Sequences 1

1)

$$1 + 2 + 3 + 4 + 5 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$$

(a) (i) Show that this formula is true for the sum of the first 8 natural numbers. [2]

(ii) Find the sum of the first 400 natural numbers. [1]

(b) (i) Show that $2 + 4 + 6 + 8 + \ldots\ldots\ldots\ldots\ldots + 2n = n(n + 1)$. [1]

(ii) Find the sum of the first 200 even numbers. [1]

(iii) Find the sum of the first 200 odd numbers. [1]

(c) (i) Use the formula at the beginning of the question to find the sum of the first $2n$ natural numbers. [1]
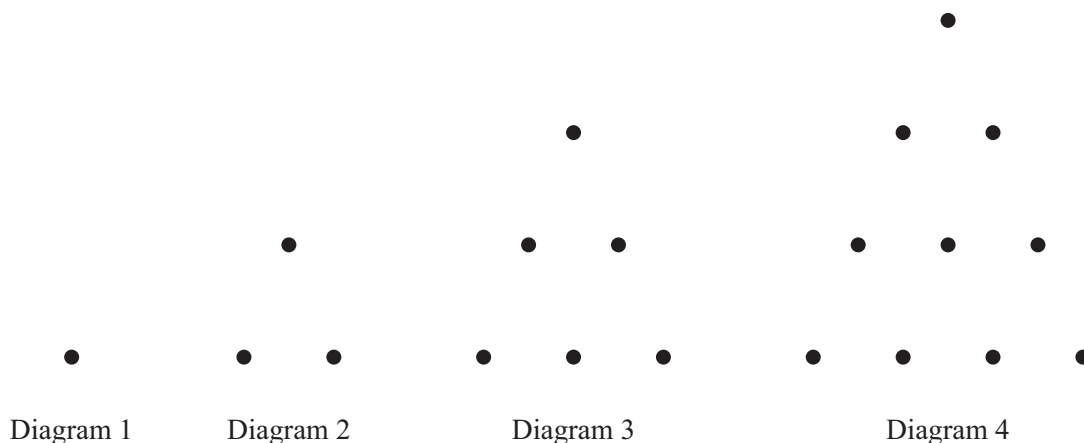
(ii) Find a formula, in its simplest form, for

$$1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots\ldots\ldots + (2n - 1).$$

Show your working. [2]

Sequences 1

2)

Diagram 1   Diagram 2   Diagram 3   Diagram 4

The first four terms in a sequence are 1, 3, 6 and 10.
They are shown by the number of dots in the four diagrams above.

(a) Write down the next four terms in the sequence.

Answer(a) .... , ... , ... , ...   [2]

(b) **(i)** The sum of the two consecutive terms 3 and 6 is 9.
The sum of the two consecutive terms 6 and 10 is 16.

Complete the following statements using different pairs of terms.

The sum of the two consecutive terms .... and .... is ....

The sum of the two consecutive terms .... and .... is ....   [1]

(ii) What special name is given to these sums?

Answer(b)(ii) ....   [1]
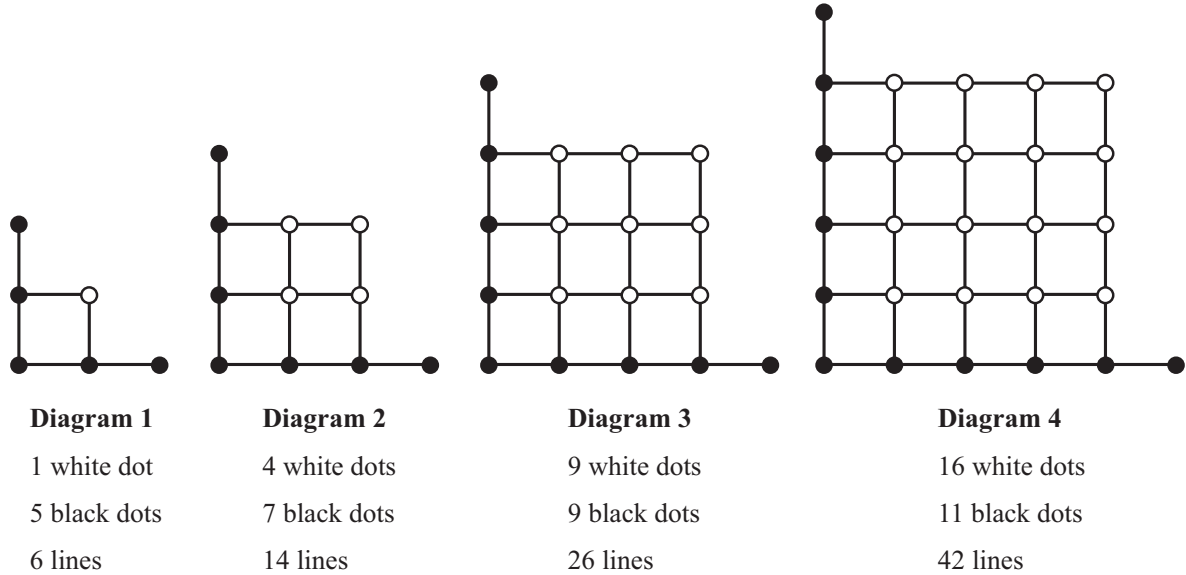
(c) **(i)** The formula for the nth term in the sequence 1, 3, 6, 10... is $\dfrac{n(n+1)}{k}$,

where k is an integer.

