu}{\sigma} = -0.8416, \quad \frac{8 - \mu}{\sigma} = 1.282$$

A1A1

$$3 - \mu = -0.8416\sigma$$

$$8 - \mu = 1.282\sigma$$

$$5 = 2.1236\sigma$$

$$\sigma = 2.35, \quad \mu = 4.99$$

A1A1 N4

[6]

16. $X \sim N(\mu, \sigma^2)$, $P(X > 90) = 0.15$ and $P(X < 40) = 0.12$ (M1)

Finding standardized values 1.036, -1.175 (A1A1)

Setting up the equations $1.036 = \frac{90 - \mu}{\sigma}$, $-1.175 = \frac{40 - \mu}{\sigma}$ (M1)

$$\mu = 66.6, \quad \sigma = 22.6$$

A1A1

[6]

17. (i) $P(X > 3200) = P(Z > 0.4)$ (M1)

$$= 1 - 0.6554 = 34.5\% (= 0.345)$$

(A1) (N2)

(ii) $P(2300 < X < 3300) = P(-1.4 < Z < 0.6)$ (M1)

$$= 0.4192 + 0.2257$$

$$= 0.645$$

(A1)

$$P(\text{both}) = (0.645)^2 = 0.416$$

(A1) (N2)

(iii) $0.7422 = P(Z < 0.65)$ (A1)

$$\frac{d - 3000}{500} = 0.65 \quad (\text{A1})$$

$d = \$3325$ (= \$3330 to 3 s.f.) (Accept \$3325.07) (A1) (N3)

[8]

18. (a) $z = \frac{185 - 170}{20} = 0.75$ (M1)(A1)

$P(Z < 0.75) = 0.773$ (A1) (N3)

(b) $z = -0.47$ (may be implied) (A1)

$$-0.47 = \frac{d - 170}{20} \quad (\text{M1})$$

$d = 161$ (A1) (N3)

[6]

19. (a) (i) $a = -1$ (A1)

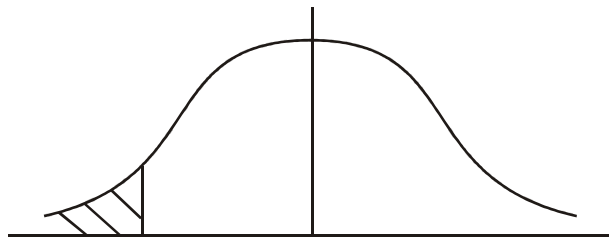
$b = 0.5$ (A1)

(ii) (a) 0.841 (A2)

(b) $0.6915 - 0.1587$ (or $0.8413 - 0.3085$) (M1)

$= 0.533$ (3 sf) (A1)(N2) 6

(b) (i) Sketch of normal curve (A1)(A1)



(ii) $c = 0.647$ (A2) 4

[10]

20. $X \sim N(80, 8^2)$

(a) $P(X < 72) = P(Z < -1)$ (M1)
 $= 1 - 0.8413$
 $= 0.159$ (A1)

OR

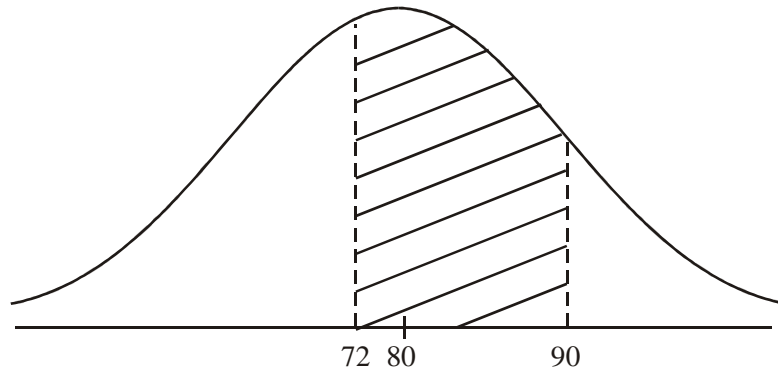
$P(X < 72) = 0.159$ (G2) 2

(b) (i) $P(72 < X < 90) = P(-1 < Z < 1.25)$ (M1)
 $= 0.3413 + 0.3944$ (A1)
 $= 0.736$ (A1)

OR

$P(72 < X < 90) = 0.736$ (G3)

(ii)



(A1)(A1) 5

Note: Award (A1) for a normal curve and (A1) for the shaded area, which should not be symmetrical.

