

SL - Integration Volume of Revolution Answers

0 min
0 marks

1. attempt to set up integral expression M1
e.g. $\pi \int \sqrt{16-4x^2}^2 dx, 2\pi \int_0^2 (16-4x^2), \int \sqrt{16-4x^2}^2 dx$
 $\int 16dx = 16x, \int 4x^2 dx = \frac{4x^3}{3}$ (seen anywhere) A1A1
evidence of substituting limits into the integrand (M1)
e.g. $\left(32 - \frac{32}{3}\right) - \left(-32 + \frac{32}{3}\right), 64 - \frac{64}{3}$
volume = $\frac{128\pi}{3}$ A2 N3
- [6]
2. (a) evidence of valid approach (M1)
e.g. $f(x) = 0$, graph
 $a = -1.73, b = 1.73$ ($a = -\sqrt{3}, b = \sqrt{3}$) A1A1 N3
- (b) attempt to find max (M1)
e.g. setting $f'(x) = 0$, graph
 $c = 1.15$ (accept (1.15, 1.13)) A1 N2
- (c) attempt to substitute either limits or the function into formula M1
e.g. $V = \pi \int_0^c [f(x)]^2 dx, \pi \int [x \ln(4-x^2)]^2, \pi \int_0^{1.149\dots} y^2 dx$
 $V = 2.16$ A2 N2

- (d) valid approach recognizing 2 regions (M1)
e.g. finding 2 areas
- correct working (A1)
e.g. $\int_0^{-1.73\dots} f(x)dx + \int_0^{1.149\dots} f(x)dx; -\int_{-1.73\dots}^0 f(x)dx + \int_0^{1.149\dots} f(x)dx$
- area = 2.07 (accept 2.06) A2 N3
[12]

3. attempt to substitute into formula $V = \int \pi y^2 dx$ (M1)
- integral expression A1
e.g. $\pi \int_0^a (\sqrt{x})^2 dx, \pi \int x$
- correct integration (A1)
e.g. $\int x dx = \frac{1}{2} x^2$
- correct substitution $V = \pi \left[\frac{1}{2} a^2 \right]$ (A1)
equating **their** expression to 32π M1
e.g. $\pi \left[\frac{1}{2} a^2 \right] = 32\pi$
 $a^2 = 64$
 $a = 8$ A2 N2
[7]

4. (a) finding the limits $x = 0, x = 5$ (A1)
integral expression A1
e.g. $\int_0^5 f(x)dx$
- area = 52.1 A1 N2
- (b) evidence of using formula $v = \int \pi y^2 dx$ (M1)
correct expression A1
e.g. volume = $\pi \int_0^5 x^2 (x-5)^4 dx$
volume = 2340 A2 N2

(c) area is $\int_0^a x(a-x)dx$ A1

$$= \left[\frac{ax^2}{2} - \frac{x^3}{3} \right]_0^a \quad \text{A1A1}$$

substituting limits (M1)

$$\text{e.g. } \frac{a^3}{2} - \frac{a^3}{3}$$

setting expression equal to area of R (M1)
correct equation A1

$$\text{e.g. } \frac{a^2}{2} - \frac{a^3}{3} = 52.1, a^3 = 6 \times 52.1,$$

$$a = 6.79$$

A1 N3

[14]

5. (a) finding derivative (A1)

$$\text{e.g. } f'(x) = \frac{1}{2}x^{-\frac{1}{2}}, \frac{1}{2\sqrt{x}}$$

correct value of derivative or its negative reciprocal (seen anywhere) A1

$$\text{e.g. } \frac{1}{2\sqrt{4}}, \frac{1}{4}$$

gradient of normal = $-\frac{1}{\text{gradient of tangent}}$ (seen anywhere) A1

$$\text{e.g. } -\frac{1}{f'(4)} = -4, -2\sqrt{x}$$

substituting into equation of line (for normal) M1

$$\text{e.g. } y - 2 = -4(x - 4)$$

$$y = -4x + 18 \quad \text{AG NO}$$

(b) recognition that $y = 0$ at A (M1)

$$\text{e.g. } -4x + 18 = 0$$

$$x = \frac{18}{4} \left(= \frac{9}{2} \right) \quad \text{A1 N2}$$

(c) splitting into two appropriate parts (areas and/or integrals) (M1)

correct expression for area of R

$$\text{e.g. area of } R = \int_0^4 \sqrt{x}dx + \int_4^{4.5} (-4x + 18)dx, \int_0^4 \sqrt{x}dx + \frac{1}{2} \times 0.5 \times 2 \text{ (triangle)}$$

A2 N3

Note: Award A1 if dx is missing.

