Binomial Distribution Answers

0 min
0 marks

1. (a) $E(X) = 2$ A1 N1

(b) evidence of appropriate approach involving binomial (M1)

e.g. $\binom{10}{3}(0.2)^3$, $(0.2)^3(0.8)^7$, $X \sim B(10, 0.2)$

$P(X = 3) = 0.201$ A1 N2

(c) **METHOD 1**

$P(X \le 3) = 0.10737 + 0.26844 + 0.30199 + 0.20133$ (= 0.87912...) (A1)
evidence of using the complement (seen anywhere) (M1)
e.g. 1 – any probability, $P(X > 3) = 1 - P(X \le 3)$

$P(X > 3) = 0.121$ A1 N2

METHOD 2

recognizing that $P(X > 3) = P(X \ge 4)$ (M1)
e.g. summing probabilities from $X = 4$ to $X = 10$

correct expression or values (A1)

e.g. $\sum_{r=4}^{10}\binom{10}{r}(0.2)^{10-r}(0.8)^r$

0.08808 + 0.02642 + 0.005505 + 0.000786 + 0.0000737 + 0.000004 + 0.0000001

$P(X > 3) = 0.121$ A1 N2

[6]

2. (a) evidence of binomial distribution (may be seen in parts (b) or (c)) (M1)

e.g. np, 100×0.04

mean = 4 A1 N2

(b) $P(X = 6) = \binom{100}{6}(0.04)^6(0.96)^{94}$ (A1)

$= 0.105$ A1 N2

(c) for evidence of appropriate approach (M1)

e.g. complement, $1 - P(X = 0)$

$P(X = 0) = (0.96)^{100} = 0.01687...$ (A1)

$P(X \geq 1) = 0.983$ A1 N2

[7]

3. (a) evidence of using binomial probability (M1)

e.g. $P(X = 2) = \binom{7}{2}(0.18)^2(0.82)^5$

$P(X = 2) = 0.252$ A1 N2

(b) **METHOD 1**

evidence of using the complement M1

e.g. $1 - (P(X \leq 1))$

$P(X \leq 1) = 0.632$ (A1)

$P(X \geq 2) = 0.368$ A1 N2

METHOD 2

evidence of attempting to sum probabilities M1

e.g. P(2 heads) + P(3 heads) + ... + P(7 heads), 0.252 + 0.0923 + …

correct values for each probability (A1)

e.g. 0.252 + 0.0923 + 0.0203 + 0.00267 + 0.0002 + 0.0000061

$P(X \geq 2) = 0.368$ A1 N2

[5]

4. (a) (i) Attempt to find $P(3H) = \left(\frac{1}{3}\right)^3$ (M1)

$= \frac{1}{27}$ A1 N2

(ii) Attempt to find P(2H, 1T) (M1)

$= 3\left(\frac{1}{3}\right)^2 \frac{2}{3}$ A1

$= \frac{2}{9}$ A1 N2

(b) (i) Evidence of using np $\left(\frac{1}{3}\times 12\right)$ (M1)

expected number of heads = 4 A1 N2

(ii) 4 heads, so 8 tails (A1)

E(winnings) = 4 × 10 – 8 × 6 (= 40 – 48) (M1)

= –$ 8 A1 N1

[10]

5. (a) $X \sim B(100, 0.02)$

$E(X) = 100 \times 0.02 = 2$ A1 N1

(b) $P(X = 3) = \binom{100}{3}(0.02)^3\,(0.98)^{97}$ (M1)

= 0.182 A1 N2

(c) **METHOD 1**

$P(X > 1) = 1 - P(X \le 1) = 1 - (P(X = 0) + P(X = 1))$ M1

$= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ (M1)

= 0.597 A1 N2

METHOD 2

$P(X > 1) = 1 - P(X \le 1)$ (M1)

= 1 – 0.40327 (A1)