(c) 4% fail in less than x months
 $\Rightarrow x = 80 - 8 \times \Phi^{-1}(0.96)$ (M1)
 $= 80 - 8 \times 1.751$ (A1)
 $= 66.0$ months (A1)

OR

$x = 66.0$ months (G3) 3

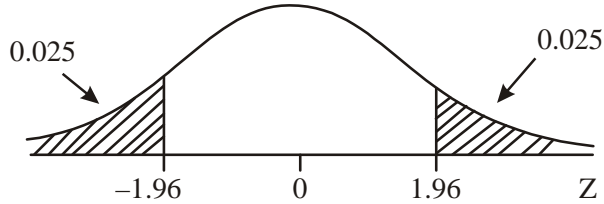
[10]

21. (a) $P(M \geq 350) = 1 - P(M < 350) = 1 - P\left(Z < \frac{350 - 310}{30}\right)$ (M1)
 $= 1 - P(Z < 1.333) = 1 - 0.9088$
 $= 0.0912$ (accept 0.0910 to 0.0920) (A1)

OR

$P(M \geq 350) = 0.0912$ (G2)

(b)



$$P(Z < 1.96) = 1 - 0.025 = 0.975 \quad (\text{A1})$$

$$1.96(30) = 58.8 \quad (\text{M1})$$

$$310 - 58.8 < M < 310 + 58.8 \Rightarrow a = 251, b = 369 \quad (\text{A1})$$

OR

$$251 < M < 369 \quad (\text{G3})$$

Note: Award (G1) if only one of the end points is correct.

[5]

22. (a) (These answers may be obtained from a calculator or by finding z in each case and the corresponding area.)

$$M \sim N(750, 625)$$

$$(i) \quad P(M < 740 \text{ g}) = 0.345 \quad (\text{G2})$$

OR

$$z = -0.4 \quad P(z < -0.4) = 0.345 \quad (\text{A1})(\text{A1})$$

$$(ii) \quad P(M > 780 \text{ g}) = 0.115 \quad (\text{G2})$$

OR

$$z = 1.2 \quad P(z > 1.2) = 1 - 0.885 = 0.115 \quad (\text{A1})(\text{A1})$$

$$(iii) \quad P(740 < M < 780) = 0.540 \quad (\text{G1})$$

OR

$$1 - (0.345 + 0.115) = 0.540 \quad (\text{A1}) \quad 5$$

(b) Independent events

$$\text{Therefore, } P(\text{both} < 740) = 0.345^2 \quad (\text{M1})$$

$$= 0.119 \quad (\text{A1}) \quad 2$$

(c) 70% have mass < 763 g (G1)

Therefore, 70% have mass of at least $750 - 13$

$$x = 737 \text{ g} \quad (\text{A1}) \quad 2$$

[9]

23.

Note: Where accuracy is not specified, accept answers with greater than 3 sf accuracy, provided they are correct as far as 3 sf

(a) $z = \frac{197 - 187.5}{9.5} = 1.00$ (M1)

$P(Z > 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$
 $= 0.159$ (3 sf) (A1)
 $= 15.9\%$ (A1)

OR

$P(H > 197) = 0.159$ (G2)
 $= 15.9\%$ (A1) 3

(b) Finding the 99th percentile

$\Phi(a) = 0.99 \Rightarrow a = 2.327$ (accept 2.33) (A1)
 $\Rightarrow 99\%$ of heights under $187.5 + 2.327(9.5) = 209.6065$ (M1)
 $= 210$ (3 sf) (A1)

OR

99% of heights under $209.6 = 210$ cm (3 sf) (G3)
Height of standard doorway = $210 + 17 = 227$ cm (A1) 4

[7]

24. (a) Let X be the random variable for the IQ.