- (d) correct expression for the volume from $x = 0$ to $x = 4$ (A1)

$$e.g. V = \int_0^4 \pi [f(x)^2] dx, \int_0^4 \pi \sqrt{x^2} dx, \int_0^4 \pi x dx$$

$$V = \left[\frac{1}{2} \pi x^2 \right]_0^4 \quad \text{A1}$$

$$V = \pi \left(\frac{1}{2} \times 16 - \frac{1}{2} \times 0 \right) \quad \text{(A1)}$$

$$V = 8\pi \quad \text{A1}$$

finding the volume from $x = 4$ to $x = 4.5$

EITHER

recognizing a cone (M1)

$$e.g. V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (2)^2 \times \frac{1}{2} \quad \text{(A1)}$$

$$= \frac{2\pi}{3} \quad \text{A1}$$

$$\text{total volume is } 8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi \right) \quad \text{A1 N4}$$

OR

$$V = \pi \int_4^{4.5} (-4x+18)^2 dx \quad \text{(M1)}$$

$$= \int_4^{4.5} \pi (16x^2 - 144x + 324) dx$$

$$= \pi \left[\frac{16}{3}x^3 - 72x^2 + 324x \right]_4^{4.5} \quad \text{A1}$$

$$= \frac{2\pi}{3} \quad \text{A1}$$

$$\text{total volume is } 8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi \right) \quad \text{A1 N4}$$

[17]

6. (a) (i) range of f is $[-1, 1]$, $(-1 \leq f(x) \leq 1)$ A2 N2

$$(ii) \sin^3 x = 1 \Rightarrow \sin x = 1 \quad \text{A1}$$

justification for one solution on $[0, 2\pi]$ R1

$$e.g. x = \frac{\pi}{2}, \text{ unit circle, sketch of } \sin x$$

$$1 \text{ solution (seen anywhere)} \quad \text{A1 N1}$$

(b) $f'(x) = 3 \sin^2 x \cos x$

A2 N2

(c) using $V = \int_a^b \pi y^2 dx$

(M1)

$$V = \int_0^{\frac{\pi}{2}} \pi \left(\sqrt{3} \sin x \cos^2 x \right)^2 dx \quad (\text{A1})$$

$$= \pi \int_0^{\frac{\pi}{2}} 3 \sin^2 x \cos x dx \quad \text{A1}$$

$$V = \pi \left[\sin^3 x \right]_0^{\frac{\pi}{2}} \left(= \pi \left(\sin^3 \left(\frac{\pi}{2} \right) - \sin^3 0 \right) \right) \quad \text{A2}$$

evidence of using $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$ (A1)

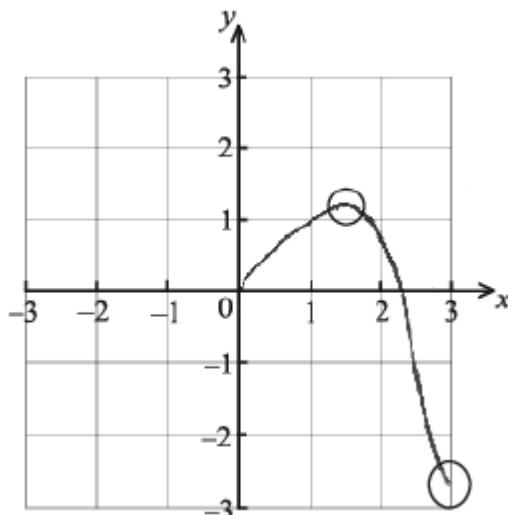
e.g. $\pi(1 - 0)$

$V = \pi$

A1 N1

[14]

7. (a)



A1A2 N3

Notes: Award A1 for correct domain, $0 \leq x \leq 3$.

