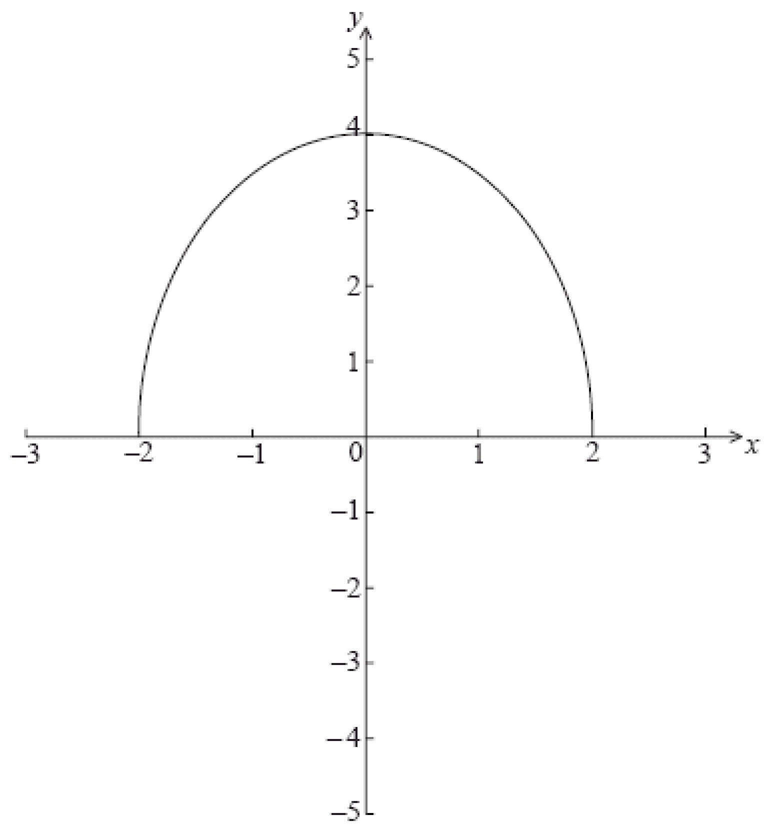
# SL - Integration Volume of Revolution

194 min
194 marks

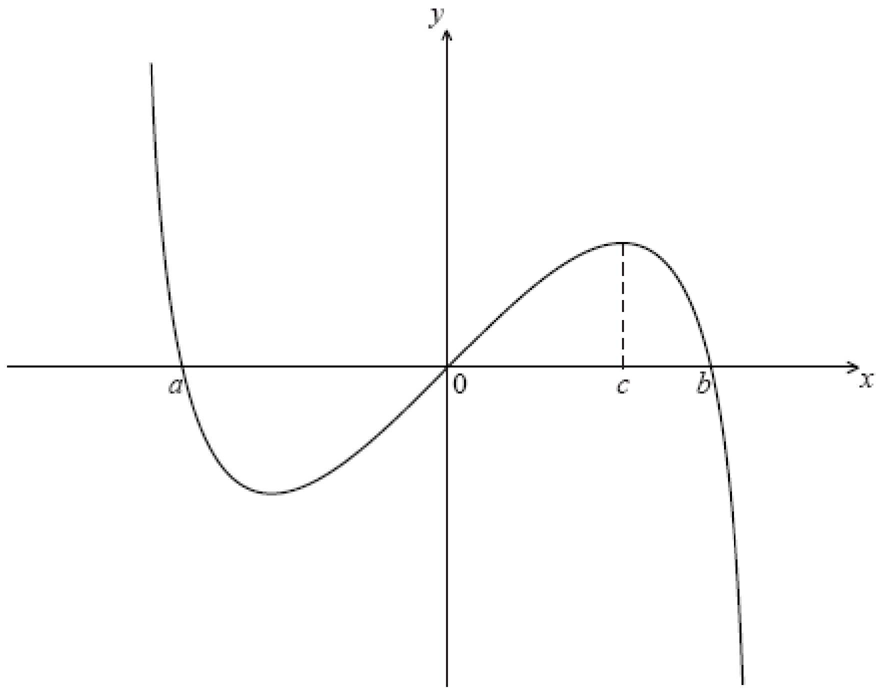
**1.** The graph of $f(x) = \sqrt{16 - 4x^2}$ , for $-2 \le x \le 2$, is shown below.



The region enclosed by the curve of $f$ and the $x$-axis is rotated 360° about the $x$-axis.
Find the volume of the solid formed.

**(Total 6 marks)**

**2.** Let $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of $f$ is shown below.




The graph of $f$ crosses the $x$-axis at $x = a$, $x = 0$ and $x = b$.

