

SL Differentiation Kinematics

0 min
0 marks

1. (a) $v = 1$ A1 N1 1

(b) (i) $\frac{d}{dt}(2t) = 2$ A1

$\frac{d}{dt}(\cos 2t) = -2 \sin 2t$ A1A1

Note: Award A1 for coefficient 2 and A1 for $-\sin 2t$.

evidence of considering acceleration = 0 (M1)

e.g. $\frac{dv}{dt} = 0, 2 - 2 \sin 2t = 0$

correct manipulation A1

e.g. $\sin 2k = 1, \sin 2t = 1$

$2k = \frac{\pi}{2} \left(\text{accept } 2t = \frac{\pi}{2} \right)$ A1

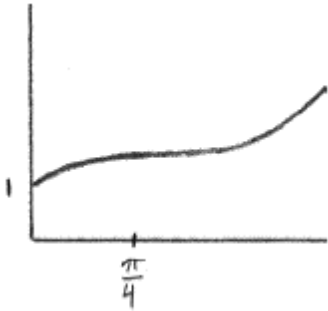
$k = \frac{\pi}{4}$ AG N0

(ii) attempt to substitute $t = \frac{\pi}{4}$ into v (M1)

e.g. $2 \left(\frac{\pi}{4} \right) + \cos \left(\frac{2\pi}{4} \right)$

$v = \frac{\pi}{2}$ A1 N2 8

(c)



A1A1A2 N4 4

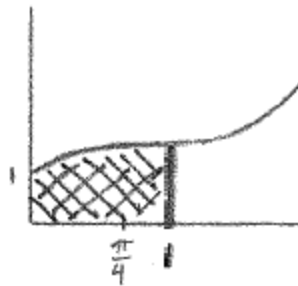
Notes: Award A1 for y-intercept at $(0, 1)$, A1 for curve having zero gradient at $t = \frac{\pi}{4}$, A2 for shape that is concave down to the left of $\frac{\pi}{4}$ **and** concave up to the right of $\frac{\pi}{4}$. If a correct curve is drawn without indicating $t = \frac{\pi}{4}$, do not award the second A1 for the zero gradient, but award the final A2 if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.

(d) (i) correct expression

A2

e.g. $\int_0^1 (2t + \cos 2t) dt, \left[t^2 + \frac{\sin 2t}{2} \right]_0^1, 1 + \frac{\sin 2}{2}, \int_0^1 v dt$

(ii)



A1 3

Note: The line at $t = 1$ needs to be clearly after $t = \frac{\pi}{4}$.

[16]

2. (a) $f(x) = -10(x+4)(x-6)$

A1A1 N2 2

(b)	METHOD 1			
	attempting to find the x -coordinate of maximum point	(M1)		
	<i>e.g.</i> averaging the x -intercepts, sketch, $y' = 0$, axis of symmetry			
	attempting to find the y -coordinate of maximum point	(M1)		
	<i>e.g.</i> $k = -10(1+4)(1-6)$			
	$f(x) = -10(x-1)^2 + 250$	A1A1	N4	4
	METHOD 2			
	attempt to expand $f(x)$	(M1)		
	<i>e.g.</i> $-10(x^2 - 2x - 24)$			
	attempt to complete the square	(M1)		
	<i>e.g.</i> $-10((x-1)^2 - 1 - 24)$			
	$f(x) = -10(x-1)^2 + 250$	A1A1	N4	4
(c)	attempt to simplify	(M1)		
	<i>e.g.</i> distributive property, $-10(x-1)(x-1) + 250$			
	correct simplification	A1		
	<i>e.g.</i> $-10(x^2 - 6x + 4x - 24)$, $-10(x^2 - 2x + 1) + 250$			
	$f(x) = 240 + 20x - 10x^2$	AG	N0	2
(d)	(i) valid approach	(M1)		
	<i>e.g.</i> vertex of parabola, $v'(t) = 0$			
	$t = 1$	A1	N2	
	(ii) recognizing $a(t) = v'(t)$	(M1)		
	$a(t) = 20 - 20t$	A1A1		
	speed is zero $\Rightarrow t = 6$	(A1)		
	$a(6) = -100 \text{ (m s}^{-2}\text{)}$	A1	N3	7
	[15]			