Award A2 for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.

(b) $a = 2.31$

A1 N1

(c) evidence of using $V = \pi \int [f(x)]^2 dx$ (M1)

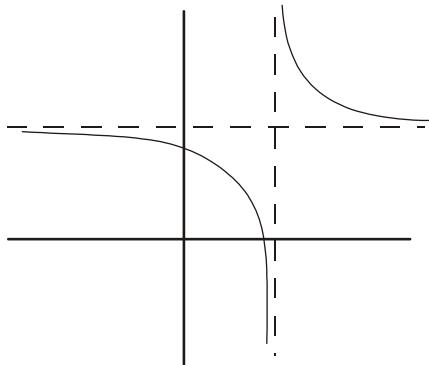
fully correct integral expression A2

e.g. $V = \pi \int_0^{2.31} [x \cos(x - \sin x)]^2 dx, V = \pi \int_0^{2.31} [f(x)]^2 dx$

$V = 5.90$ A1 N2

[8]

8. (a)



A1A1A1 N3

Notes: Award A1 for both asymptotes shown.

The asymptotes need not be labelled.

Award A1 for the left branch in approximately correct position,

A1 for the right branch in approximately correct position.

(b) (i) $y = 3, x = \frac{5}{2}$ (must be equations) A1A1 N2

(ii) $x = \frac{14}{6} \left(\frac{7}{3} \text{ or } 2.33, \text{ also accept } \left(\frac{14}{6}, 0 \right) \right)$ A1 N1

(iii) $y = \frac{14}{6} \quad (y=2.8) \left(\text{accept } \left(0, \frac{14}{5} \right) \text{ or } (0, 2.8) \right)$ A1 N1

(c) (i) $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2} \right) dx = 9x +$

$3\ln(2x-5) - \frac{1}{2(2x-5)} + C$ A1A1A1

A1A1 N5

(ii) Evidence of using $V = \int_a^b \pi y^2 dx$ (M1)

Correct expression A1

$$eg \int_3^a \pi \left(3 + \frac{1}{2x-5}\right)^2 dx, \pi \int_3^a \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx,$$

$$\left[9x + 3\ln(2x-5) - \frac{1}{2(2x-5)}\right]_3^a$$

$$\text{Substituting } \left(9a + 3\ln(2a-5) - \frac{1}{2(2a-5)}\right) - \left(27 + 3\ln 1 - \frac{1}{2}\right) \quad A1$$

Setting up an equation (M1)

$$9a - \frac{1}{2(2a-5)} - 27 + \frac{1}{2} + 3\ln(2a-5) - 3\ln 1 = \left(\frac{28}{3} + 3\ln 3\right)$$

Solving gives $a = 4$ A1 N2

[17]

9. (a) (i) $p = 2$ A1 N1
(ii) $q = 1$ A1 N1

(b) (i) $f(x) = 0$ (M1)

$$2 - \frac{3x}{x^2 - 1} = 0 \quad (2x^2 - 3x - 2 = 0) \quad A1$$

$$x = -\frac{1}{2}, x = 2 \quad A1 \quad N2$$

$$\left(-\frac{1}{2}, 0\right)$$

(ii) Using $V = \int_a^b \pi y^2 dx$ (limits not required) (M1)

$$V = \boxed{\frac{1}{2}} \pi \int_2^0 \left(2 - \frac{3x}{x^2 - 1}\right)^2 dx \quad A2$$

$$V = 2.52 \quad A1 \quad N2$$

(c)	(i)	Evidence of appropriate method <i>eg</i> Product or quotient rule	M1
		Correct derivatives of $3x$ and $x^2 - 1$	A1A1
		Correct substitution	A1
		$eg \frac{-3(x^2 - 1) - (-3x)(2x)}{(x^2 - 1)^2}$	
		$f'(x) = \frac{-3x^2 + 3 + 6x^2}{(x^2 - 1)^2}$	A1
		$f'(x) = \frac{3x^2 + 3}{(x^2 - 1)^2} = \frac{3(x^2 + 1)}{(x^2 - 1)^2}$	AG N0