(a) Find the value of $a$ and of $b$.

**(3)**

The graph of $f$ has a maximum value when $x = c$.

(b) Find the value of $c$.

**(2)**

(c) The region under the graph of $f$ from $x = 0$ to $x = c$ is rotated 360° about the $x$-axis. Find the volume of the solid formed.

**(3)**

(d) Let $R$ be the region enclosed by the curve, the $x$-axis and the line $x = c$, between $x = a$ and $x = c$.

Find the area of $R$.

**(4)**

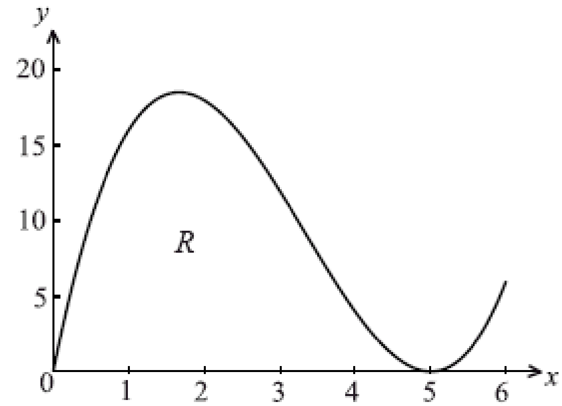
**(Total 12 marks)**

**3.** The graph of $y = \sqrt{x}$ between $x = 0$ and $x = a$ is rotated 360° about the $x$-axis.

The volume of the solid formed is $32\pi$. Find the value of $a$.

**(Total 7 marks)**

**4.** Let $f(x) = x(x - 5)^2$, for $0 \le x \le 6$. The following diagram shows the graph of $f$.




Let $R$ be the region enclosed by the $x$-axis and the curve of $f$.

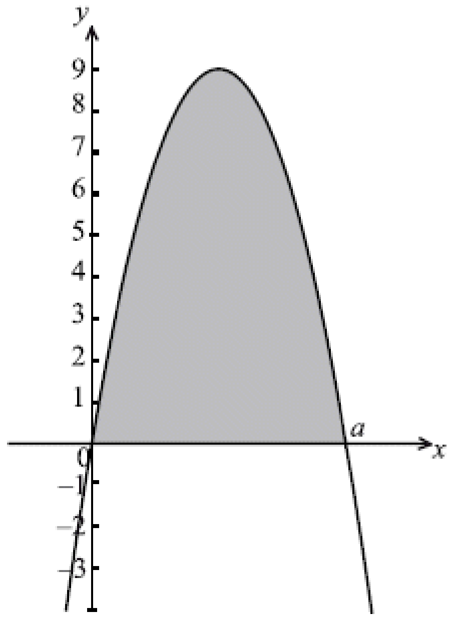
(a) Find the area of $R$.

**(3)**

(b) Find the volume of the solid formed when $R$ is rotated through 360° about the $x$-axis.

**(4)**

(c) The diagram below shows a part of the graph of a quadratic function $g(x) = x(a - x)$. The graph of $g$ crosses the $x$-axis when $x = a$.




The area of the shaded region is equal to the area of $R$. Find the value of $a$.

**(7)**

**(Total 14 marks)**

**5.** Let $f(x) = \sqrt{x}$ . Line $L$ is the normal to the graph of $f$ at the point (4, 2).

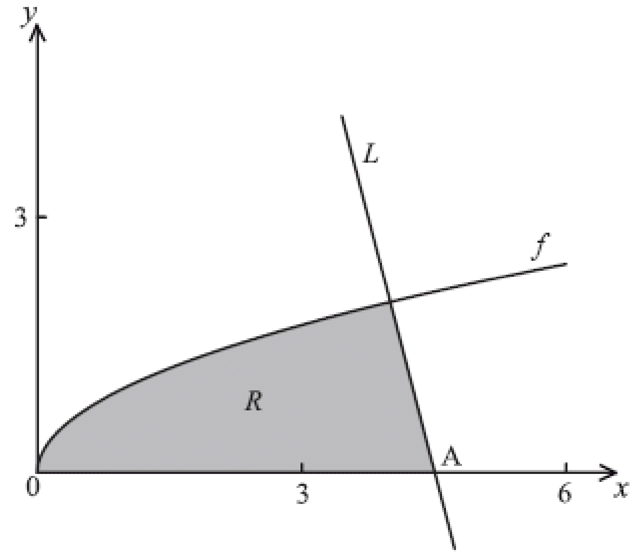
(a) Show that the equation of $L$ is $y = -4x + 18$.