$X \sim N(100, 225)$

$P(90 < X < 125) = P(-0.67 < Z < 1.67)$ (M1)
 $= 0.701$

70.1 percent of the population (accept 70 percent). (A1)

OR

$P(90 < X < 125) = 70.1\%$ (G2) 2

(b) $P(X \geq 125) = 0.0475$ (or 0.0478) (M1)

$P(\text{both persons having IQ} \geq 125) = (0.0475)^2$ (or $(0.0478)^2$) (M1)
 $= 0.00226$ (or 0.00228) (A1) 3

(c) Null hypothesis (H_0): mean IQ of people with disorder is 100 (M1)

Alternative hypothesis (H_1): mean IQ of people with disorder is less than 100 (M1)

$$P(\bar{X} < 95.2) = P\left(Z < \left(\frac{95.2 - 100}{\frac{15}{\sqrt{25}}}\right)\right) = P(Z < -1.6) = 1 - 0.9452$$
$$= 0.0548 \quad (\text{A1})$$

The probability that the sample mean is 95.2 and the null hypothesis true is $0.0548 > 0.05$. Hence the evidence is not sufficient. (R1) 4

[9]

25. (a) $Z = \frac{25 - 25.7}{0.50} = -1.4$ (M1)

$$P(Z < -1.4) = 1 - P(Z < 1.4)$$
$$= 1 - 0.9192$$
$$= 0.0808 \quad (\text{A1})$$

OR

$$P(W < 25) = 0.0808 \quad (\text{G2}) \quad 2$$

(b) $P(Z < -a) = 0.025 \Rightarrow P(Z < a) = 0.975$
 $\Rightarrow a = 1.960$ (A1)

$$\frac{25 - \mu}{0.50} = -1.96 \Rightarrow \mu = 25 + 1.96(0.50)$$
$$= 25 + 0.98 = 25.98 \quad (\text{A1})$$
$$= 26.0 \text{ (3 sf)} \quad (\text{AG})$$

OR

$$\frac{25.0 - 26.0}{0.50} = -2.00 \quad (\text{M1})$$

$$P(Z < -2.00) = 1 - P(Z < 2.00)$$
$$= 1 - 0.9772 = 0.0228 \quad (\text{A1})$$
$$\approx 0.025 \quad (\text{A1})$$

OR

$$\mu = 25.98 \quad (\text{G2})$$

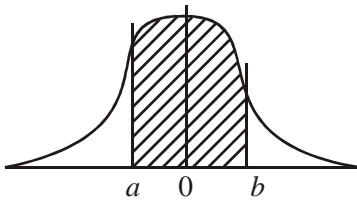
$$\Rightarrow \text{mean} = 26.0 \text{ (3 sf)} \quad (\text{A1})(\text{AG}) \quad 3$$

(c) Clearly, by symmetry $\mu = 25.5$ (A1)
 $Z = \frac{25.0 - 25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma$ (M1)
 $\Rightarrow \sigma = 0.255 \text{ kg}$ (A1) 3

(d) On average, $\frac{\text{cement saving}}{\text{bag}} = 0.5 \text{ kg}$ (A1)
 $\frac{\text{cost saving}}{\text{bag}} = 0.5(0.80) = \0.40 (M1)
 To save \$5000 takes $\frac{5000}{0.40} = 12500 \text{ bags}$ (A1) 3

[11]

26. (a) Let X be the lifespan in hours
 $X \sim N(57, 4.4^2)$



(i) $a = -0.455$ (3 sf) (A1)
 $b = 0.682$ (3 sf) (A1)

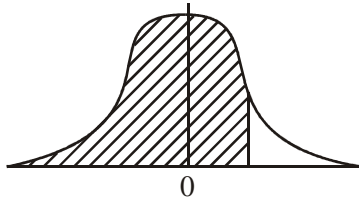
(ii) (a) $P(X > 55) = P(Z > -0.455)$
 $= 0.675$ (A1)

(b) $P(55 \leq X \leq 60) = P\left(\frac{2}{4.4} \leq Z \leq \frac{3}{4.4}\right)$
 $\approx P(0.455 \leq Z \leq 0.682)$
 $\approx 0.6754 + 0.752 - 1$ (A1)
 $= 0.428$ (3sf) (A1)

OR

$P(55 \leq X \leq 60) = 0.428$ (3 sf) (G2) 5

(b) 90% have died \Rightarrow shaded area = 0.9 (M1)