(ii) **METHOD 1**

Evidence of using $f'(x) = 0$ at max/min	(M1)
$3(x^2 + 1) = 0$ ($3x^2 + 3 = 0$)	A1
no (real) solution	R1
Therefore, no maximum or minimum.	AG N0

METHOD 2

Evidence of using $f'(x) = 0$ at max/min	(M1)
Sketch of $f'(x)$ with good asymptotic behaviour	A1
Never crosses the x -axis	R1
Therefore, no maximum or minimum.	AG N0

METHOD 3

Evidence of using $f'(x) = 0$ at max/min	(M1)
Evidence of considering the sign of $f'(x)$	A1
$f'(x)$ is an increasing function ($f'(x) > 0$, always)	R1
Therefore, no maximum or minimum.	AG N0

(d) For using integral

(M1)

$$\text{Area} = \int_0^a g(x) dx \left(\text{or } \int_0^a f'(x) dx \text{ or } \int_0^a \frac{3x^2 + 3}{(x^2 - 1)^2} dx \right) \quad \text{A1}$$

$$\text{Recognizing that } \int_0^a g(x) dx = f(x) \Big|_0^a \quad \text{A2}$$

Setting up equation (seen anywhere) (M1)

Correct equation A1

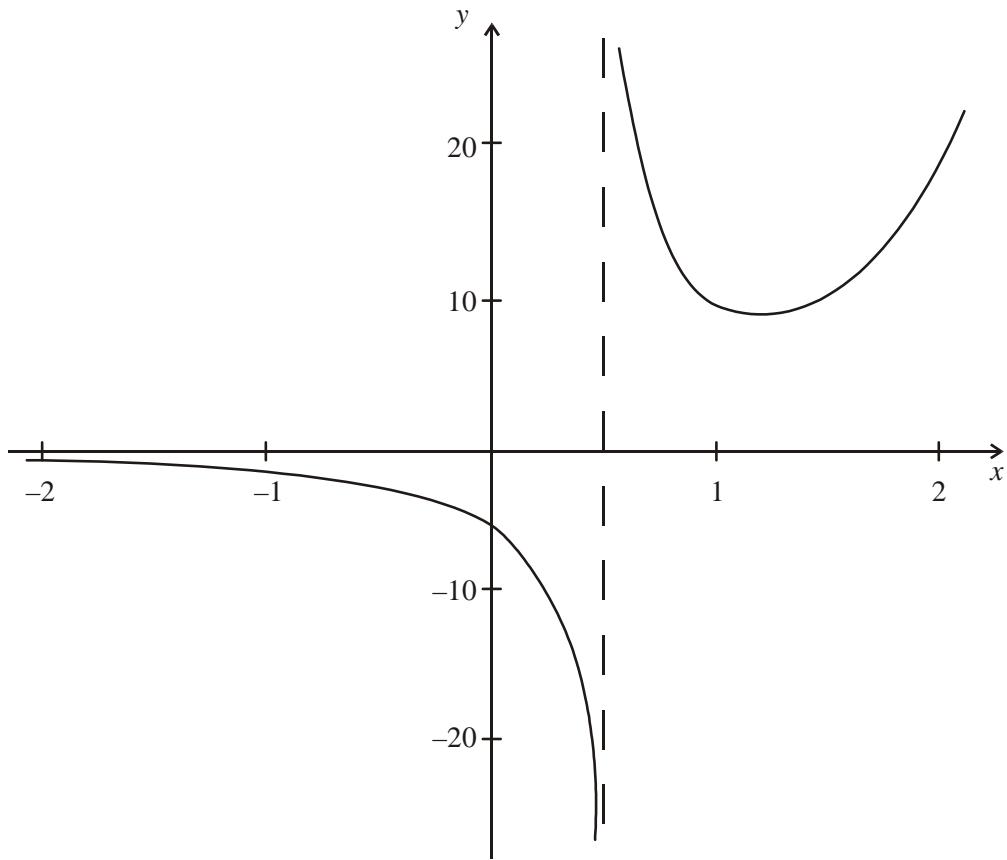
$$eg \int_0^a \frac{3x^2 + 3}{(x^2 - 1)^2} dx = 2, \left[2 - \frac{3a}{a^2 - 1} \right] - [2 - 0] = 2, 2a^2 + 3a - 2 = 0$$

$$a = \frac{1}{2} \quad a = -2$$

$$a = \frac{1}{2} \quad \text{A1} \quad \text{N2}$$

[24]