**(4)**

(b) Point A is the $x$-intercept of $L$. Find the $x$-coordinate of A.

**(2)**

In the diagram below, the shaded region $R$ is bounded by the $x$-axis, the graph of $f$ and the line $L$.




(c) Find an expression for the area of $R$.

**(3)**

(d) The region $R$ is rotated 360° about the $x$-axis. Find the volume of the solid formed, giving your answer in terms of $\pi$.

**(8)**

**(Total 17 marks)**

**6.** Let $f : x \alpha \ \sin^3 x$.

(a) (i) Write down the range of the function $f$.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

**(5)**

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.

**(2)**

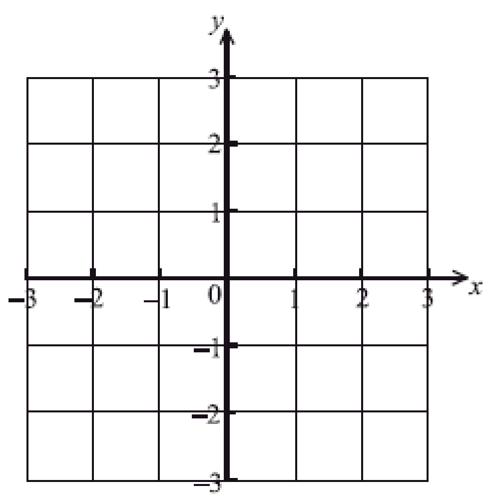
(c) Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of $g$ is revolved through $2\pi$ about the $x$-axis.

**(7)**

**(Total 14 marks)**

**7.** Let $f(x) = x \cos(x - \sin x)$, $0 \le x \le 3$.

(a) Sketch the graph of $f$ on the following set of axes.




**(3)**

(b) The graph of $f$ intersects the $x$-axis when $x = a$, $a \ne 0$. Write down the value of $a$.

**(1)**

(c) The graph of $f$ is revolved 360° about the $x$-axis from $x = 0$ to $x = a$.
Find the volume of the solid formed.

**(4)**
**(Total 8 marks)**

**8.** The function $f(x)$ is defined as $f(x) = 3 + \dfrac{1}{2x-5}$, $x \neq \dfrac{5}{2}$.

(a) Sketch the curve of $f$ for $-5 \le x \le 5$, showing the asymptotes.

**(3)**

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the $x$-intercept;

(iii) the value of the y-intercept.

**(4)**

(c) The region enclosed by the curve of $f$, the $x$-axis, and the lines $x = 3$ and $x = a$, is revolved through 360° about the $x$-axis. Let $V$ be the volume of the solid formed.

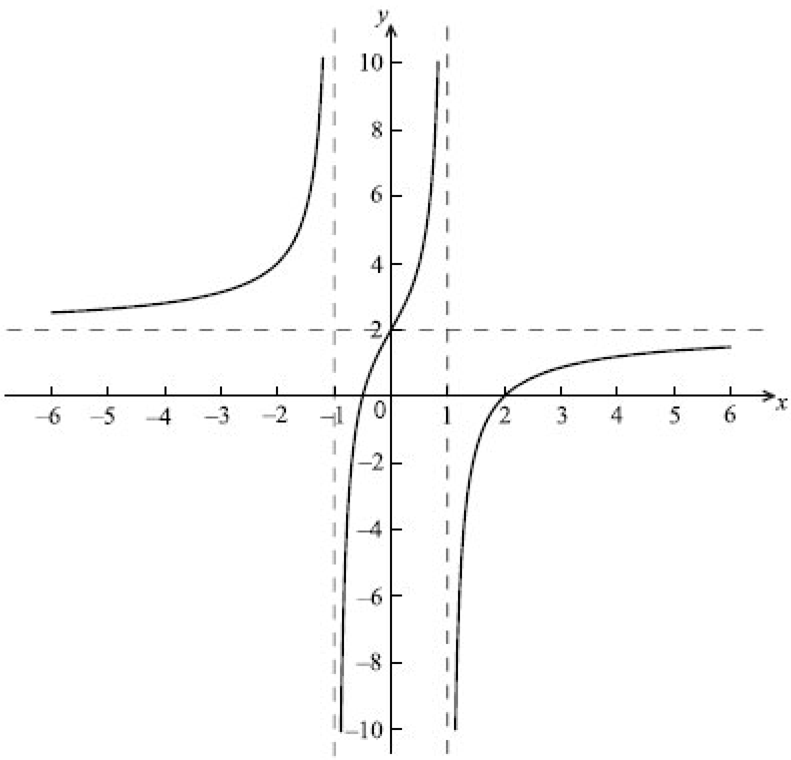
(i) Find $\displaystyle\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx$.

