SEQUENCES AND SERIES – PRACTICE MARKING SCHEME

1. (a) $a = 100 \quad d = 25$

$T_{17} = 100 + (17 - 1)\times 25$ (M1)

$= \$500$ (A1) (C2)

(b) $S_n = \frac{n}{2}(a + l)$

$S_{17} = \frac{17}{2}(100 + 500)$ (M1)

$= \$5100$ (A1)

Note: *Allow follow through from candidate's answer for T_{17}, which is l*

OR

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$S_{17} = \frac{17}{2}\{2\times 100 + (17 - 1)\times 25\}$ (M1)

$= \$5100$ (A1) (C2)

OR

Table

[4]

2. (a) For obtaining an equation in r^2, can be implied (M1)

$28 = 7r^2$ (A1)

$r = 2$ (A1) (C3)

(b) For using their value of r in the GP sum formula (M1)

For obtaining 114681 (accept fewer s.f. up to 115000) (M1) (A1) (C3)

[6]

3. (a) Choice A = $100 \times 12 = \$1200$ (A1)

Choice B = $1100\left(1+\frac{12}{1200}\right)^{12} = \$1239.51$ (M1) (A1)

Choice C = $75 + 80 + \ldots$ (M1)

$= \frac{12}{2}\{2\times 75 + 11\times 5\} = \$1230$ (A1)

Choice D = $80 + 80 \times 1.05 + 80 \times 1.05^2 + \ldots$ (M1)

$= \frac{80(1.05^{12}-1)}{(1.05-1)} = \$1273.37$ (M1) (A1)

Note: *Award method marks if candidate works out each amount. But the answer has to be accurate to receive the mark of (A1).*

8

(b) Option D because the total allowance is the highest (C1) (R1) 2

(c) $1200\left(1+\frac{r}{100}\right)^2 = 1452$ (A1)

$\left(1+\frac{r}{100}\right)^2 = \frac{1452}{1200} = 1.21$ (A1)

$\left(1+\frac{r}{100}\right) = \sqrt{1.21} = 1.1$ (A1)

$\frac{r}{100} = 0.1$

$r = 10\%$ (A1) 4

[14]

4. (a) $100 + 15 \times 10$ (M1)

$= 250$ (A1)

OR

250 (using table function of the GDC) (G2) (C2)

(b) $100(1.08)^{10}$ (M1)

$= 215.89$ (A1)

OR

215.89 (using table function of the GDC) (G2) (C2)

(c) $100 + 15x = 100(1.08)^x$ (M1)

After 16 years (A1)

Note: *Candidate can use trial and error so not necessary to see the first line to award (A2).*

OR

16 years (using table function of the GDC). (G2) (C2)

[6]

5. (a) $u_1 + 3d = 12$ (A1)(A1)

$u_1 + 9d = 42$ (A1)(A1) (C4)

Note: *Award (A1) for left hand side correct, (A1) for right hand side correct.*

(b) $6d = 30$ (A1)

$d = 5$ (A1)

$u_1 = -3$ (M1)(A1) (C4)

Note: *Follow through (ft) from candidate's equations.*

[8]

6. (a) $u_1 = -16$, $u_1 + 10d = 39$ (M1)

$-16 + 10d = 39$ (A1)

Note: *Award (M1) for correct formula, (A1) for correct numbers.*

$10d = 39 + 16 = 55$ (A1)

$d = 5.5$ (A1) (C4)

(b) $u_1 r^2 = 12$ (M1)

$u_1 r^4 = \frac{16}{3}$ (A1)

Note: *Award (M1) for correct formula, (A1) for correct numbers.*

$r^2 = \frac{\left(\frac{16}{3}\right)}{12} = \frac{16}{36} = \frac{4}{9}$ (M1)

$r = \frac{2}{3}$ (A1) (C4)

[8]

7. 4^{th} term $= a + 3d$

8^{th} term $= a + 7d$

20^{th} term $= a + 19d$ (M2)

Note: *Award (M1) for each correct answer up to a maximum of [2 marks].*

$a + 7d = 2(a + 3d)$

$a + 19d = 4000$ (M1)

Note: *Award (M1) for any one correct equation.*

$d = 200$ (A1)

[4]

8. (a) (i) $a = \$250$ (A1)
$d = \$200$ (A1)
$T_{10} = 250 + 9 \times 200$
$= 2050$ (A1)

Note: *Award the marks for the values of 'a' and 'd' if they are correctly substituted into the formula without being explicitly stated.*