10. (a)



A1A1A1 N3

Note: Award A1 for the left branch asymptotic to the x-axis and crossing the y-axis,
A1 for the right branch approximately the correct shape,
A1 for a vertical asymptote at
approximately $x = \frac{1}{2}$.

(b) (i) $x = \frac{1}{2}$ (must be an equation)

A1 N1

(ii) $\int_0^2 f(x) dx$

A1 N1

(iii) Valid reason

R1 N1

eg reference to area undefined or discontinuity

Note: GDC reason **not** acceptable.

(c) (i) $V = \pi \int_1^{1.5} f(x)^2 dx$ A2 N2

(ii) $V = 105$ (accept 33.3π) A2 N2

(d) $f'(x) = 2e^{2x-1} - 10(2x-1)^{-2}$ A1A1A1A1 N4

(e) (i) $x = 1.11$ (accept $(1.11, 7.49)$) A1 N1

(ii) $p = 0, q = 7.49$ (accept $0 \leq k < 7.49$) A1A1 N2

[17]

11. (a) Attempting to use the formula $V = \int_a^b \pi y^2 dx$ (M1)

$$\text{Volume} = \pi \int_0^2 (2x - x^2)^2 dx \quad \text{A2 N3}$$

(b) $\text{Volume} = \pi \int_0^2 (4x^2 - 4x^3 + x^4) dx$ (A1)

$$= \pi \left[4 \frac{x^3}{3} - 4 \frac{x^4}{4} + \frac{x^5}{5} \right]_0^2 \quad \text{(A1)}$$

$$= \frac{16\pi}{15} \text{ or } 3.35 \quad (\text{accept } 1.07\pi) \quad \text{A1 N3}$$

[6]

12. (a) (i) $f'(x) = -\frac{3}{2}x + 1$ A1A1 N2

(ii) For using the derivative to find the gradient of the tangent (M1)

$$f'(2) = -2 \quad (\text{A1})$$

Using negative reciprocal to find the gradient of the normal $\left(\frac{1}{2}\right)$ M1

$$y - 3 = \frac{1}{2}(x - 2) \quad \left(\text{or } y = \frac{1}{2}x + 2 \right) \quad \text{A1 N3}$$

(iii) Equating $-\frac{3}{4}x^2 + x + 4 = \frac{1}{2}x + 2$ (or sketch of graph) M1

$$3x^2 - 2x - 8 = 0 \quad (\text{A1})$$

$$(3x + 4)(x - 2) = 0$$

$$x = -\frac{4}{3} (-= -1.33) \quad (\text{accept } -\frac{4}{3}, \frac{4}{3} \text{ or } x = -\frac{4}{3}, x = 2) \quad \text{A1 N2}$$

(b) (i) Any **completely** correct expression (accept absence of dx) A2

$$\text{eg } \int_{-1}^2 \left(-\frac{3}{4}x^2 + x + 4 \right) dx, \left[-\frac{1}{4}x^3 + \frac{1}{2}x^2 + 4x \right]_{-1}^2 \quad \text{N2}$$

(ii) Area $= \frac{45}{4} (= 11.25)$ (accept 11.3) A1 N1

(iii) Attempting to **use** the formula for the volume (M1)

$$\text{eg } \int_{-1}^2 \pi \left(-\frac{3}{4}x^2 + x + 4 \right) dx, \pi \int_{-1}^2 \left(-\frac{3}{4}x^2 + x + 4 \right)^2 dx \quad \text{A2 N3}$$

(c) $\int_1^k f(x) dx = \left[-\frac{1}{4}x^3 + \frac{1}{2}x^2 + 4x \right]_1^k \quad \text{A1A1A1}$

Note: Award A1 for $-\frac{1}{4}x^3$, A1 for $\frac{1}{2}x^2$, A1 for $4x$.