(ii) Hence, given that $V = \pi\left(\dfrac{28}{3} + 3\ln 3\right)$, find the value of $a$.

**(10)**
**(Total 17 marks)**

**9.** Let $f(x) = p - \dfrac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of $f$, including the asymptotes, is shown below.




(a) The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$. Write down the value of

(i) $p$;

(ii) $q$.

**(2)**

(b) Let $R$ be the region bounded by the graph of $f$, the $x$-axis, and the $y$-axis.

(i) Find the negative $x$-intercept of $f$.

(ii) Hence find the volume obtained when $R$ is revolved through 360° about the $x$-axis.

**(7)**

(c) (i) Show that $f'(x) = \dfrac{3(x^2+1)}{(x^2-1)^2}$.

(ii) Hence, show that there are no maximum or minimum points on the graph of $f$.

**(8)**

(d) Let $g(x) = f'(x)$. Let $A$ be the area of the region enclosed by the graph of g and the $x$-axis, between $x = 0$ and $x = a$, where $a > 0$. Given that $A = 2$, find the value of $a$.

**(7)**

**(Total 24 marks)**

**10.** Consider the function $f(x)\ e^{(2x-1)} + \left(\frac{5}{(2x-1)}\right)$, $x \neq \frac{1}{2}$.

(a) Sketch the curve of $f$ for $-2 \leq x \leq 2$, including any asymptotes.

**(3)**

(b) (i) Write down the equation of the vertical asymptote of $f$.

(ii) Write down which one of the following expressions does **not** represent an area between the curve of $f$ and the $x$-axis.

$$\int_1^2 f(x)\mathrm{d}x$$

$$\int_0^2 f(x)\mathrm{d}x$$

(iii) Justify your answer.

**(3)**

(c) The region between the curve and the $x$-axis between $x = 1$ and $x = 1.5$ is rotated through 360° about the $x$-axis. Let $V$ be the volume formed.

(i) Write down an expression to represent $V$.

(ii) Hence write down the value of $V$.

**(4)**

(d) Find $f'(x)$.

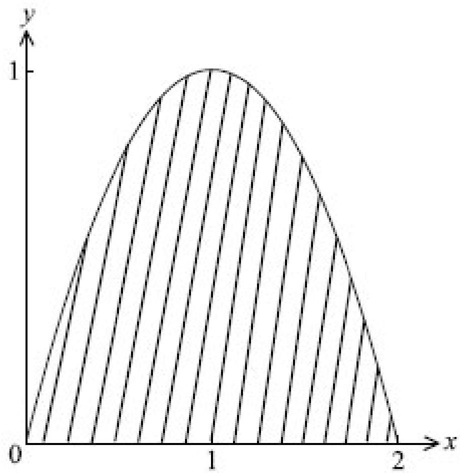
**(4)**

(e) (i) Write down the value of $x$ at the minimum point on the curve of $f$.

(ii) The equation $f(x) = k$ has no solutions for $p \leq k < q$. Write down the value of $p$ and of $q$.

**(3)**
**(Total 17 marks)**

**11.** A part of the graph of $y = 2x - x^2$ is given in the diagram below.




The shaded region is revolved through 360° about the $x$-axis.

(a) Write down an expression for this volume of revolution.

(b) Calculate this volume.

**(Total 6 marks)**

**12.** Let $f(x) = -\frac{3}{4}x^2 + x + 4$.

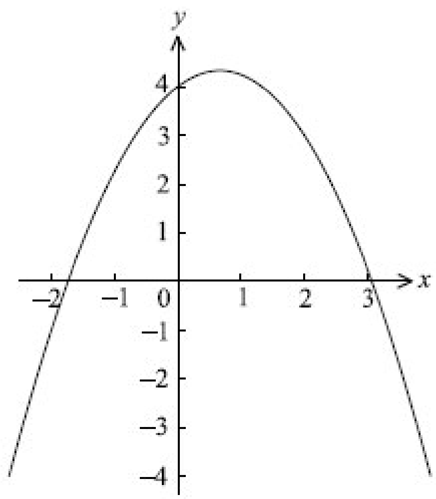
(a) (i) Write down $f'(x)$.

(ii) Find the equation of the normal to the curve of $f$ at (2, 3).

(iii) This normal intersects the curve of $f$ at (2, 3) and at one other point P. Find the $x$-coordinate of P.

**(9)**

Part of the graph of $f$ is given below.




(b) Let $R$ be the region under the curve of $f$ from $x = -1$ to $x = 2$.

(i) Write down an expression for the area of $R$.

