IB Questionbank Mathematical Studies 3rd edition

quadratics ans

0 min
0 marks

1. (a) $y = 2x$ (A1)

(b) $y = 2x + 8$ (follow through from part (a)) (A1)

(c) $2x + 8 = 0$ (or other method) (M1)
(–4, 0) (follow through from part (b)) (A1)

[4]

2. (a) A; $y = 0,\ 3x = 24 \Rightarrow x = 8$
A(8, 0) (A1)

B; $x = 0,\ 4y = 24 \Rightarrow y = 6$
B(0, 6) (A1) 2

(b) M; $x_m = \dfrac{8+0}{2} = 4,\ y_m = \dfrac{0+6}{2} = 3$ (A1) 2

M(4, 3) (A1)

(c) L_2: gradient = $\dfrac{3--2}{4-0} = \dfrac{5}{4}$ (A1)

$y = \dfrac{5}{4}x - 2$ (or equivalent) (A1) 2

(d) (i) M(4, 3), C(0, –2)

$MC = \sqrt{(4-0)^2 + (3-(-2))^2}$ (M1)

$= \sqrt{41}$

$= 6.40$ (A1)

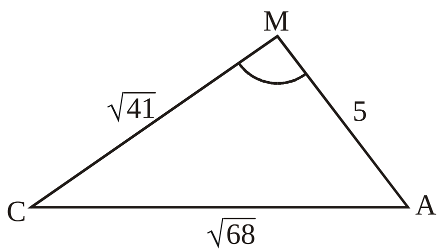
(ii) A(8, 0), C(0, –2)

$AC = \sqrt{8^2 + (-2)^2}$ (M1)

$= \sqrt{68}$

$= 8.25$ (A1) 4

(e) (i)



$\cos M = \dfrac{5^2 + (\sqrt{41})^2 - (\sqrt{68})^2}{2 \times 5\sqrt{41}}$ (M1)

$= \dfrac{25 + 41 - 68}{10\sqrt{41}}$ (M1)

$C\hat{M}A = 91.8°$ (3 s.f.) (A1)

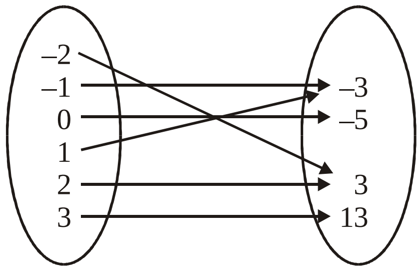
(ii) Area of $\Delta CMA = \frac{1}{2}\sqrt{41} \times 5 \sin 91.8°$ (M1)

$= 15.99991171...$

$= 16.0$ (3 s.f.) (A1) 5

[15]

3. (a)



For six single lines going to correct *y* (*y*-value can be repeated) (M1)
Correct diagram (*y*-values not repeated) (A1) (C2)

(b) $x \in \{-2, -1, 0, 1, 2, 3\}$ (A2) (C2)

Note: *Award (A1) if one value omitted.*

(c) $y \in \{-3, -5, 3, 13\}$ (A2) (C2)

[6]

4. (a) Put $x = 0$ to find $y = -2$ (M1)
Coordinates are (0, –2) (A1) (C2)

Note: *Award (M1)(A0) for –2 if working is shown. If not, award (M0)(A0).*

(b) Factorise fully, $y = (x - 2)(x + 1)$. (A1)(A1)
$y = 0$ when $x = -1, 2$. (A1)(A1)
Coordinates are A(–1, 0), B(2, 0). (A1)(A1) (C6)

Note: *Award (C2) for each correct x value if no method shown and full coordinates not given. If the quadratic formula is used correctly award (M1)(A1)(A1)(A1)(A1)(A1). If the formula is incorrect award only the last (A1)(A1) as ft.*

[8]

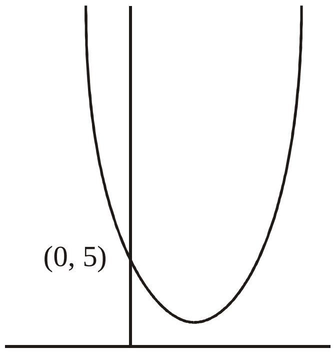
5. (a) $(x + 2)(x - 4)$ (A1)

(b) (i) (–2, 0) (A1)

(ii) (1, –9) (A1)(A1)

[4]

6. (a)



(A3) (C3)

Notes: *Award (A1) for point (0,5) indicated.*
Award (A2) for correct shape.