Hence $t = 57 + (4.4 \times 1.282)$ (M1)
 $= 57 + 5.64$ (A1)
 $= 62.6$ hours (A1)

OR $t = 62.6$ hours (G3) 5

[10]

27. (a) *Note: Candidates using tables may get slightly different answers, especially if they do not interpolate. Accept these answers.*

$$P(\text{speed} > 50) = 0.3 = 1 - \Phi\left(\frac{50 - \mu}{10}\right) \quad (\text{A1})$$

Hence, $\frac{50 - \mu}{10} = \Phi^{-1}(0.7)$ (M1)

$$\mu = 50 - 10\Phi^{-1}(0.7) \quad (\text{M1})$$

$$= 44.75599 \dots \dots = 44.8 \text{ km/h (3 sf) (accept 44.7)} \quad (\text{AG}) \quad 3$$

(b) H_1 : “the mean speed has been reduced by the campaign”. (A1) 1

(c) One-tailed; because H_1 involves only “<”. (A2) 2

(d) For a one-tailed test at 5% level, critical region is $Z < \mu_m - 1.64\sigma_m$ (accept $-1.65\sigma_m$) (M1)

Now, $\mu_m = \mu = 44.75\dots$; $\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$ (allow ft) (A1)

So test statistic is $44.75\dots - 1.64 \times 2 = 41.47$ (A1)

Now $41.3 < 41.47$ so reject H_0 , yes. (A1) 4

[10]

28. (a) Area $A = 0.1$ (A1) 1

- (b) **EITHER** Since $p(X \geq 12) = p(X \leq 8)$, (M1)
then 8 and 12 are symmetrically disposed around the (M1)(R1)
mean.

$$\text{Thus mean} = \frac{8+12}{2} \quad (\text{M1})$$

$$= 10 \quad (\text{A1})$$

Notes: If a candidate says simply "by symmetry $\mu = 10$ " with no further explanation award [3 marks] (M1, A1, R1). As a full explanation is requested award an additional (A1) for saying since $p(X < 8) = p(X > 12)$ and another (A1) for saying that the normal curve is symmetric.

OR $p(X \geq 12) = 0.1 \Rightarrow p\left(Z \geq \frac{12-\mu}{\sigma}\right) = 0.1 \quad (\text{M1})$

$$\Rightarrow p\left(Z \leq \frac{12-\mu}{\sigma}\right) = 0.9$$

$$p(X \leq 8) = 0.1 \Rightarrow p\left(Z \leq \frac{8-\mu}{\sigma}\right) = 0.1$$

$$\Rightarrow p\left(Z \leq \frac{\mu-8}{\sigma}\right) = 0.9 \quad (\text{A1})$$

$$\text{So } \frac{12-\mu}{\sigma} = \frac{\mu-8}{\sigma} \quad (\text{M1})$$

$$\Rightarrow 12 - \mu = \mu - 8 \quad (\text{M1})$$

$$\Rightarrow \mu = 10 \quad (\text{A1})$$

5

(c) $\Phi\left(\frac{12-10}{\sigma}\right) = 0.9 \quad (\text{A1})(\text{M1})(\text{A1})$

Note: Award (A1) for $\left(\frac{12-10}{\sigma}\right)$, (M1) for standardizing, and

(A1) for 0.9.

$$\Rightarrow \frac{2}{\sigma} = 1.282 \text{ (or } 1.28) \quad (\text{A1})$$

$$\sigma = \frac{2}{1.282} \left(\text{or } \frac{2}{1.28}\right) \quad (\text{A1})$$

$$= 1.56 \text{ (3 sf)} \quad (\text{AG})$$

5

Note: Working backwards from $\sigma = 1.56$ to show it leads the given data should receive a maximum of [3 marks] if done correctly.

(d) $p(X \leq 11) = p\left(Z \leq \frac{11-10}{1.561}\right)$ (or 1.56) (M1)(A1)

Note: Award (M1) for standardizing and (A1) for $\left(\frac{11-10}{1.561}\right)$.

$= p(Z \leq 0.6407)$ (or 0.641 or 0.64) (A1)

$= \Phi(0.6407)$ (M1)

$= 0.739$ (3 sf) (A1)

5

[16]