3. evidence of integrating the acceleration function (M1)

e.g. $\int \left(\frac{1}{t} + 3 \sin 2t \right) dt$

correct expression $\ln t - \frac{3}{2} \cos 2t + c$ A1A1

evidence of substituting (1, 0) (M1)

e.g. $0 = \ln 1 - \frac{3}{2} \cos 2 + c$

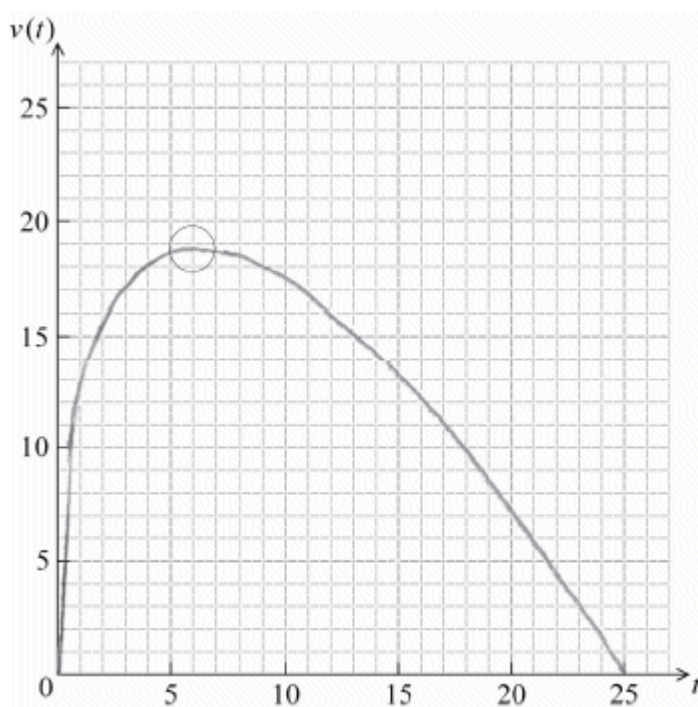
$c = -0.624 \left(= \frac{3}{2} \cos 2 - \ln 1 \text{ or } \frac{3}{2} \cos 2 \right)$ (A1)

$v = \ln t - \frac{3}{2} \cos 2t - 0.624 \left(= \ln t - \frac{3}{2} \cos 2t + \frac{3}{2} \cos 2 \text{ or } \ln t - \frac{3}{2} \cos 2t + \frac{3}{2} \cos 2 - \ln 1 \right)$ (A1)

$v(5) = 2.24$ (accept the exact answer $\ln 5 - 1.5 \cos 10 + 1.5 \cos 2$) A1 N3

[7]

4. (a)



A1A1A1 N3

Note: Award A1 for approximately correct shape, A1 for right endpoint at (25, 0) and A1 for maximum point in circle.

- (b) (i) recognizing that d is the area under the curve (M1)
e.g. $\int v(t)$
 correct expression in terms of t , with correct limits A2 N3
e.g. $d = \int_0^9 (15\sqrt{t} - 3t)dt, d = \int_0^9 vdt$
- (ii) $d = 148.5$ (m) (accept 149 to 3 sf) A1 N1

[7]

5. (a)

Function	Graph
displacement	A
acceleration	B

A2A2 N4

- (b) $t = 3$ A2 N2

[6]

6. **Note:** In this question, do not penalize absence of units.