(b) (1.5, 0.5) (A1)(A1) (C2)

(c) $x = 1.5$ (A1) (C1)

[6]

7. (a) $x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$ (A1)

$x = 2$ (A1)

$x = 3$ (A1)

(b) (2, 0)

(3, 0) (A1)

Notes: *Follow through from part (a). Both must be correct and written as coordinates for (A1)*

[4]

8. (a) $(x - 2)(x - 4)$ (A1)(A1) (C2)

(b) $x = 2$, $x = 4$ (A1)(ft)(A1)(ft) (C2)

(c) $x = 0.807$, $x = 6.19$ (A1)(A1) (C2)

Note: *Award maximum of (A0)(A1) if coordinate pairs given.*

OR

(M1) for an attempt to solve $x^2 - 7x + 5 = 0$ via formula with correct values substituted. (M1)

$x = \dfrac{7 \pm \sqrt{29}}{2}$ (A1) (C2)

[6]

9. (a) $\dfrac{0+6}{2} = 3 \quad h = 3$ (M1)(A1) (C2)

Note: *Award (M1) for any correct method.*

(b) $y = ax(x-6)$ (A1)

$8 = 3a(-3)$ (A1)(ft)

$a = -\dfrac{8}{9}$ (A1)(ft)

$y = -\dfrac{8}{9}x(x-6)$ (A1)(ft)

Notes: *Award (A1) for correct substitution of b = 6 into equation.*
Award (A1)(ft) for substitution of their point V into the equation.

OR

$y = a(x-3)^2 + 8$ (A1)(ft)

Note: *Award (A1)(ft) for correct substitution of their h into the equation.*

$0 = a(6-3)^2 + 8$ **OR** $0 = a(0-3)^2 + 8$ (A1)

Note: *Award (A1) for correct substitution of an x-intercept.*

$a = -\dfrac{8}{9}$ (A1)(ft)

$y = -\dfrac{8}{9}(x-3)^2 + 8$ (A1)(ft) (C4)

[6]

10. (a) $x = 0$, $x = 4$ (A1)(A1) (C2)

Notes: *Accept 0 and 4*

(b) $x = 2$ (A1)(A1) (C2)

Note: *Award (A1) for x = constant, (A1) for 2.*

(c) $x = –2$ (A1) (C1)

Note: *Accept –2*

(d) $y \geq –4$ ($f(x) \geq –4$) (A1) (C1)

Notes: *Accept alternative notations.*
Award (A0) for use of strict inequality.

[6]

11. (a) $q = 4$ (A1) (C1)

(b) $2.5 = \frac{r}{4}$ (M1)

$r = 10$ (A1) (C2)