(ii) Calculate this area.

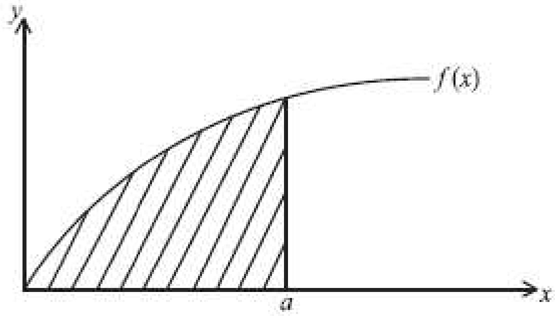
(iii) The region $R$ is revolved through 360° about the $x$-axis. Write down an expression for the volume of the solid formed.

**(6)**

(c) Find $\int_1^k f(x)\,dx$, giving your answer in terms of $k$.

**(6)**
**(Total 21 marks)**

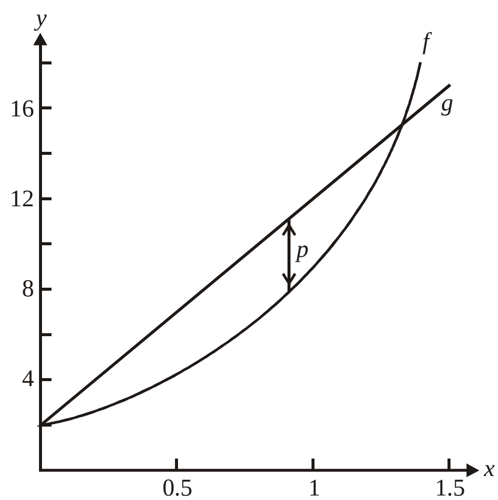
**13.** The shaded region in the diagram below is bounded by $f(x) = \sqrt{x}$, $x = a$, and the $x$-axis. The shaded region is revolved around the $x$-axis through 360°. The volume of the solid formed is $0.845\pi$.




Find the value of $a$.

**(Total 6 marks)**

**14.** The diagram below shows the graphs of $f(x) = 1 + e^{2x}$, $g(x) = 10x + 2$, $0 \le x \le 1.5$.

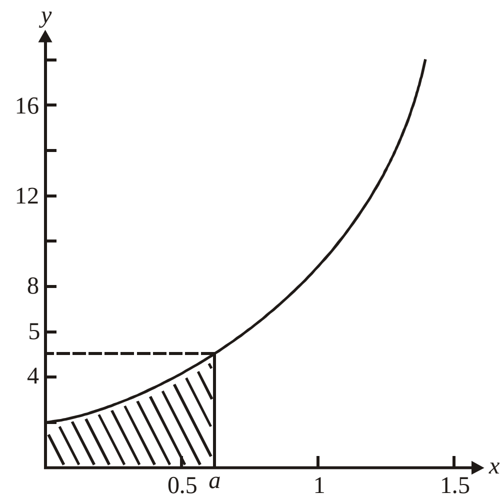



(a) (i) Write down an expression for the vertical distance $p$ between the graphs of $f$ and $g$.

(ii) Given that $p$ has a maximum value for $0 \le x \le 1.5$, find the value of $x$ at which this occurs.

**(6)**

The graph of $y = f(x)$ only is shown in the diagram below. When $x = a$, $y = 5$.




(b) (i) Find $f^{-1}(x)$.

(ii) **Hence** show that $a = \ln 2$.

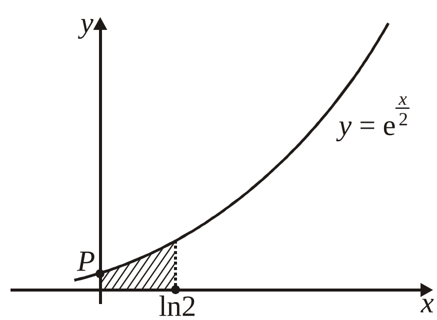
**(5)**

(c) The region shaded in the diagram is rotated through 360° about the $x$-axis. Write down an expression for the volume obtained.

**(3)**

**(Total 14 marks)**

**15.** The diagram shows part of the graph of $y = e^{\frac{x}{2}}$.



(a) Find the coordinates of the point $P$, where the graph meets the $y$-axis.

**(2)**

The shaded region between the graph and the $x$-axis, bounded by $x = 0$ and $x = \ln 2$, is rotated through 360° about the $x$-axis.

(b) Write down an integral which represents the volume of the solid obtained.

**(4)**

(c) Show that this volume is $\pi$.

**(5)**

**(Total 11 marks)**