- (a) (i) $s = \int (40 - at)dt$ (M1)
 $s = 40t - \frac{1}{2}at^2 + c$ (A1)(A1)
 substituting $s = 100$ when $t = 0$ ($c = 100$) (M1)
 $s = 40t - \frac{1}{2}at^2 + 100$ A1 N5

- (ii) $s = 40t - \frac{1}{2}at^2$ A1 N1

- (b) (i) stops at station, so $v = 0$ (M1)
 $t = \frac{40}{a}$ (seconds) A1 N2

(ii)	evidence of choosing formula for s from (a) (ii)	(M1)
	substituting $t = \frac{40}{a}$	(M1)
	$e.g. 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$	
	setting up equation	M1
	$e.g. 500 = s, 500 = 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}, 500 = \frac{1600}{a} - \frac{800}{a}$	
	evidence of simplification to an expression which obviously leads to $a = \frac{8}{5}$	A1
	$e.g. 500a = 800, 5 = \frac{8}{a}, 1000a = 3200 - 1600$	
	$a = \frac{8}{5}$	AG N0

(c) **METHOD 1**

$v = 40 - 4t$, stops when $v = 0$	
$40 - 4t = 0$	(A1)
$t = 10$	A1
substituting into expression for s	M1
$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$	
$s = 200$	A1
since $200 < 500$ (allow FT on their s , if $s < 500$)	R1
train stops before the station	AG N0

METHOD 2

from (b) $t = \frac{40}{4} = 10$	A2
substituting into expression for s	
$e.g. s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$	M1
$s = 200$	A1
since $200 < 500$,	R1
train stops before the station	AG N0

METHOD 3

a is deceleration

A2

$$4 > \frac{8}{5}$$

A1

so stops in shorter time

(A1)

so less distance travelled

R1

so stops before station

AG N0

[17]

7. evidence of anti-differentiation

(M1)

e.g. $s = \int (6e^{3x} + 4) dx$

$$s = 2e^{3t} + 4t + C$$

A2A1

substituting $t = 0$,

(M1)

$$7 = 2 + C$$

A1

$$C = 5$$

$$s = 2e^{3t} + 4t + 5$$

A1 N3

[7]

8. (a) (i) range of f is $[-1, 1]$, $(-1 \leq f(x) \leq 1)$

A2 N2

(ii) $\sin^3 x = 1 \Rightarrow \sin x = 1$

A1

justification for one solution on $[0, 2\pi]$

R1

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere)

A1 N1

(b) $f'(x) = 3 \sin^2 x \cos x$

A2 N2

(c) using $V = \int_a^b \pi y^2 dx$ (M1)

$$V = \int_0^{\frac{\pi}{2}} \pi \left(\sqrt{3} \sin x \cos^{\frac{1}{2}} x \right)^2 dx \quad (A1)$$

$$= \pi \int_0^{\frac{\pi}{2}} 3 \sin^2 x \cos x dx \quad A1$$

$$V = \pi \left[\sin^3 x \right]_0^{\frac{\pi}{2}} \left(= \pi \left(\sin^3 \left(\frac{\pi}{2} \right) - \sin^3 0 \right) \right) \quad A2$$

evidence of using $\sin \frac{\pi}{2} = 1$ and $\sin 0 = 0$ (A1)

e.g. $\pi(1 - 0)$

$V = \pi$ A1 N1

[14]

9. (a) substituting $t = 0$ (M1)

e.g. $a(0) = 0 + \cos 0$

$a(0) = 1$ A1 N2

(b) evidence of integrating the acceleration function (M1)

e.g. $\int (2t + \cos t) dt$

correct expression $t^2 + \sin t + c$ A1A1

Note: If "+c" is omitted, award no further marks.

evidence of substituting (0, 2) into indefinite integral (M1)

e.g. $2 = 0 + \sin 0 + c$, $c = 2$

$v(t) = t^2 + \sin t + 2$ A1 N3

(c) $\int (t^2 + \sin t + 2) dt = \frac{t^3}{3} - \cos t + 2t$ A1A1A1

Note: Award A1 for each correct term.