Find the value of k.

Answer(c)(i) $k =$ ....   [1]

3)

Diagram 1	**Diagram 2**	**Diagram 3**	**Diagram 4**
1 white dot	4 white dots	9 white dots	16 white dots
5 black dots	7 black dots	9 black dots	11 black dots
6 lines	14 lines	26 lines	42 lines

The four diagrams above are the first four of a pattern.

(a) Diagram 5 has been started below.
Complete this diagram and write down the information about the numbers of dots and lines.
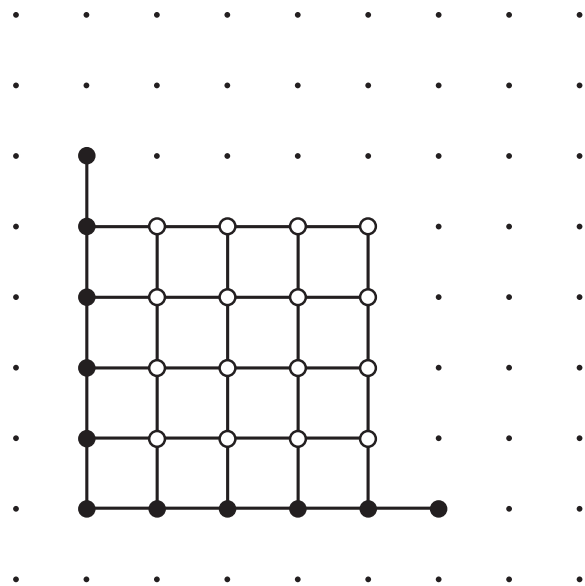

Diagram 5

.... white dots

.... black dots

.... lines

[4]

Sequences 1

(b) Complete the information about the number of dots and lines in Diagram 8.

Answer(b) .... white dots

.... black dots

.... lines   [3]

(c) Complete the information about the number of dots in Diagram *n*.
Give your answers in terms of *n*.

Answer(c) .... white dots

.... black dots   [2]

(d) The number of lines in diagram *n* is $k(n^2 + n + 1)$.

Find

(i) the value of *k*,

Answer(d)(i) $k =$ ....   [1]

(ii) the number of lines in Diagram 100.

Answer(d)(ii)   ....   [1]
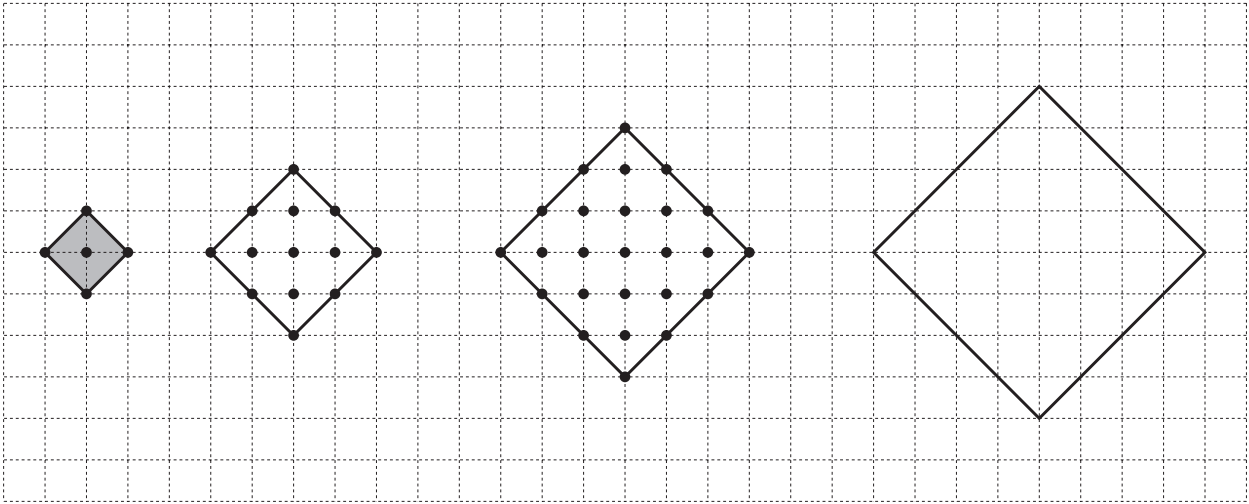
4)

Diagram 1   Diagram 2   Diagram 3   Diagram 4

The diagrams show squares and dots on a grid.