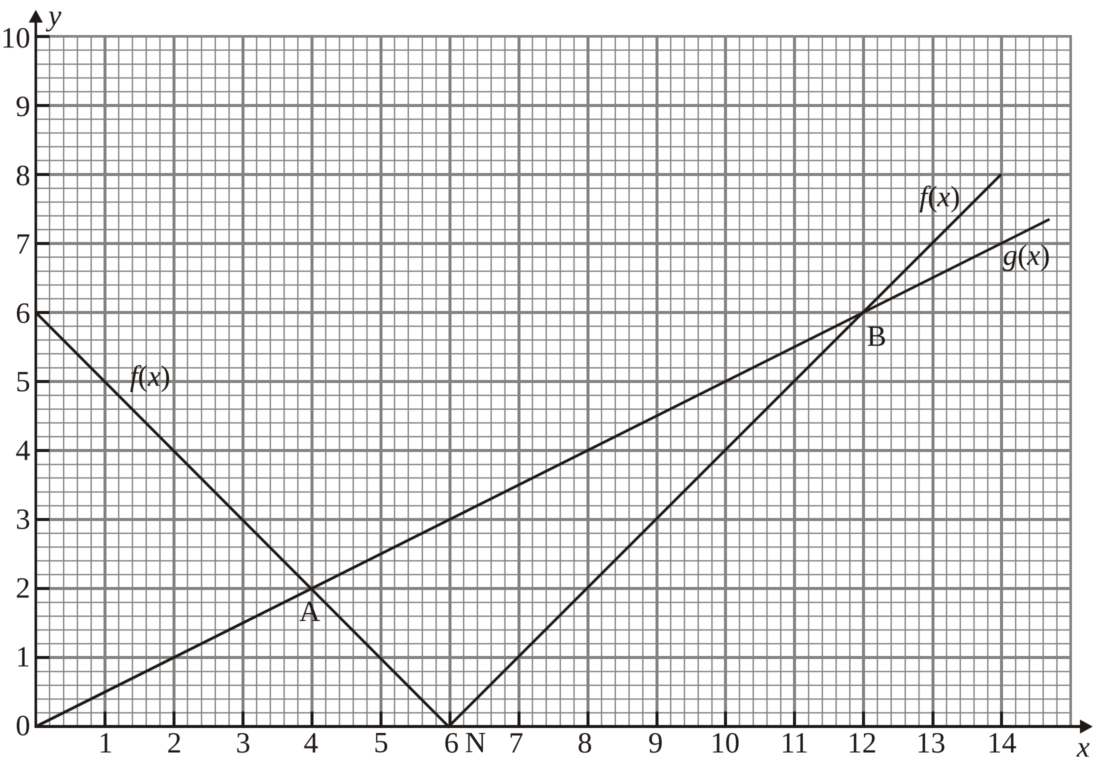
(c) –8.5 (A1)(ft) (C1)

(d) $–8.5 \leq y \leq 104$ (A1)(ft)(A1)(ft) (C2)

Notes: *Award (A1)(ft) for their answer to part (c) with correct inequality signs, (A1)(ft) for 104. Follow through from their values of q and r. Accept 104 ±2 if read from graph.*

[6]

12. (a)



(A5) 5

Note: *Award (A1) for a correctly labelled graph, (A1) for correct scales, (A1) for line f(x) = 6 – x drawn correctly, (A1) for line f(x) = x – 6 drawn correctly, (A1) for g(x) =* $\frac{1}{2}x$ *drawn correctly.*

(b) (i) Points named on the graph (A and B can be inversed) (A1)

(ii) A(4, 2), B(12, 6) (A1)(A1) 3

(c) Midpoint = $\left(\frac{12+4}{2}, \frac{6+2}{2}\right)$ (M1)

= (8, 4) (A1) 2

Note: *Allow (A2) for reading from the graph but* ***both*** *coordinates must be correct.*

(d) Gradient = $\frac{4-0}{8-6} = 2$ (A1)

$y = mx + c$
$0 = 2 \times 6 + c$ (M1)
$c = -12$ (A1)
Equation is $y = 2x - 12$ (or correct alternatives).
Ft from candidate's previous work. (A1) 4

[14]

13. (a) $220 = 2(W + x)$ (M1)

Therefore $W = \frac{220-2x}{2}$ or $110 - x$ (A1)

(b) Area = $x(110 - x)$ *(allow follow through from part (a))* (A1)

(c) Area = 70(110 – 70) = 2800 m^2 *(allow follow through from part (b))* (A1)

[4]

14. (a) $x(x - k)$ (A1) (C1)

(b) $x = 0$ **or** $x = k$ (A1) (C1)

Note: *Both correct answers only*

(c) $k = 3$ (A1) (C1)

(d) Vertex at $x = \frac{-(-3)}{2(1)}$ (M1)

Note: *(M1) for correct substitution in formula*

$x = 1.5$ (A1)(ft)
$y = –2.25$ (A1)(ft)

OR

$f'(x) = 2x – 3$ (M1)

Note: *(M1) for correct differentiation*

$x = 1.5$ (A1)(ft)
$y = –2.25$ (A1)(ft)