evidence of using $v(3) - v(0)$ (M1)

correct substitution A1

e.g. $(9 - \cos 3 + 6) - (0 - \cos 0 + 0)$, $(15 - \cos 3) - (-1)$

$16 - \cos 3$ (accept $p = 16$, $q = -1$) A1A1 N3

- (d) reference to motion, reference to first 3 seconds
e.g. displacement in 3 seconds, distance travelled in 3 seconds

R1R1 N2

[16]

10. (a) $a = \frac{dv}{dt}$
 $= -10 \text{ (m s}^{-2}\text{)}$

(M1)

A1 N2

(b) $s = \int v \, dt$
 $= 50t - 5t^2 + c$
 $40 = 50(0) - 5(0) + c \Rightarrow c = 40$
 $s = 50t - 5t^2 + 40$

(M1)

A1

A1

A1 N2

Note: Award (M1) and the first A1 in part (b) if c is missing, but do **not** award the final 2 marks.

[6]

11. Evidence of integration

(M1)

$$s = -0.5 e^{-2t} + 6t^2 + c$$

A1A1

Substituting $t = 0, s = 2$

(M1)

$$\text{eg } 2 = -0.5 + c$$

$$c = 2.5$$

(A1)

$$s = -0.5 e^{-2t} + 6t^2 + 2.5$$

A1 N4

[6]

12. (a) Evidence of using $a = \frac{dv}{dt}$

(M1)

$$\text{eg } 3e^{3t-2}$$

$$a(1) = 3e \quad (= 8.15)$$

A1 N2

- (b) Attempt to solve $e^{3t-2} = 22.3$

(M1)

$$\text{eg } (3t - 2) (\ln e) = \ln 22.3, \text{ sketch}$$

$$t = 1.70$$

A1 N2

(c) Evidence of using $s = \int v dt$ (limits not required) M1

$$e.g. \int e^{3t-2} dt, \frac{1}{3} [e^{3t-2}]_0^1$$

$$\frac{1}{3} (e^1 - e^{-2}) \left[= \frac{1}{3} (e - e^{-2}) = 0.861 \right] \quad \text{A1} \quad \text{N1}$$

[6]

13. Finding anti-derivative of $4t^3 - 2t$ (M1)

$$s = t^4 - t^2 + c \quad \text{A1A1}$$

$$\text{Substituting correctly } 8 = 2^4 - 2^2 + c \quad \text{A1}$$

*Note: Exception to the **FT** rule. Allow full **FT** on incorrect integration.*

$$c = -4 \quad (\text{A1})$$

$$s = t^4 - t^2 - 4 \quad \text{A1} \quad \text{N3}$$

[6]

14. (a) $S_{\min} = 6.05$ (accept (1, 6.05)) A1 N1

$$(b) \quad \frac{ds}{dt} = -15 \sin 3t + 2t \quad \text{A1}$$

$$a = \frac{d^2s}{dt^2} \quad (\text{M1})$$

$$a = \frac{d^2s}{dt^2} = -45 \cos 3t + 2 \quad (\text{Exception to } \mathbf{FT} \text{ rule : allow } \mathbf{FT}$$

$$\text{from } \frac{ds}{dt}) \quad \text{A1} \quad \text{N2}$$

(c) **EITHER**

Maximum value of a when $\cos 3t$ is minimum *ie* $\cos 3t = -1$ (A1)

OR

At maximum $\frac{da}{dt} = 0$ ($135 \sin 3t = 0$) (A1)

THEN

$t = \frac{\pi}{3}$ (accept 1.05; do **not** accept 60°) A1 N2

[6]

15. $s = \int v \, dt$ (M1)

$$s = \frac{1}{2} e^{2t-1} + c \quad \text{A1A1}$$

Substituting $t = 0.5$

$$\frac{1}{2} + c = 10$$

$$c = 9.5 \quad \text{(A1)}$$

Substituting $t = 1$ M1

$$s = \frac{1}{2} e + 9.5 (= 10.9 \text{ to } 3 \text{ s.f.}) \quad \text{A1 N3}$$

[6]

16. (a) $a = \frac{dv}{dt}$ (M1)

$$= -10 \quad \text{A1 3}$$

(b) $s = \int v \, dt$ (M1)

$$= 50t - 5t^2 + c \quad \text{A1}$$

$$40 = 50(0) - 5(0) + c \Rightarrow c = 40 \quad \text{A1}$$

$$s = 50t - 5t^2 + 40 \quad \text{A1 3}$$

Note: Award (M1) and the first (A1) in part (b) if c is missing, but do not award the final 2 marks.

