IB Questionbank Maths SL

SL Differentiation Kinematics

279 min
264 marks

1. The velocity v m s^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \leq t \leq 2$.

(a) Write down the velocity of the particle when $t = 0$.

(1)

When $t = k$, the acceleration is zero.

(b) (i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

(8)

(c) When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$.

Sketch a graph of v against t.

(4)

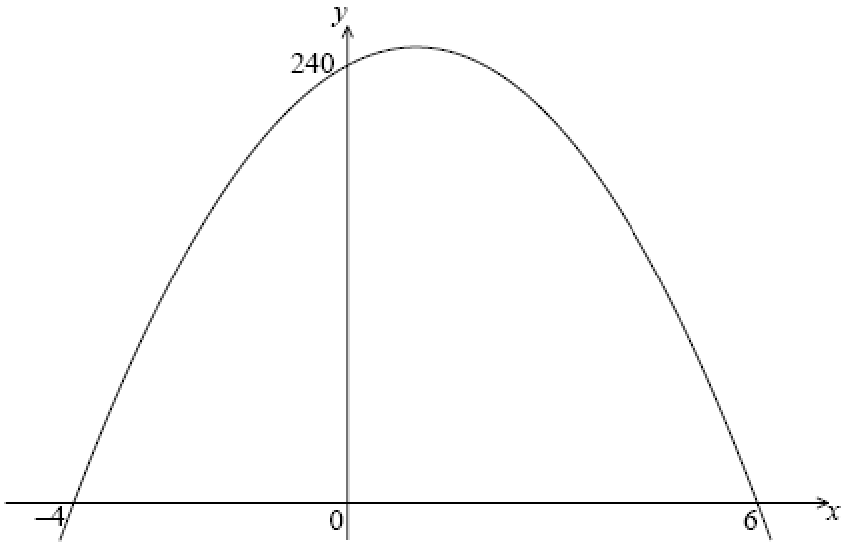
(d) Let d be the distance travelled by the particle for $0 \leq t \leq 1$.

(i) Write down an expression for d.

(ii) Represent d on your sketch.

(3)
(Total 16 marks)

2. The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at (–4, 0) and (6, 0) and the y-intercept is at (0, 240).

(a) Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.

(2)

(b) Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$.

(4)

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.

(2)

A particle moves along a straight line so that its velocity, v m s^{-1}, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$.

(d) (i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

(7)
(Total 15 marks)

3. The acceleration, a m s^{-2}, of a particle at time t seconds is given by

$$a = \frac{1}{t} + 3\sin 2t, \text{ for } t \ge 1.$$

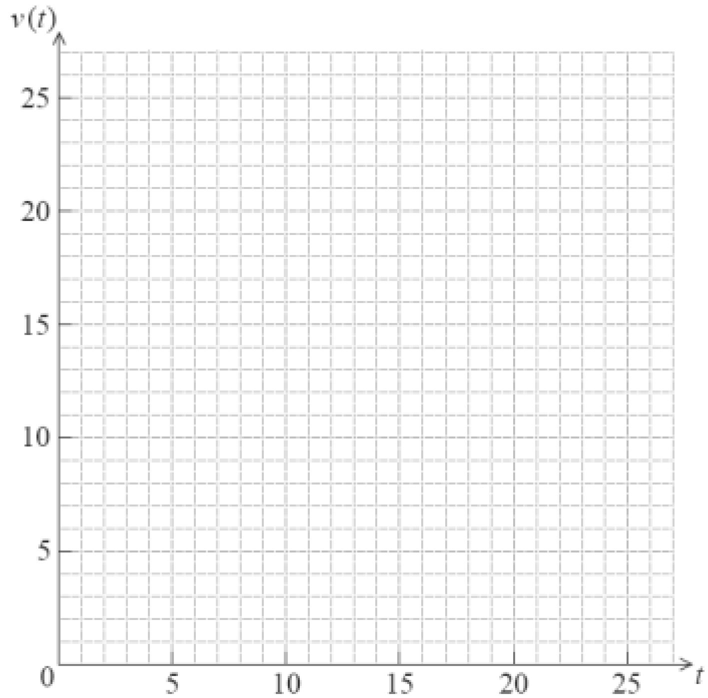
The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

(Total 7 marks)

4. The velocity v m s^{-1} of an object after t seconds is given by $v(t) = 15\sqrt{t} - 3t$, for $0 \le t \le 25$.

(a) On the grid below, sketch the graph of v, clearly indicating the maximum point.