(ii) $a = \$10$ (A1)
$r = 2$ (A1)
$T_0 = 10 \times 2^9$
$= 5120$ (A1) 6

Note: *Award the marks for the values of 'a' and 'r' if they are correctly substituted into the formula without being explicitly stated.*

(b) $S_{10} = \frac{10}{2}(250 + 2050)$ (M1)
$= 11500$ (A1)

OR

$S_{10} = \frac{10}{2}\{2 \times 250 + (10 - 1) \times 200\}$ (M1)
$= 11500$ (A1) 2

(c) Option One: \$10000 (A1)
Option Two: \$11500
Option Three: $S_{10} = \frac{10(2^{10} - 1)}{2 - 1}$ (M1)
$= 10\,230$ (A1)
Therefore, Option Two would be best. (R1) 4

[12]

9. (a) (i) 2 minutes + 6 seconds + 6 seconds = 2 minutes 12 seconds (M1)(A1)

(ii) $2(1.05)^2 = 2.205$ (M1)(AG) 3

(b) $2 + 2 \times 1.05 + 2 \times 1.05^2 + ... + 2 \times 1.05^9$
$= \frac{2(1.05^{10} - 1)}{(1.05 - 1)} = 25.2$ minutes (or 25 minutes 12 seconds) (M1)(A1)(A1) 3

[6]

10. (a) The sixth number is 22 (C1)

(b) $u_{200} = 2 + 199 \times 4$ (M1)(A1)(A1)
$= 798$ (A1) (C4)

Note: *Award (A1) for a = 2 stated or used, (A1) for d = 4 stated or used.*

(c) $S_{90} = \frac{90}{2}(2 \times 2 + 89 \times 4)$ or $\frac{90}{2}(2 + 358)$ (M1)(A1)
$= 16\,200$ (A1) (C3)

[8]

11. (a) $u_n = 2(0.9)^7 = 0.957$ m (M1)(A1) 2

Note: *Award (M1) for substitution into formula, list or suitable diagram.*

(b) $S_n = \dfrac{2(1-(0.9)^5)}{1-(0.9)} = 8.19$ m (M1)(M1)(A1)

Note: *Award (M1) for substitution into formula, list or suitable diagram.*

Total distance travelled = 2 × 8.19 = 16.4 m. (A1) 4

[6]

12. (a) $u_1 = 59$ $u_2 = 55$ (A1)(A1) 2

(b) $63 - 4n = -13$ $-4n = -76$ $n = 19$ (M1)(A1) or (G2) 2

(c) $63 - 4k + 63 - 4(k + 1) = 34$
$-8k = -88$ $k = 11$ (M1)(M1)(A1) 3

Note: *Award (M1) for the terms 15 and 19.*

[7]

13. (a) $4n - 3$ (A1)

(b) 397 (A1)

(c) $S_{100} = \dfrac{100}{2}[(2 \times 1) + (99 \times 4)]$ or $50(1 + 397)$ (M1)
= 19 900 (A1)

[4]

14. (a) Let the population at the end of 1999 be x.
$\dfrac{44100}{x} = \dfrac{x}{40000}$
$x = 42\,000$ (A1)

(b) $r = \dfrac{44100}{42000}$ (M1)
$r = 1.05$
$u_n = u_1 r^{n-1}$ (M1)
$44\,100 = u_1(1.05)^6$
$u_1 = 32\,908$ (or 32 900 to 3 s.f.) (A1)

[4]

15. (a) $u_6 = u_1 + 5d = 24$

$u_1 + 5 \times 8 = 24$ (M1)(A1)

$u_1 = 24 - 40$

$= -16$ (A1) (C3)

(b) $S_n = \frac{n}{2}(2 \times -16 + (n-1)8)$ (M1)(A1)

$600 = \frac{n}{2}(-32 + 8n - 8)$ (A1)

$1200 = -40n + 8n^2$

$150 = -5n + n^2$ (A1)

$(n - 15)(n + 10) = 0$

$n = 15$ or $\cancel{-10}$ (A1) (C5)

Note: *Not all the steps of working out need to be shown.*

[8]

16. (a) $r = \frac{2500}{2000}$ (M1)

$= 1.25$ (A1) (C2)

(b) $S_6 = \frac{2000(1.25^6 - 1)}{1.25 - 1}$ (M1)

Note: *Award (M1) for any appropriate method*

$= 22517.57813.....$

$= 22518$ (to the nearest dollar) (A1) (C2)

[4]