[6]

17. (a) $s = 25t - \frac{4}{3}t^3 + c$ (M1)(A1)(A1)

Note: Award no further marks if “c” is missing.

Substituting $s = 10$ and $t = 3$ (M1)

$$10 = 25 \times 3 - \frac{4}{3}(3)^3 + c$$

$$10 = 75 - 36 + c$$

$$c = -29$$
 (A1)

$$s = 25t - \frac{4}{3}t^3 - 29$$
 (A1) (N3)

(b) **METHOD 1**

s is a maximum when $v = \frac{ds}{dt} = 0$ (may be implied) (M1)

$$25 - 4t^2 = 0$$
 (A1)

$$t^2 = \frac{25}{4}$$

$$t = \frac{5}{2}$$
 (A1) (N2)

METHOD 2

Using maximum of s ($12\frac{2}{3}$, may be implied) (M1)

$$25t - \frac{4}{3}t^3 - 29 = 12\frac{2}{3}$$
 (A1)

$$t = 2.5$$
 (A1) (N2)

(c) $25t - \frac{4}{3}t^3 - 29 > 0$ (accept equation) (M1)

$$m = 1.27, n = 3.55$$
 (A1)(A1) (N3)

[12]

18. (a) $d = \int_0^4 (4t + 5 - 5e^{-t}) dt$ (M1)(A1)(A1) (C3)

Note: Award (M1) for \int , (A1) for **both** limits, (A1) for $4t + 5 - 5e^{-t}$

(b) $d = [2t^2 + 5t + 5e^{-t}]_0^4$ (A1)(A1)

Note: Award (A1) for $2t^2 + 5t$, (A1) for $5e^{-t}$.

$$= (32 + 20 + 5e^{-4}) - (5)$$

$$= 47 + 5e^{-4} \text{ (47.1, 3sf)} \quad (A1) \quad (C3)$$

[6]

19. (a) Velocity is $\frac{ds}{dt}$. (M1)

$$\frac{ds}{dt} = 10 - t \quad (A1)$$

$$10 \text{ (m s}^{-1}\text{)} \quad (A1) \quad (C3)$$

(b) The velocity is zero when $\frac{ds}{dt} = 0$ (M1)

$$10 - t = 0$$

$$t = 10 \text{ (secs)} \quad (A1) \quad (C2)$$

(c) $s = 50 \text{ (metres)}$ (A1) (C1)

Note: Do not penalize absence of units.

[6]

20. (a) (i) When $t = 0$, $v = 50 + 50e^0$ (A1)
 $= 100 \text{ m s}^{-1}$ (A1)

(ii) When $t = 4$, $v = 50 + 50e^{-2}$ (A1)
 $= 56.8 \text{ m s}^{-1}$ (A1) 4

(b) $v = \frac{ds}{dt} \Rightarrow s = \int v \, dt$

$$\int_0^4 (50 + 50e^{-0.5t}) \, dt \quad (A1)(A1)(A1) \quad 3$$

Note: Award (A1) for each limit in the correct position and (A1) for the function.