OR

for finding the midpoint of their 0 and 3 (M1)
$x = 1.5$ (A1)(ft)
$y = –2.25$ (A1)(ft) (C3)

Note: *If final answer is given as (1.5, –2.25) award a maximum of (M1)(A1)(A0)*

[6]

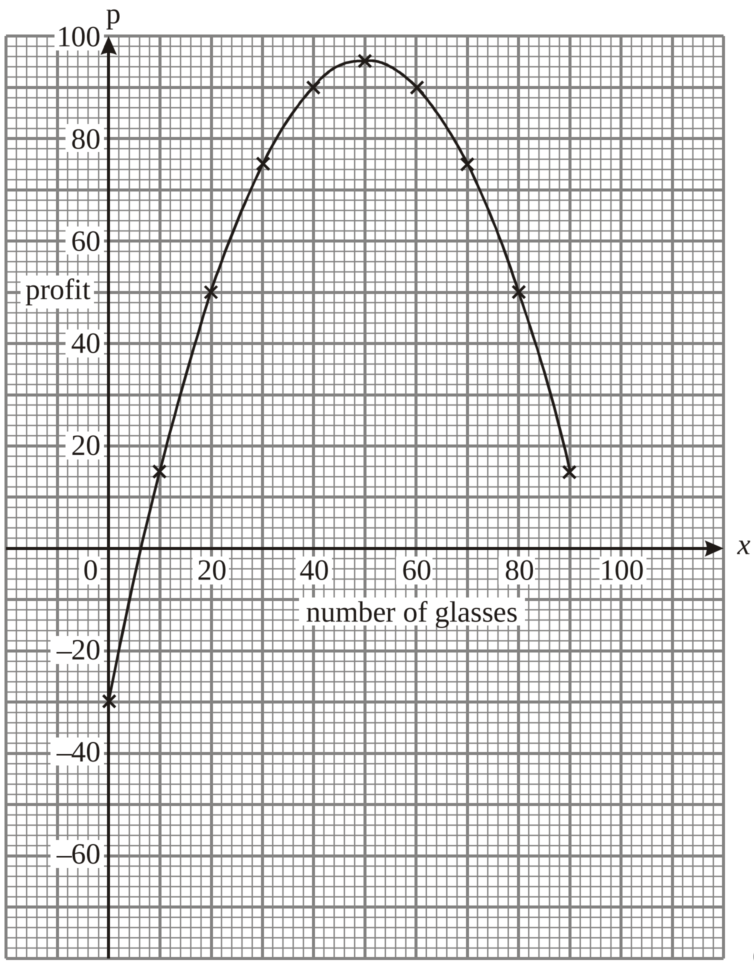
15. (a) (A3)

x	0	10	20	30	40	50	60	70	80	90
P	**–30**	15	**50**	75	90	**95**	**90**	75	50	**15**

Note: *Award ½-mark for each correct bold entry, and round down.*

If a candidate obtains (A0) here but has clearly shown the method of substituting in the values of x into the formula award (M1)

(b)



(A2)(A2)(A1)

Note: *For graph, follow through from candidate's table*

Notes*: Award (A2) for axes, (A2) for plotting points and (A1) for a smooth curve.*

Axes: *Award ½-mark for each of the following and then round down:*
horizontal axis labelled with "x" or "Numbers of glasses..."
vertical axis labelled with "P" of "Profit"
horizontal scale $\rightarrow$ consistent and presents values $0 \rightarrow 90$
vertical scale as for horizontal but represents their range of values for P.

Points: *Award (A2) for 0 or 1 error*
Award (A1) for 2 or 3 errors
Award (A0) otherwise

(c) (i) maximum profit = 95 swiss francs (A1)

(ii) 50 glasses (A1)

(iii) 67 ± 2 (A1)
33 ± 2 (A1)

(iv) 30 swiss francs (A1)

Note: *Award no marks for –30 swiss francs*

Note: *Follow through from candidate's graph*

(d) Fiona's share = $\frac{3}{6}$ (M1)

Profit from 40 glasses = 90 swiss francs

Fiona's profit = $\frac{1}{2} \times 90$

= 45 (A1)

[15]