IB Questionbank Mathematical Studies 3rd edition

## **Quadratics**

## 112 min 110 marks

**1.** The following diagram shows a straight line *l*.



- (a) Find the equation of the line *l*.
- (b) The line *n* is parallel to *l* and passes through the point (0, 8). Write down the equation of the line *n*.
- (c) The line *n* crosses the horizontal axis at the point P. Find the coordinates of P.

(Total 4 marks)

2. The line  $L_1$  shown on the set of axes below has equation 3x + 4y = 24.  $L_1$  cuts the *x*-axis at A and cuts the *y*-axis at B.





(a) Write down the coordinates of A and B.

M is the midpoint of the line segment [AB].

(b) Write down the coordinates of M.

The line  $L_2$  passes through the point M and the point C (0, -2).

(c) Write down the equation of  $L_2$ .

(2)

(d) Find the length of

(i)	MC;	
		(2)

(ii) AC. (2)

(2)

(2)

- (e) The length of AM is 5. Find
  - (i) the size of angle CMA;
  - (ii) the area of the triangle with vertices C, M and A.

(2) (Total 15 marks)

(3)

3. (a) Represent the function  $y = 2x^2 - 5$ , where  $x \in \{-2, -1, 0, 1, 2, 3\}$  by a mapping diagram.



- (b) List the elements of the domain of this function.
- (c) List the elements of the range of this function.

(Total 6 marks)

4. The graph of the function  $y = x^2 - x - 2$  is drawn below.



- (a) Write down the coordinates of the point C.
- (b) Calculate the coordinates of the points A and B.

(Total 8 marks)

5. The diagram shows the graph of  $y = x^2 - 2x - 8$ . The graph crosses the *x*-axis at the point A, and has a vertex at B.



- (a) Factorize  $x^2 2x 8$ .
- (b) Write down the coordinates of each of these points
  - (i) A;
  - (ii) B.

(Total 4 marks)

- 6. (a) Sketch the graph of the function  $y = 2x^2 6x + 5$ .
  - (b) Write down the coordinates of the local maximum or minimum of the function.
  - (c) Find the equation of the axis of symmetry of the function.

(Total 6 marks)

- 7. (a) Solve the equation  $x^2 5x + 6 = 0$ .
  - (b) Find the coordinates of the points where the graph of  $y = x^2 5x + 6$  intersects the *x*-axis.

## (Total 4 marks)

- 8. Let  $f(x) = x^2 6x + 8$ .
  - (a) Factorise  $x^2 6x + 8$ .
  - (b) Hence, or otherwise, solve the equation  $x^2 6x + 8 = 0$ .

Let g(x) = x + 3.

(c) Write down the solutions to the equation f(x) = g(x).

(2) (Total 6 marks)

(2)

(2)

9. A quadratic curve with equation y = ax(x - b) is shown in the following diagram.



The x-intercepts are at (0, 0) and (6, 0), and the vertex V is at (h, 8).

(a) Find the value of *h*.

(2)

(b) Find the equation of the curve.

(4) (Total 6 marks) **10.** The following is the graph of the quadratic function y = f(x).



- (a) Write down the solutions to the equation f(x) = 0.
- (b) Write down the equation of the axis of symmetry of the graph of f(x).
- (c) The equation f(x) = 12 has two solutions. One of these solutions is x = 6. Use the symmetry of the graph to find the other solution.
  - (1)

(d) The minimum value for y is -4. Write down the range of f(x).

(1) (Total 6 marks)

(2)

(2)

11. The graph of  $y = 2x^2 - rx + q$  is shown for  $-5 \le x \le 7$ .



The graph cuts the y-axis at (0, 4).

(a) Write down the value of q.

The axis of symmetry is x = 2.5.

- (b) Find the value of *r*.
- (c) Write down the minimum value of *y*.
- (d) Write down the range of *y*.

(2) (Total 6 marks)

(1)

(2)

(1)

**12.** Two functions are defined as follows

$$f(x) = \begin{cases} 6 - x \text{ for } 0 \le x < 6\\ x - 6 \text{ for } x \ge 6 \end{cases}$$
$$g(x) = \frac{1}{2}x$$

- (a) Draw the graphs of the functions *f* and *g* in the interval  $0 \le x \le 14$ ,  $0 \le y \le 8$  using a scale of 1 cm to represent 1 unit on both axes.
- (b) (i) Mark the intersection points A and B of f(x) and g(x) on the graph.
  - (ii) Write down the coordinates of A and B.
- (c) Calculate the midpoint M of the line AB.
- (d) Find the equation of the straight line which joins the points M and N. (4)

(Total 14 marks)

(5)

(3)

(2)

13. The perimeter of this rectangular field is 220 m. One side is x m as shown.



- (a) Express the width (*W*) in terms of *x*.
- (b) Write an expression, in terms of *x* only, for the area of the field.
- (c) If the length (x) is 70 m, find the area.

(Total 4 marks)

(1)

(1)

- **14.** (a) Factorise the expression  $x^2 kx$ .
  - (b) Hence solve the equation  $x^2 kx = 0$ .

The diagram below shows the graph of the function  $f(x) = x^2 - kx$  for a particular value of k.



(c) Write down the value of *k* for this function.

(1)

(d) Find the minimum value of the function y = f(x).

## (3) (Total 6 marks)

**15.** The profit (*P*) in Swiss Francs made by three students selling homemade lemonade is modelled by the function

$$P = -\frac{1}{20}x^2 + 5x - 30$$

where *x* is the number of glasses of lemonade sold.

(a) **Copy** and complete the table below

x	0	10	20	30	40	50	60	70	80	90
Р		15			90			75	50	

(3)

(5)

(b) On graph paper draw axes for *x* and *P*, placing *x* on the horizontal axis and *P* on the vertical axis. Use suitable scales. Draw the graph of *P* against *x* by plotting the points. Label your graph.

(c) Use your graph to find

(i)	the maximum possible profit;	(1)
(ii)	the number of glasses that need to be sold to make the maximum profit;	(1)
(iii)	the number of glasses that need to be sold to make a profit of 80 Swiss Francs;	(2)
(iv)	the amount of money initially invested by the three students.	(1)

(d) The three students Baljeet, Jane and Fiona share the profits in the ratio of 1:2:3 respectively. If they sold 40 glasses of lemonade, calculate Fiona's share of the profits.