= 0.597 A1 N2

Note: *Award marks as follows for finding P(X ≥ 1), if working shown.*

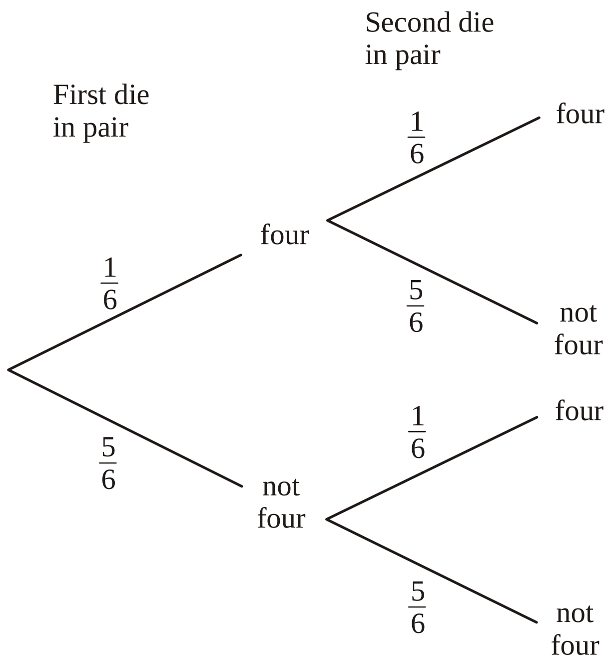
$P(X \ge 1)$ A0

$= 1 - P(X \le 2) = 1 - 0.67668$ M1(FT)

= 0.323 A1(FT) N0

[6]

6. (a)



A1A1A1 N3

***Note**: Award A1 for **each pair** of complementary probabilities.*

(b) $\mathrm{P}(E) = \frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6} \quad \left(=\frac{5}{36}+\frac{5}{36}\right)$ (A2)

$= \frac{10}{36} \quad \left(=\frac{5}{18} \text{ or } 0.278\right)$ A1 N3

(c) Evidence of recognizing the binomial distribution (M1)

eg $X \sim \mathrm{B}\left(5, \frac{5}{18}\right)$ or $p=\frac{5}{18}, q=\frac{13}{18}$

$\mathrm{P}(X = 3) = \binom{5}{3}\left(\frac{5}{18}\right)^3\left(\frac{13}{18}\right)^2$ (or other evidence of correct setup) (A1)

$= 0.112$ A1 N3

(d) **METHOD 1**

Evidence of using the complement — M1

eg $P(X \geq 3) = 1 - P(X \leq 2)$

Correct value $1 - 0.865$ — (A1)

$= 0.135$ — A1 N2

METHOD 2

Evidence of adding correct probabilities — M1

eg $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

Correct values $0.1118 + 0.02150 + 0.001654$ — (A1)

$= 0.135$ — A1 N2

[12]

7. ***Note:*** *Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.*

$W \sim N(2.5, 0.3^2)$

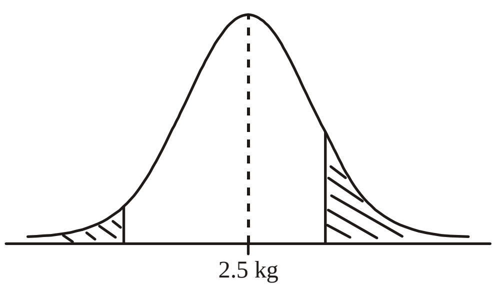
(a) (i) $z = -1.67$ (accept 1.67) — (A1)

$P(W < 2) = 0.0478$ (accept answers between 0.0475 and 0.0485) — A1 N2

(ii) $z = 1$ — (A1)

$P(W > 2.8) = 0.159$ — A1 N2

(iii)



A1A1 N2

Note: *Award A1 for a vertical line to left of mean and shading to left, A1 for vertical line to right of mean and shading to right.*

(iv) Evidence of appropriate calculation — M1

eg $1 - (0.047790 + 0.15866)$, $0.8413 - 0.0478$

$P = 0.7936$ — AG N0

Note: *The final value may vary depending on what level of accuracy is used.*

Accept their value in subsequent parts.

(b) (i) $X \sim B(10, 0.7935...)$

Evidence of calculation M1

eg $P(X = 10) = (0.7935...)^{10}$

$P(X = 10) = 0.0990$ (3 sf) A1 N1

(ii) **METHOD 1**

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

$P(X \leq 6) = 0.1325...$ (or $P(X = 1) + ... + P(X = 6)$) (A1)

evidence of using the complement (M1)

eg $P(X \geq 7) = 1 - P(X \leq 6)$, $P(X \geq 7) = 1 - P(X < 7)$

$P(X \geq 7) = 0.867$ A1 N3

METHOD 2

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

For adding terms from $P(X = 7)$ to $P(X = 10)$ (M1)

$P(X \geq 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030$ (A1)

$= 0.867$ A1 N3

[13]

8. METHOD 1 Use of the GDC

(a) Evidence of using the binomial facility, M1

that is set up with $P = \frac{1}{2}$ and $n = 5$.

