Revision for Series Binomial Logs ANSWERS

1) 2. (a) $u_4 = u_1 + 3d$ or 16 = -2 + 3d(M1) $d = \frac{16 - (-2)}{3}$ (M1) (A1)(C3)(b) $u_n = u_1 + (n-1)6$ or 11998 = -2 + (n-1)6(M1) $n = \frac{11998 + 2}{6} + 1$ (A1)(A1) (C3)[6 marks] M02/520/S(1)M+

(b) $(3x^2)^3 \left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1) $\binom{9}{6} 3^3 x^6 \frac{1}{x^6} \left(\text{or } 84 \times 3^3 x^6 \frac{1}{x^6} \right)$ (A1)(A1)(C4)[6 marks]

M02/520/S(1)M+

3) **QUESTION 1**

7.

(a) 10

2)

Arithmetic sequence d = 3 (may be implied) (M1)(A1)n = 1250(A2) $S = \frac{1250}{2}(3+3750)$ $\left(\text{or } S = \frac{1250}{2}(6+1249\times3)\right)$ (M1)= 2345625(C6)(A1)

M03/520/S(1)M+

N3

(A2)

(C2)

4) **QUESTION 2**

> $\ln a^3 b = 3 \ln a + \ln b$ (A1)(A1) $\ln a^3 b = 3p + q$ N3 A1 (b) $\ln \frac{\sqrt{a}}{b} = \frac{1}{2} \ln a - \ln b$ (A1)(A1) $\ln \frac{\sqrt{a}}{h} = \frac{1}{2} p - q$

> > N06/5/MATME/SP1/ENG/TZ0/XX+

A1

(a) METHOD 1

$$5^{x+1} = 5^4$$
 A1
 $x+1=4$ (A1)
 $x=3$ A1 N2

METHOD 2

Taking logs A1

e.g.
$$x+1 = \log_5 625$$
, $(x+1)\log 5 = \log 625$
 $x+1 = \frac{\log 625}{\log 5}$ $(x+1=4)$ (A1)

 $x=3$ A1 N2

(b) METHOD 1

Attempt to re-arrange equation	(M1)	
$3x + 5 = a^2$	A1	
$x = \frac{a^2 - 5}{3}$	AI	N2

METHOD 2

Change base to give $\log(3x+5) = \log a^2$	(M1)	
$3x + 5 = a^2$	AI	
$x = \frac{a^2 - 5}{3}$	AI	N2

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Note: Throughout this question, the first and last terms are interchangeable.

(a)	For recognizing the arithmetic sequence	(M1)	
	$u_1 = 1$, $n = 20$, $u_{20} = 20$ ($u_1 = 1$, $n = 20$, $d = 1$)	(A1)	
	Evidence of using sum of an AP	M1	
	$S_{20} = \frac{(1+20)20}{2}$ (or $S = \frac{20}{2}(2\times1+19\times1)$)	A1	
	$S_{20} = 210$	AG	N0
			[4 marks]

(b) Let there be n cans in bottom row

Evidence of using $S_n = 3240$ (M1)

e.g.
$$\frac{(1+n)n}{2} = 3240$$
, $\frac{n}{2}(2+(n-1)) = 3240$, $\frac{n}{2}(2n+(n-1)(-1)) = 3240$

$$n = 80 \text{ or } n = -81$$
 (A1)

$$n = 80$$
 A1 N2

[4 marks]

A1

(c) (i) Evidence of using
$$S = \frac{(1+n)n}{2}$$
 (M1)

$$S = n^2 + n A1$$

$$n^2 + n - 2S = 0 AG N0$$

(ii) METHOD 1

Substituting S = 2100

e.g.
$$n^2 + n - 4200 = 0$$
, $2100 = \frac{(1+n)n}{2}$

EITHER

$$n = 64.3, n = -65.3$$

Any valid reason which includes reference to integer being needed, and pointing out that integer not possible here.

R1* N1***

e.g. n must be a (positive) integer, this equation does not have integer solutions.

OR

Valid reason which includes reference to integer being needed, and pointing out that integer not possible here. R1 N1

e.g. this discriminant is not a perfect square, therefore no integer solution as needed.

METHOD 2

Trial and error

$$S_{64} = 2080, S_{65} = 2145$$
 A1A1

Any valid reason which includes reference to integer being needed, and pointing out that integer not possible here.

R1 N1

R1

[6 marks]

Total [14 marks]

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(b) (i) Evidence of using the sum of an AP
$$MI$$

$$e.g. \frac{20}{2} 2 \times 3 + (20-1) \times 3$$

$$\sum_{n=1}^{20} 3n = 630 \qquad AI \qquad NI$$

(ii) METHOD 1

Correct calculation for
$$\sum_{n=1}^{100} 3n$$
 (A1)
e.g. $\frac{100}{2} (2 \times 3 + 99 \times 3), 15150$

$$\sum_{n=21}^{100} 3n = 14520 AI N2$$

METHOD 2

Recognising that first term is 63, the number of terms is 80 (A1)(A1) e.g. $\frac{80}{2}$ (63 + 300), $\frac{80}{2}$ (126 + 79 × 3) $\sum_{n=14}^{100} 3n = 14520$ A1 N2

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8) QUESTION 13

(a) For finding second, third and fourth terms correctly (A1)(A1)(A1)

Second term
$$\binom{4}{1}e^3\left(\frac{1}{e}\right)$$
, third term $\binom{4}{2}e^2\left(\frac{1}{e}\right)^2$, fourth term $\binom{4}{3}e\left(\frac{1}{e}\right)^3$

For finding first and last terms, **and** adding them to **their** three terms (A1) $\left(e + \frac{1}{e}\right)^4 = \binom{4}{0}e^4 + \binom{4}{1}e^3\left(\frac{1}{e}\right) + \binom{4}{2}e^2\left(\frac{1}{e}\right)^2 + \binom{4}{3}e\left(\frac{1}{e}\right)^3 + \binom{4}{4}\left(\frac{1}{e}\right)^4 \\
\left(e + \frac{1}{e}\right)^4 = e^4 + 4e^3\left(\frac{1}{e}\right) + 6e^2\left(\frac{1}{e}\right)^2 + 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4 \left(=e^4 + 4e^2 + 6 + \frac{4}{e^2} + \frac{1}{e^4}\right)$ N4

(b)
$$\left(e - \frac{1}{e}\right)^4 = e^4 - 4e^3\left(\frac{1}{e}\right) + 6e^2\left(\frac{1}{e}\right)^2 - 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4 \left(=e^4 - 4e^2 + 6 - \frac{4}{e^2} + \frac{1}{e^4}\right)$$
 (A1)
Adding gives $2e^4 + 12 + \frac{2}{e^4}$ $\left(\text{accept } 2\binom{4}{0}e^4 + 2\binom{4}{2}e^2\left(\frac{1}{e}\right)^2 + 2\binom{4}{4}\left(\frac{1}{e}\right)^4\right)$ A1 N2

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For using
$$u_3 = u_1 r^2 = 8$$
 (M1)
 $8 = 18r^2$ (A1)
 $r^2 = \frac{8}{18} \left(= \frac{4}{9} \right)$ (A1)
 $r = \pm \frac{2}{3}$ (A1)(A1)
 $S_{\infty} = \frac{u_1}{1-r}$,
 $S_{\infty} = 54, \frac{54}{5}$ (=10.8) (A1)(A1) (C3)(C3)

N05/5/MATME/SP1/ENG/TZ0/XX/M+