(c) Distance travelled in 4 seconds $= \int_0^4 (50 + 50e^{-0.5t}) dt$

$$= [50t - 100e^{-0.5t}]_0^4 \quad (\text{A1})$$

$$= (200 - 100e^{-2}) - (0 - 100e^0)$$

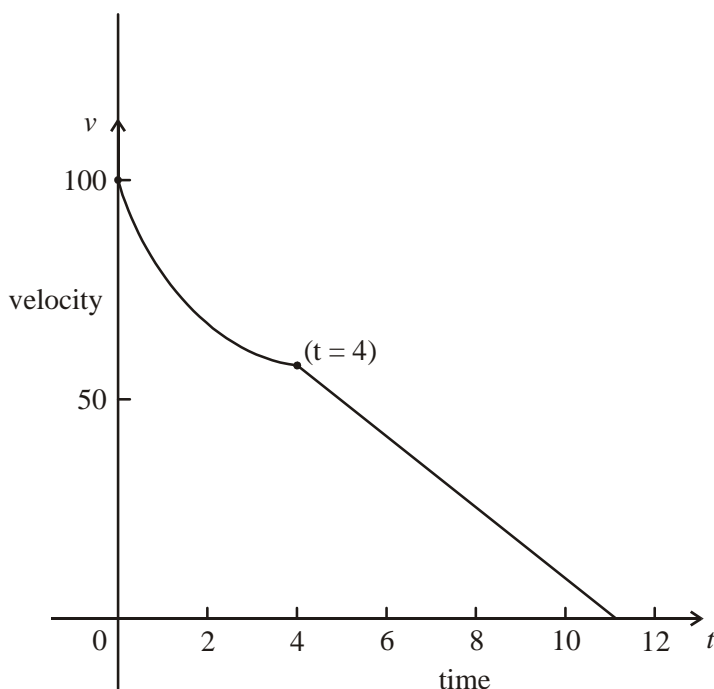
$$= 286 \text{ m (3 sf)} \quad (\text{A1})$$

Note: Award first (A1) for $[50t - 100e^{-0.5t}]$, ie limits not required.

OR

Distance travelled in 4 seconds = 286 m (3 sf) (G2) 2

(d)



Notes: Award (A1) for the exponential part, (A1) for the straight line through (11, 0),
Award (A1) for indication of time on x-axis **and** velocity on y-axis,
(A1) for scale on x-axis **and** y-axis.
Award (A1) for marking the point where $t = 4$.

5

(e) Constant rate $= \frac{56.8}{7}$ (M1)

$= 8.11 \text{ m s}^{-2}$ (A1) 2

Note: Award (M1)(A0) for -8.11 .

$$(f) \quad \text{distance} = \frac{1}{2} (7)(56.8) \quad (\text{M1})$$

$$= 199 \text{ m} \quad (\text{A1}) \quad 2$$

Note: Do not award **ft** in parts (e) and (f) if candidate has not used a straight line for $t = 4$ to $t = 11$ or if they continue the exponential beyond $t = 4$.

[18]

$$21. \quad (a) \quad \frac{ds}{dt} = 30 - at \Rightarrow s = 30t - a \frac{t^2}{2} + C \quad (\text{A1})(\text{A1})(\text{A1})$$

Note: Award (A1) for $30t$, (A1) for $a \frac{t^2}{2}$, (A1) for C .

$$t = 0 \Rightarrow s = 30(0) - a \frac{(0^2)}{2} + C = 0 + C \Rightarrow C = 0 \quad (\text{M1})$$

$$\Rightarrow s = 30t - \frac{1}{2} at^2 \quad (\text{A1}) \quad 5$$

$$(b) \quad (i) \quad \text{vel} = 30 - 5(0) = 30 \text{ m s}^{-1} \quad (\text{A1})$$

$$(ii) \quad \text{Train will stop when } 0 = 30 - 5t \Rightarrow t = 6 \quad (\text{M1})$$

$$\text{Distance travelled} = 30t - \frac{1}{2} at^2$$

$$= 30(6) - \frac{1}{2} (5) (6^2) \quad (\text{M1})$$

$$= 90\text{m} \quad (\text{A1})$$

$$90 < 200 \Rightarrow \text{train stops before station.} \quad (\text{R1})(\text{AG}) \quad 5$$

$$(c) \quad (i) \quad 0 = 30 - at \Rightarrow t = \frac{30}{a} \quad (A1)$$

$$(ii) \quad 30\left(\frac{30}{a}\right) - \frac{1}{2}(a)\left(\frac{30}{a}\right)^2 = 200 \quad (M1)(M1)$$

Note: Award (M1) for substituting $\frac{30}{a}$, (M1) for setting equal to 200.

$$\Rightarrow \frac{900}{a} - \frac{450}{a} = \frac{450}{a} = 200 \quad (A1)$$

$$\Rightarrow a = \frac{450}{200} = \frac{9}{4} = 2.25 \text{ m s}^{-2} \quad (A1) \quad 5$$

Note: Do not penalize lack of units in answers.

