IGCSE Additional Mathematics Ch 5

Remainder Theorem Test

Student Name: ANSWERS

Time allowed: 50 minutes

READ THESE INSTRUCTIONS FIRST
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 34.

1.

The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x) = 0$ are -2, $1 + \sqrt{3}$ and $1 - \sqrt{3}$.

(i) Express $f(x)$ as a cubic polynomial in x with integer coefficients. [3]

(ii) Find the remainder when $f(x)$ is divided by $x - 3$. [2]

(iii) Solve the equation $f(-x) = 0$. [2]

2.

The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x) = 0$ are $1, k$ and k^2. It is given that $f(x)$ has a remainder of 7 when divided by $x - 2$.

(i) Show that $k^3 - 2k^2 - 2k - 3 = 0$. [3]

(ii) Hence find a value for k and show that there are no other real values of k which satisfy this equation. [5]

3.

(a) The expression $f(x) = x^3 + ax^2 + bx + c$ leaves the same remainder, R, when it is divided by $x + 2$ and when it is divided by $x - 2$.

(i) Evaluate b. [2]

$f(x)$ also leaves the same remainder, R, when divided by $x - 1$.

(ii) Evaluate a. [2]

$f(x)$ leaves a remainder of 4 when divided by $x - 3$.

(iii) Evaluate c. [1]

(b) Solve the equation $x^3 + 3x^2 = 2$, giving your answers to 2 decimal places where necessary. [5]

4.

(a) The remainder when the expression $x^3 - 11x^2 + kx - 30$ is divided by $x - 1$ is 4 times the remainder when this expression is divided by $x - 2$. Find the value of the constant k. [4]

(b) Solve the equation $x^3 - 4x^2 - 8x + 8 = 0$, expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

Remainder Theorem Test

1.i. $f(x) = (x+2)(x-1-\sqrt{3})(x-1+\sqrt{3})$

$= (x+2)(x^2 - x + \cancel{x\sqrt{3}} - x - \cancel{x\sqrt{3}} + 1 - 3)$

$= (x+2)(x^2 - 2x - 2)$

$= x^3 - \cancel{2x^2} - 2x + \cancel{2x^2} - 4x - 4$

3 $= \underline{\underline{x^3 - 6x - 4}}$

ii. $f(3) = 27 - 18 - 4 = 5$

2 $\therefore$ Remainder $= \underline{\underline{5}}$

iii. $f(-x) = (-x)^3 - 6(-x) - 4$

$= -x^3 + 6x - 4$

$\therefore$ Require $-x^3 + 6x - 4 = 0$

or $x^3 - 6x + 4 = 0$

$f(2) = 0 \therefore (x-2)$ is a factor

$\therefore (x-2)(x^2 + 2x - 2) = 0$

4 $\therefore x = 2$ or $x = \dfrac{-2 \pm \sqrt{(4+8)}}{2}$

$= -1 \pm \sqrt{3}$

2 $\therefore \underline{\underline{x = 2, -1 \pm \sqrt{3}}}$

2. i. $f(x) = (x-1)(x-K)(x-K^2)$

$f(2) = 7 \quad \therefore \quad (2-1)(2-K)(2-K^2) = 7$

$\therefore \quad 4 - 2K^2 - 2K + K^3 = 7$

3 $\quad \therefore \quad K^3 - 2K^2 - 2K - 3 = 0$, as required.

ii. when $K = 1$, this $= 1 - 2 - 2 - 3 = -6$

when $K = -1$, this $= -1 - 2 + 2 - 3 = -4$

when $K = 3$, this $= 27 - 18 - 6 - 3 = 0 \quad \therefore (K-3)$ is a factor.

$\therefore \quad (K-3)(K^2 + K + 1) = 0$

for $K^2 + K + 1 = 0$, $K = \dfrac{-1 \pm \sqrt{(1-4)}}{2}$

$\therefore$ no real solutions

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5 $\quad \therefore \quad \underline{\underline{K = 3}}$ and there are no other real values.

3a.i. $f(-2) = f(2)$

$\therefore -8 + 4a - 2b + c = 8 + 4a + 2b + c$

$\therefore 4b = -16$

2 $\therefore \underline{\underline{b = -4}}$

ii. $f(1) = f(2)$

$\therefore 1 + a - 4 + c = 8 + 4a - 8 + c$

$\therefore 3a = -3$

2 $\therefore \underline{\underline{a = -1}}$

iii. $f(3) = 4$

$\therefore 27 - 9 - 12 + c = 4$

1 $\therefore \underline{\underline{c = -2}}$

b. $x^3 + 3x^2 - 2 = 0$

since $(-1)^3 + 3(-1)^2 - 2 = 0$, $(x+1)$ is a factor

$\therefore (x+1)(x^2 + 2x - 2) = 0$

$\therefore x = -1$ or $x = \dfrac{-2 \pm \sqrt{(2^2 + 4 \times 2)}}{2}$

$= -1 \pm \sqrt{3}$

5 $\therefore \underline{\underline{x = -1, -2.73 \text{ or } 0.73 \quad (2 \text{ dp.})}}$

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4. a. $f(1) = 4 \times f(2)$

$\therefore 1 - 11 + K - 30 = 4(8 - 44 + 2K - 30)$

$\therefore K - 40 = 8K - 264$

$\therefore 7K = 224$

4 $\therefore \underline{\underline{K = 32}}$

b. Let $f(x) = x^3 - 4x^2 - 8x + 8$

$f(2) = 8 - 16 - 16 + 8 = -16$ $\therefore (x-2)$ not factor

$f(-2) = -8 - 16 + 16 + 8 = 0$ $\therefore (x+2)$ is a factor

$\therefore f(x) = (x+2)(x^2 - 6x + 4)$

$\therefore$ for $x^2 - 6x + 4 = 0$,

$$x = \frac{6 \pm \sqrt{(36 - 16)}}{2}$$

$$= 3 \pm \sqrt{5}$$

5 $\therefore$ Solutions for $f(x) = 0$ are $\underline{\underline{x = -2,\ 3 \pm \sqrt{5}}}$

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