Substituting $\left(-\frac{1}{4}k^3 + \frac{1}{2}k^2 + 4k \right) - \left(-\frac{1}{4} + \frac{1}{2} + 4 \right)$ (M1)(A1)

$$= -\frac{1}{4}k^3 + \frac{1}{2}k^2 + 4k - 4.25 \quad \text{A1 N3}$$

13. Using $V = \int \pi y^2 dx$ (M1)

Correctly integrating $\int \left(x^{\frac{1}{2}} \right)^2 dx = \frac{x^2}{2}$ A1

$$V = \pi \left[\frac{x^2}{2} \right]_0^a \quad \text{A1}$$

$$= \frac{\pi a^2}{2} \quad (\text{A1})$$

Setting up **their** equation $\left(\frac{1}{2} \pi a^2 = 0.845 \pi \right)$ M1

$$a^2 = 1.69$$

$$a = 1.3$$

A1 N2

[6]

14. (a) (i) $p = (10x + 2) - (1 + e^{2x})$ A2 2

Note: Award (A1) for $(l + e^{2x}) - (10x + 2)$.

$$(ii) \quad \frac{dp}{dx} = 10 - 2e^{2x} \quad \text{A1A1}$$

$$\frac{dp}{dx} = 0 \quad (10 - 2e^{2x} = 0) \quad \text{M1}$$

$$x = \frac{\ln 5}{2} \quad (= 0.805) \quad \text{A1} \quad 4$$

(b) (i) **METHOD 1**

$$x = 1 + e^{2x} \quad \text{M1}$$

$$\ln(x - 1) = 2y \quad \text{A1}$$

$$f^{-1}(x) = \frac{\ln(x - 1)}{2} \left(\text{Allow } y = \frac{\ln(x - 1)}{2} \right) \quad \text{A1} \quad 3$$

METHOD 2

$$y - 1 = e^{2x} \quad \text{A1}$$

$$\frac{\ln(y - 1)}{2} = x \quad \text{M1}$$

$$f^{-1}(x) = \frac{\ln(x - 1)}{2} \left(\text{Allow } y = \frac{\ln(x - 1)}{2} \right) \quad \text{A1} \quad 3$$

$$\begin{aligned}
 \text{(ii)} \quad a &= \frac{\ln(5-1)}{2} \left(= \frac{1}{2} \ln 2^2 \right) & \text{M1} \\
 &= \frac{1}{2} \times 2 \ln 2 & \text{A1} \\
 &= \ln 2 & \text{AG} \quad 2
 \end{aligned}$$

$$\text{(c)} \quad \text{Using } V = \int_a^b \pi y^2 \, dx \quad (\text{M1})$$

$$\text{Volume} = \int_0^{\ln 2} \pi (1 + e^{2x})^2 \, dx \quad \left(\text{or} \int_0^{0.805} \pi (1 + e^{2x})^2 \, dx \right) \quad \text{A2} \quad 3$$

[14]

$$\text{15. (a)} \quad y = e^{x/2} \text{ at } x = 0 \quad y = e^0 = 1 \quad P(0, 1) \quad (\text{A1})(\text{A1}) \quad 2$$

$$\text{(b)} \quad V = \pi \int_0^{\ln 2} (e^{x/2})^2 \, dx \quad (\text{A4}) \quad 4$$

Notes: Award (A1) for π
 (A1) for each limit
 (A1) for $(e^{x/2})^2$.

$$\begin{aligned}
 \text{(c)} \quad V &= \int_0^{\ln 2} e^x \, dx & (\text{A1}) \\
 &= \pi [e^x]_0^{\ln 2} & (\text{A1}) \\
 &= \pi [e^{\ln 2} - e^0] & (\text{A1}) \\
 &= \pi [2 - 1] = \pi & (\text{A1})(\text{A1}) \\
 &= \pi & (\text{AG}) \quad 5
 \end{aligned}$$

[11]