[15]

$$22. \quad (a) \quad (i) \quad v(0) = 50 - 50e^0 = 0 \quad (A1)$$

$$(ii) \quad v(10) = 50 - 50e^{-2} = 43.2 \quad (A1) \quad 2$$

$$(b) \quad (i) \quad a = \frac{dv}{dt} = -50(-0.2e^{-0.2t}) \quad (M1)$$

$$= 10e^{-0.2t} \quad (A1)$$

$$(ii) \quad a(0) = 10e^0 = 10 \quad (A1) \quad 3$$

$$(c) \quad (i) \quad t \rightarrow \infty \Rightarrow v \rightarrow 50 \quad (A1)$$

$$(ii) \quad t \rightarrow \infty \Rightarrow a \rightarrow 0 \quad (A1)$$

$$(iii) \quad \text{when } a = 0, v \text{ is constant at } 50 \quad (R1) \quad 3$$

$$(d) \quad (i) \quad y = \int v dt \quad (M1)$$

$$= 50t - \frac{e^{-0.2t}}{-0.2} + k \quad (A1)$$

$$= 50t + 250e^{-0.2t} + k \quad (AG)$$

$$(ii) \quad 0 = 50(0) + 250e^0 + k = 250 + k \quad (M1)$$

$$\Rightarrow k = -250 \quad (A1)$$

$$\begin{aligned}
\text{(iii)} \quad & \text{Solve } 250 = 50t + 250e^{-0.2t} - 250 & (M1) \\
& \Rightarrow 50t + 250e^{-0.2t} - 500 = 0 \\
& \Rightarrow t + 5e^{-0.2t} - 10 = 0 \\
& \Rightarrow t = 9.207 \text{ s} & (G2) \quad 7
\end{aligned}$$

[15]

$$\begin{aligned}
23. \quad (a) \quad & \text{When } t = 0, & (M1) \\
& h = 2 + 20 \times 0 - 5 \times 0^2 = 2 \quad h = 2 & (A1) \quad 2
\end{aligned}$$

$$\begin{aligned}
(b) \quad & \text{When } t = 1, & (M1) \\
& h = 2 + 20 \times 1 - 5 \times 1^2 & (A1) \\
& = 17 & (AG) \quad 2
\end{aligned}$$

$$\begin{aligned}
(c) \quad (i) \quad & h = 17 \Rightarrow 17 = 2 + 20t - 5t^2 & (M1) \\
(ii) \quad & 5t^2 - 20t + 15 = 0 & (M1) \\
& \Leftrightarrow 5(t^2 - 4t + 3) = 0 \\
& \Leftrightarrow (t - 3)(t - 1) = 0 & (M1) \\
& \text{Note: Award (M1) for factorizing or using the formula} \\
& \Leftrightarrow t = 3 \text{ or } 1 & (A1) \quad 4 \\
& \text{Note: Award (A1) for } t = 3
\end{aligned}$$

$$\begin{aligned}
(d) \quad (i) \quad & h = 2 + 20t - 5t^2 \\
& \Rightarrow \frac{dh}{dt} = 0 + 20 - 10t \\
& = 20 - 10t & (A1)(A1)
\end{aligned}$$

$$\begin{aligned}
(ii) \quad & t = 0 & (M0) \\
& \Rightarrow \frac{dh}{dt} = 20 - 10 \times 0 = 20 & (A1)
\end{aligned}$$

$$\begin{aligned}
(iii) \quad & \frac{dh}{dt} = 0 & (M1) \\
& \Leftrightarrow 20 - 10t = 0 \Leftrightarrow t = 2 & (A1)
\end{aligned}$$

$$\begin{aligned}
(iv) \quad & t = 2 & (M1) \\
& \Rightarrow h = 2 + 20 \times 2 - 5 \times 2^2 = 22 \Rightarrow h = 22 & (A1) \quad 7
\end{aligned}$$