$P(X = 3) = 0.3125 \quad \left(0.313, \frac{5}{16}\right)$ A2 N2

(b) Evidence of set up, with $1 - P(X = 0)$ M1

$= 0.969 \quad \left(= \frac{31}{32}\right)$ A2 N2

METHOD 2 Use of the formula

(a) Evidence of binomial formula (M1)

$P(X = 3) = \binom{5}{3}\left(\frac{1}{2}\right)^5$ A1

$= \frac{5}{16} \quad (= 0.313)$ A1 N2

(b) **METHOD 1**

P(at least one head) = $1 - P(X = 0)$ (M1)

$= 1 - \left(\frac{1}{2}\right)^5$ A1

$= \frac{31}{32} \ (= 0.969)$ A1 N2

METHOD 2

P(at least one head) = $P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$ (M1)

$= 0.15625 + 0.3125 + 0.3125 + 0.15625 + 0.03125$ A1

$= 0.969$ A1 N2

[6]

9. (a) $X \sim B(100, 0.02)$
$E(X) = 100 \times 0.02 = 2$ A1 1

(b) $P(X = 3) = \binom{100}{3}(0.02)^3(0.98)^{97}$ (M1)

$= 0.182$ A1 2

(c) **METHOD 1**

$P(X > 1) = 1 - P(X \le 1) = 1 - (P(X = 0) + P(X = 1)$ M1
$= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ (M1)
$= 0.597$ A1 2

METHOD 2

$P(X > 1) = 1 - P(X \le 1)$ (M1)
$= 1 - 0.40327$ (A1)
$= 0.597$ A1 2

Note*: Award marks as follows for finding P(X > 1), if working shown.*

$P(X \ge 1)$ A0
$= 1 - P(X<2) = 1 - 0.67668$ M1(ft)
$= 0.323$ A1(ft) 2

[6]

10. $p(\text{Red}) = \dfrac{35}{40} = \dfrac{7}{8}$ $p(\text{Black}) = \dfrac{5}{40} = \dfrac{1}{8}$

(a) (i) $p(\text{one black}) = \binom{8}{1}\left(\dfrac{1}{8}\right)^1\left(\dfrac{7}{8}\right)^7$ (M1)(A1)

$= 0.393$ to 3 sf (A1) 3

(ii) $p(\text{at least one black}) = 1 - p(\text{none})$ (M1)

$= 1 - \binom{8}{0}\left(\dfrac{1}{8}\right)^0\left(\dfrac{7}{8}\right)^8$ (A1)

$= 1 - 0.344$

$= 0.656$ (A1) 3

(b) 400 draws: expected number of blacks $= \dfrac{400}{8}$ (M1)

$= 50$ (A1) 2

[8]

11. (a) $p\ (4\text{ heads}) = \binom{8}{4}\left(\dfrac{1}{2}\right)^4\left(\dfrac{1}{2}\right)^{8-4}$ (M1)

$= \dfrac{8\times7\times6\times5}{1\times2\times3\times4}\times\left(\dfrac{1}{2}\right)^8$

$= \dfrac{70}{256} \cong 0.273$ (3 sf) (A1) 2

(b) $p\ (3\text{ heads}) = \binom{8}{3}\left(\dfrac{1}{2}\right)^3\left(\dfrac{1}{2}\right)^{8-3} = \dfrac{8\times7\times6}{1\times2\times3}\times\left(\dfrac{1}{2}\right)^8$

$= \dfrac{56}{256} \cong 0.219$ (3 sf) (A1) 1

(c) p (5 heads) = p (3 heads) (by symmetry) (M1)

p (3 or 4 or 5 heads) = p (4) + 2p (3) (M1)

$= \dfrac{70+2\times56}{256} = \dfrac{182}{256}$

≈ 0.711 (3 sf) (A1) 3

[6]