(3)

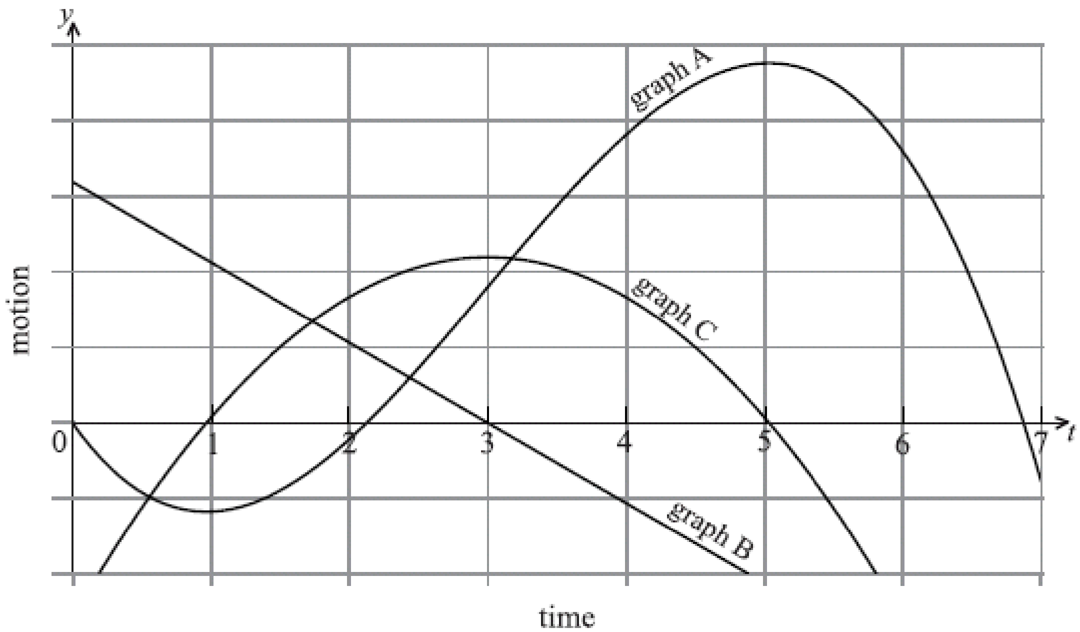
Let d be the distance travelled in the first nine seconds.

(b) (i) Write down an expression for d.

(ii) Hence, write down the value of d.

(4)
(Total 7 marks)

5. The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t.



(a) Complete the following table by noting which graph A, B or C corresponds to each function.

Function	Graph
displacement	
acceleration	

(4)

(b) Write down the value of t when the velocity is greatest.

(2)
(Total 6 marks)

6. *In this question s represents displacement in metres and t represents time in seconds.*

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t.

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t.

(6)

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

(ii) Hence show that $a = \frac{8}{5}$.

(6)

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station.

(5)
(Total 17 marks)

7. A particle moves along a straight line so that its velocity, v m s^{-1} at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of t.

(Total 7 marks)

8. Let $f : x \;\alpha\; \sin^3 x$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

(5)

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.

(2)

(c) Let $g(x) = \sqrt{3}\sin x(\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis.

(7)
(Total 14 marks)

9. The acceleration, a m s^{-2}, of a particle at time t seconds is given by $a = 2t + \cos t$.

(a) Find the acceleration of the particle at $t = 0$.

(2)

(b) Find the velocity, v, at time t, given that the initial velocity of the particle is 2 m s^{-1}.

(5)

(c) Find $\int_0^3 v\,\mathrm{d}t$, giving your answer in the form $p - q \cos 3$.

(7)

(d) What information does the answer to part (c) give about the motion of the particle?

(2)
(Total 16 marks)

10. The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}.

(2)

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

(4)
(Total 6 marks)

11. The velocity v of a particle at time t is given by $v = e^{-2t} + 12t$. The displacement of the particle at time t is s. Given that $s = 2$ when $t = 0$, express s in terms of t.

(Total 6 marks)

12. The velocity, $\boldsymbol{v}$, in m s^{-1} of a particle moving in a straight line is given by $v = e^{3t-2}$, where t is the time in seconds.

(a) Find the acceleration of the particle at t =1.

(b) At what value of t does the particle have a velocity of 22.3 m s^{-1}?

(c) Find the distance travelled in the first second.

(Total 6 marks)

13. The velocity, v m s^{-1}, of a moving object at time t seconds is given by $v = 4t^3 - 2t$. When $t = 2$, the displacement, s, of the object is 8 metres.

Find an expression for s in terms of t.

(Total 6 marks)

14. The displacement s metres at time t seconds is given by

$$s = 5 \cos 3t + t^2 + 10, \text{ for } t \geq 0.$$

(a) Write down the minimum value of s.

(b) Find the acceleration, a, at time t.

(c) Find the value of t when the **maximum** value of a first occurs.

(Total 6 marks)

15. The velocity v in m s^{-1} of a moving body at time t seconds is given by $v = e^{2t-1}$. When t = 0 5. the displacement of the body is 10 m. Find the displacement when t =1.

(Total 6 marks)

16. The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}.

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

(Total 6 marks)

17. A particle moves with a velocity v m s^{-1} given by $v = 25 - 4t^2$ where $t \geq 0$.

(a) The displacement, s metres, is 10 when t is 3. Find an expression for s in terms of t.

(6)

(b) Find t when s reaches its maximum value.

(3)

(c) The particle has a positive displacement for $m \leq t \leq n$. Find the value of m and the value of n.

(3)

(Total 12 marks)

18. A car starts by moving from a fixed point A. Its velocity, v m s^{-1} after t seconds is given by $v = 4t + 5 - 5e^{-t}$. Let d be the displacement from A when $t = 4$.