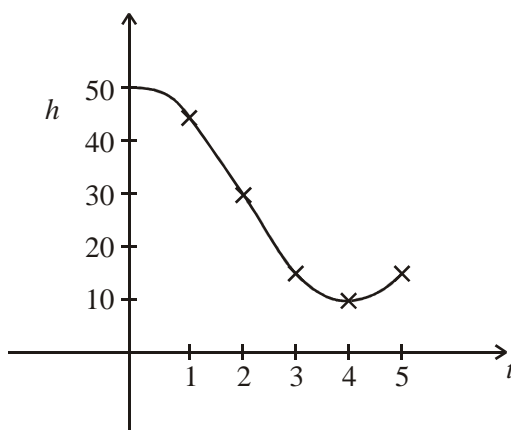
[15]

24. (a) (i) $t = 0 \text{ s} = 800$
 $t = 5 \text{ s} = 800 + 500 - 100 = 1200$ (M1)
distance in first 5 seconds = $1200 - 800$
 $= 400 \text{ m}$ (A1) 2
- (ii) $v = \frac{ds}{dt} = 100 - 8t$ (A1)
At $t = 5$, velocity = $100 - 40$ (M1)
 $= 60 \text{ m s}^{-1}$ (A1) 3
- (iii) Velocity = $36 \text{ m s}^{-1} \Rightarrow 100 - 8t = 36$ (M1)
 $t = 8$ seconds after touchdown. (A1) 2
- (iv) When $t = 8$, $s = 800 + 100(8) - 4(8)^2$ (M1)
 $= 800 + 800 - 256$ (A1)
 $= 1344 \text{ m}$ (A1) 3
- (b) If it touches down at P, it has $2000 - 1344 = 656 \text{ m}$ to stop. (M1)
To come to rest, $100 - 8t = 0 \Rightarrow t = 12.5 \text{ s}$ (M1)
Distance covered in $12.5 \text{ s} = 100(12.5) - 4(12.5)^2$ (M1)
 $= 1250 - 625$
 $= 625$ (A1)
Since $625 < 656$, it can stop safely. (R1) 5

[15]

25. (a) $t = 2 \Rightarrow h = 50 - 5(2^2) = 50 - 20$
 $= 30$ (A1)
- OR
- $h = 90 - 40(2) + 5(2^2)$
 $= 30$ (A1) 1

(b)



(A4) 4

Note: Award (A1) for marked scales on each axis, (A1) for each section of the curve.

(c) (i) $\frac{dh}{dt} = \frac{d}{dt} (50 - 5t^2)$
 $= 0 - 10t = -10t$

(A1)

(ii) $\frac{dh}{dt} = \frac{d}{dt} (90 - 40t + 5t^2)$
 $= 0 - 40 + 10t = -40 + 10t$

(A1) 2

(d) When $t = 2$ (i) $\frac{dh}{dt} = -10(2)$ or $\frac{dh}{dt} = -40 + 10 \times 2$
 $= -20$ $= -20$

(M1)

(A1) 2

(e) $\frac{dh}{dt} = 0 \Rightarrow -10t = 0 (0 \leq t \leq 2)$ or $-40 + 10t = 0 (2 \leq t \leq 5)$
 $t = 0$ or $t = 4$

(M1)

(A1)(A1) 3

(f) When $t = 4$
 $h = 90 - 40(4) + 5(4^2)$
 $= 90 - 160 + 80$
 $= 10$

(M1)

(M1)

(A1) 3