SEQUENCES AND SERIES - PRACTICE

1. A woman deposits \$100 into her son's savings account on his first birthday. On his second birthday she deposits \$125, \$150 on his third birthday, and so on.

(a) How much money would she deposit into her son's account on his 17th birthday?

(b) How much in total would she have deposited after her son's 17th birthday?

Working:

Answers:

(a) ……..

(b) ……..

(Total 4 marks)

2. A geometric sequence has all its terms positive. The first term is 7 and the third term is 28.

(a) Find the common ratio.

(b) Find the sum of the first 14 terms.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

3. On Vera's 18^{th} birthday she was given an allowance from her parents. She was given the following choices.

Choice A $100 every month of the year.
Choice B A fixed amount of $1100 at the beginning of the year, to be invested at an interest rate of 12% per annum, compounded monthly.
Choice C $75 the first month and an increase of $5 every month thereafter.
Choice D $80 the first month and an increase of 5% every month.

(a) Assuming that Vera does not spend any of her allowance during the year, calculate, for each of the choices, how much money she would have at the end of the year.

(8)

(b) Which of the choices do you think that Vera should choose? Give a reason for your answer.

(2)

(c) On her 19^{th} birthday Vera invests $1200 in a bank that pays interest at r% per annum compounded annually. Vera would like to buy a scooter costing $1452 on her 21^{st} birthday. What rate will the bank have to offer her to enable her to buy the scooter?

(4)
(Total 14 marks)

4. Two students Ann and Ben play a game. Each time Ann passes GO she receives $15. Each time Ben passes GO he receives 8% of the amount he already has. Both students start with $100.

(a) How much money will Ann have after she has passed GO 10 times?

(b) How much money will Ben have after he passes GO 10 times?

(c) How many times will the students have to pass GO for Ben to have more money than Ann?

(Total 6 marks)

5. The fourth term of an arithmetic sequence is 12 and the tenth term is 42.

(a) Given that the first term is u_1 and the common difference is d, write down two equations in u_1 and d that satisfy this information.

(b) Solve the equations to find the values of u_1 and d.

Working:

Answers:

(a) .....

.....

(b) u_1 = ...

d = ...

(Total 8 marks)

6. (a) The first term of an arithmetic sequence is –16 and the eleventh term is 39.
Calculate the value of the common difference.

(b) The third term of a geometric sequence is 12 and the fifth term is $\frac{16}{3}$.

All the terms in the sequence are positive.
Calculate the value of the common ratio.

Working:

Answers:

(a) .....

(b) .....

(Total 8 marks)

7. Mr Jones decides to increase the amount of money he spends on food by *d* GBP every year. In the first year he spends *a* GBP. In the 8th year he spends twice as much as in the 4th year. In the 20th year he spends 4000 GBP.

Find the value of *d*.

Working:

Answer:

…….....

(Total 4 marks)

8. A National Lottery is offering prizes in a new competition. The winner may choose one of the following.

Option one: \$1000 each week for 10 weeks.

Option two: \$250 in the first week, \$450 in the second week, \$650 in the third week, increasing by \$200 each week for a total of 10 weeks.

Option three: \$10 in the first week, \$20 in the second week, \$40 in the third week continuing to double for a total of 10 weeks.

(a) Calculate the amount you receive in the tenth week, if you select

(i) **option two**;

(ii) **option three**.

(6)

(b) What is the total amount you receive if you select **option two**?

(2)

(c) Which option has the greatest total value? Justify your answer by showing all appropriate calculations.

(4)

(Total 12 marks)

9. Ann and John go to a swimming pool.
They both swim the first length of the pool in 2 minutes.
The time John takes to swim a length is 6 seconds more than he took to swim the previous length.
The time Ann takes to swim a length is 1.05 times that she took to swim the previous length.

(a) (i) Find the time John takes to swim the third length.

(ii) Show that Ann takes 2.205 minutes to swim the third length.

(3)

(b) Find the time taken for Ann to swim a total of 10 lengths of the pool.

(3)
(Total 6 marks)

10. The first five terms of an arithmetic sequence are shown below.

2, 6, 10, 14, 18

(a) Write down the sixth number in the sequence.

(b) Calculate the 200^{th} term.

(c) Calculate the sum of the first 90 terms of the sequence.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 8 marks)

11. A basketball is dropped vertically. It reaches a height of 2 m on the first bounce. The height of each subsequent bounce is 90% of the previous bounce.

(a) What height does it reach on the 8th bounce?

(2)

(b) What is the total vertical distance travelled by the ball between the first and sixth time the ball hits the ground?

(4)
(Total 6 marks)

12. The n^{th} term of an arithmetic sequence is given by $u_n = 63 - 4n$.

(a) Calculate the values of the first two terms of this sequence.

(2)

(b) Which term of the sequence is –13?

(2)

(c) Two consecutive terms of this sequence, u_k and u_{k+1}, have a sum of 34. Find k.

(3)

(Total 7 marks)

13. The first four terms of an arithmetic sequence are shown below.

1, 5, 9, 13,.....

(a) Write down the n^{th} term of the sequence.

(b) Calculate the 100^{th} term of the sequence.

(c) Find the sum of the first 100 terms of the sequence.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 4 marks)

14. The population of Bangor is growing each year. At the end of 1996, the population was 40 000. At the end of 1998, the population was 44 100. Assuming that these annual figures follow a geometric progression, calculate

(a) the population of Bangor at the end of 1997;

(b) the population of Bangor at the end of 1992.

Working:

Answers:

(a) ……..

(b) ……..

(Total 4 marks)

15. The sixth term of an arithmetic sequence is 24. The common difference is 8.

(a) Calculate the first term of the sequence.

The sum of the first n terms is 600.

(b) Calculate the value of n.

Working:

Answers:

(a) .....

(b) .....

(Total 8 marks)

16. The tuition fees for the first three years of high school are given in the table below.

Year	Tuition fees (in dollars)
1	2000
2	2500
3	3125

These tuition fees form a geometric sequence.

(a) Find the common ratio, r, for this sequence.

(b) If fees continue to rise at the same rate, calculate (to the nearest dollar) the total cost of tuition fees for the first six years of high school.

Working:

Answers:

(a) ……..

(b) .....

(Total 4 marks)