Some dots are on the sides of each square and other dots are inside each square.

The area of the square (shaded) in Diagram 1 is 1 unit^2.

(a) Complete Diagram 4 by marking all the dots.   [1]

(b) Complete the columns in the table below for Diagrams 4, 5 and n.

Diagram	1	2	3	4	5	-------	n
Number of units of area	1	4	9			-------	
Number of dots inside the square	1	5	13			-------	$(n-1)^2 + n^2$
Number of dots on the sides of the square	4	8	12			-------	
Total number of dots	5	13	25			-------	

[7]

Sequences 1

5) The first and the nth terms of sequences A, B and C are shown in the table below.

(a) Complete the table for each sequence.

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence A	1					n^3
Sequence B	4					$4n$
Sequence C	4					$(n+1)^2$

[5]

(b) Find

   (i) the 8th term of sequence A,

Answer(b)(i) .... [1]

   (ii) the 12th term of sequence C.

Answer(b)(ii) .... [1]

(c) **(i)** Which term in sequence A is equal to 15 625?

Answer(c)(i) .... [1]

   (ii) Which term in sequence C is equal to 10 000?

Answer(c)(ii) .... [1]

(d) The first four terms of sequences D and E are shown in the table below.

Use the results from **part (a)** to find the 5th and the nth terms of the sequences D and E.

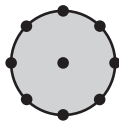|	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence D	5	16	39	80		
Sequence E	0	1	4	9		

[4]

6)

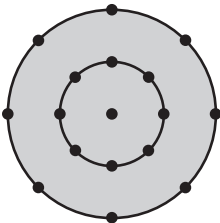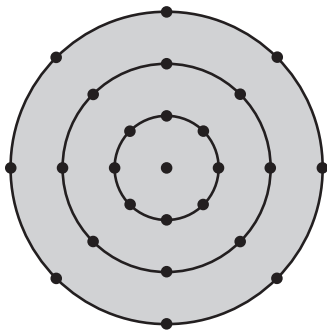Diagram 1   Diagram 2   Diagram 3

The diagrams show a sequence of dots and circles.
Each diagram has one dot at the centre and 8 dots on each circle.
The radius of the first circle is 1 unit.
The radius of each new circle is 1 unit greater than the radius of the previous circle.

(a) Complete the table for diagrams 4 and 5.

Diagram	1	2	3	4	5
Number of dots	9	17	25		
Area of the largest circle	π	4π	9π		
Total length of the circumferences of the circles	2π	6π	12π		

[4]

(b) (i) Write down, in terms of n, the number of dots in diagram n.

Answer(b)(i) .... [2]

(ii) Find n, when the number of dots in diagram n is 1097.

Answer(b)(ii) $n =$ .... [2]

(c) Write down, in terms of n and π, the area of the largest circle in

(i) diagram n,

Answer(c)(i) .... [1]

(ii) diagram $3n$.

Answer(c)(ii) .... [1]

(d) Find, in terms of n and π, the total length of the circumferences of the circles in diagram n.

Answer(d) .... [2]

7) Consecutive integers are set out in rows in a grid.

(a) This grid has 5 columns.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35

a		b
	n	
c		d

The shape drawn encloses five numbers 7, 9, 13, 17 and 19. This is the $n = 13$ shape.

In this shape, $a = 7$, $b = 9$, $c = 17$ and $d = 19$.

(i) Calculate $bc - ad$ for the $n = 13$ shape.

Answer(a)(i) .... [1]

(ii) For the 5 column grid, $a = n - 6$.

Write down b, c and d in terms of n for this grid.

Answer(a)(ii) $b =$ ....

$c =$ ....

$d =$ .... [2]

(iii) Write down $bc - ad$ in terms of n.
Show clearly that it simplifies to 20.

Answer(a)(iii)

[2]

19

(b) This grid has 6 columns. The shape is drawn for $n = 10$.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

a b
   n
c d

  (i) Calculate the value of $bc - ad$ for $n = 10$.

Answer(b)(i) .... [1]

  (ii) Without simplifying, write down $bc - ad$ in terms of n for this grid.

Answer(b)(ii) .... [2]

(c) This grid has 7 columns.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

a b
   n
c d

Show clearly that $bc - ad = 28$ for $n = 17$.

Answer(c)

[1]

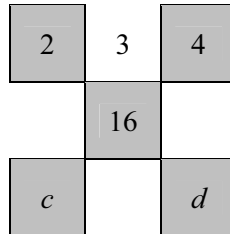**Question 10 continues on the next page.**

(d) Write down the value of $bc - ad$ when there are t columns in the grid.

Answer(d) .... [1]

(e) Find the values of c, d and $bc - ad$ for this shape.

2	3	4
	16	
c		d

Answer (e) c = ...

$d =$ ....

$bc - ad =$ .... [2]