(a) Write down an integral which represents d.

(b) Calculate the value of d.

Working:

Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(Total 6 marks)

19. The displacement s metres of a car, t seconds after leaving a fixed point A, is given by

$$s = 10t - 0.5t^2.$$

(a) Calculate the velocity when $t = 0$.

(b) Calculate the value of t when the velocity is zero.

(c) Calculate the displacement of the car from A when the velocity is zero.

Working:

Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(c) ……………………………………………..

(Total 6 marks)

20. An aircraft lands on a runway. Its velocity v m s^{-1} at time t seconds after landing is given by the equation $v = 50 + 50e^{-0.5t}$, where $0 \leq t \leq 4$.

(a) Find the velocity of the aircraft

(i) when it lands;

(ii) when $t = 4$.

(4)

(b) Write down an integral which represents the distance travelled in the first four seconds.

(3)

(c) Calculate the distance travelled in the first four seconds.

(2)

After four seconds, the aircraft slows down (decelerates) **at a constant rate** and comes to rest when $t = 11$.

(d) **Sketch** a graph of velocity against time for $0 \le t \le 11$. Clearly label the axes and mark on the graph the point where $t = 4$.

(5)

(e) Find the constant rate at which the aircraft is slowing down (decelerating) between $t = 4$ and $t = 11$.

(2)

(f) Calculate the distance travelled by the aircraft between $t = 4$ and $t = 11$.

(2)
(Total 18 marks)

21. In this question, s represents displacement in metres, and t represents time in seconds.

(a) The velocity v m s^{-1} of a moving body may be written as $v = \dfrac{ds}{dt} = 30 - at$, where a is a constant. Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t.

(5)

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.

(i) Write down its velocity as it passes the signal.

(ii) Show that it will stop before reaching the station.

(5)

(c) Train 2 slows down so that it stops at the station. Its velocity is given by $v = \frac{ds}{dt} = 30 - at$, where a is a constant.

(i) Find, in terms of a, the time taken to stop.

(ii) Use your solutions to parts (a) and (c)(i) to find the value of a.

(5)
(Total 15 marks)

22. A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t},\ t \geq 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$;

(ii) $t = 10$.

(2)

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$?

(3)

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

(iii) Explain the relationship between the answers to parts (i) and (ii).

(3)

(d) Let y metres be the distance fallen after t seconds.

(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.

(ii) Given that $y = 0$ when $t = 0$, find the value of k.

(iii) Find the time required to fall 250 m, giving your answer correct to **four** significant figures.

(7)
(Total 15 marks)

23. A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by

$$h = 2 + 20t - 5t^2,\ t \geq 0$$

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(2)

(b) Show that the height of the ball after one second is 17 metres.

(2)

(c) At a later time the ball is **again** at a height of 17 metres.

(i) Write down an equation that t must satisfy when the ball is at a height of 17 metres.

(ii) Solve the equation **algebraically**.

(4)

(d) (i) Find $\frac{dh}{dt}$.

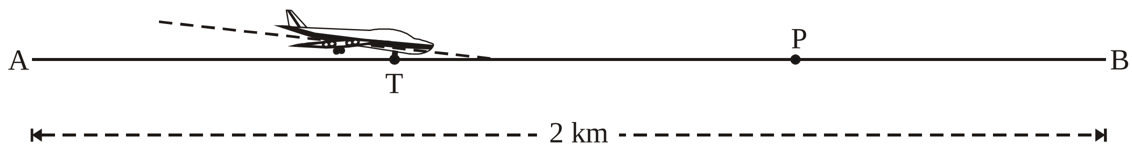
(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball.

(7)
(Total 15 marks)

24. The main runway at *Concordville* airport is 2 km long. An airplane, landing at *Concordville,* touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



Not to scale

As the airplane slows down, its distance, s, from A, is given by

$$s = c + 100t - 4t^2,$$

where t is the time in seconds after touchdown, and c metres is the distance of T from A.

(a) The airplane touches down 800 m from A, (*ie* c = 800).

(i) Find the distance travelled by the airplane in the first 5 seconds after touchdown. **(2)**

(ii) Write down an expression for the velocity of the airplane at time t seconds after touchdown, and hence find the velocity after 5 seconds. **(3)**

The airplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker; **(2)**

(iv) the distance from P to A. **(3)**

(b) Show that if the airplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway. **(5)**

(Total 15 marks)

25. A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height h metres of the rock-climber after t seconds of the fall is given by:

$$h = 50 - 5t^2, \qquad 0 \le t \le 2$$

$$h = 90 - 40t + 5t^2, \qquad 2 \le \mathrm{t} \le 5$$

(a) Find the height of the rock-climber when $t = 2$.

(1)

(b) Sketch a graph of h against t for $0 \le t \le 5$.

(4)

(c) Find $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ for:

(i) $0 \le t \le 2$

(ii) $2 \le t \le 5$

(2)

(d) Find the velocity of the rock-climber when $t = 2$.

(2)

(e) Find the times when the velocity of the rock-climber is zero.

(3)

(f) Find the minimum height of the rock-climber for $0 \le t \le 5$.

(3)
(Total 15 marks)