prob sl calc

97 min

97 marks

1. A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box.

(2)

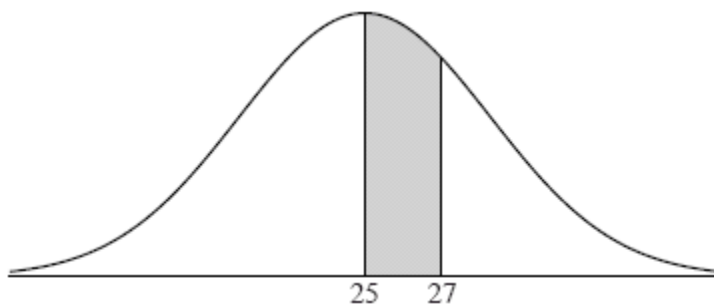(b) Find the probability that there are 15 brown eggs in the box.

(2)

(c) Find the probability that there are at least 10 brown eggs in the box.

(3)
(Total 7 marks)

2. Let the random variable X be normally distributed with mean 25, as shown in the following diagram.

The shaded region between 25 and 27 represents 30 % of the distribution.

(a)   Find P($X > 27$).

(2)

(b)   Find the standard deviation of X.

(5)
(Total 7 marks)

3.   Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a)   Find the probability that he wins exactly four games.

(2)

For game B, the probability that Evan wins is p. He plays game B seven times.

(b)   Write down an expression, in terms of p, for the probability that he wins exactly four games.

(2)

(c)   Hence, find the values of p such that the probability that he wins exactly four games is 0.15.

(3)
(Total 7 marks)

4.   The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a)   Find the probability that a player weighs more than 82 kg.

(2)

(b)   (i)   Write down the standardized value, z, for 68 kg.

(ii)   Hence, find the standard deviation of weights.

(4)

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c)　(i)　Find the set of all possible weights of players that take part in the tournament.

　　　(ii)　A player is selected at random. Find the probability that the player takes part in the tournament.

(5)

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d)　Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.
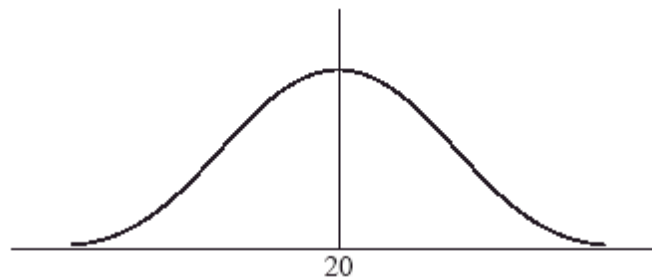
(4)
(Total 15 marks)

5.　A random variable X is distributed normally with a mean of 20 and variance 9.

(a)　Find $P(X \leq 24.5)$.

(3)

(b)　Let $P(X \leq k) = 0.85$.

　　　(i)　Represent this information on the following diagram.

20

　　　(ii)　Find the value of k.

(5)
(Total 8 marks)

6. A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $P(X \leq 475)$.

(2)

(b) Given that $P(X > a) = 0.27$, find a.

(4)
(Total 6 marks)

7. The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than 90 km h^{-1} and 12 % of them at speeds less than 40 km h^{-1}. Find μ and σ.

(Total 6 marks)

8. A fisherman catches 200 fish to sell. He measures the lengths, l cm of these fish, and the results are shown in the frequency table below.

Length l cm	$0 \leq l < 10$	$10 \leq l < 20$	$20 \leq l < 30$	$30 \leq l < 40$	$40 \leq l < 60$	$60 \leq l < 75$	$75 \leq l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish.

(3)

(b) A cumulative frequency diagram is given below for the lengths of the fish.
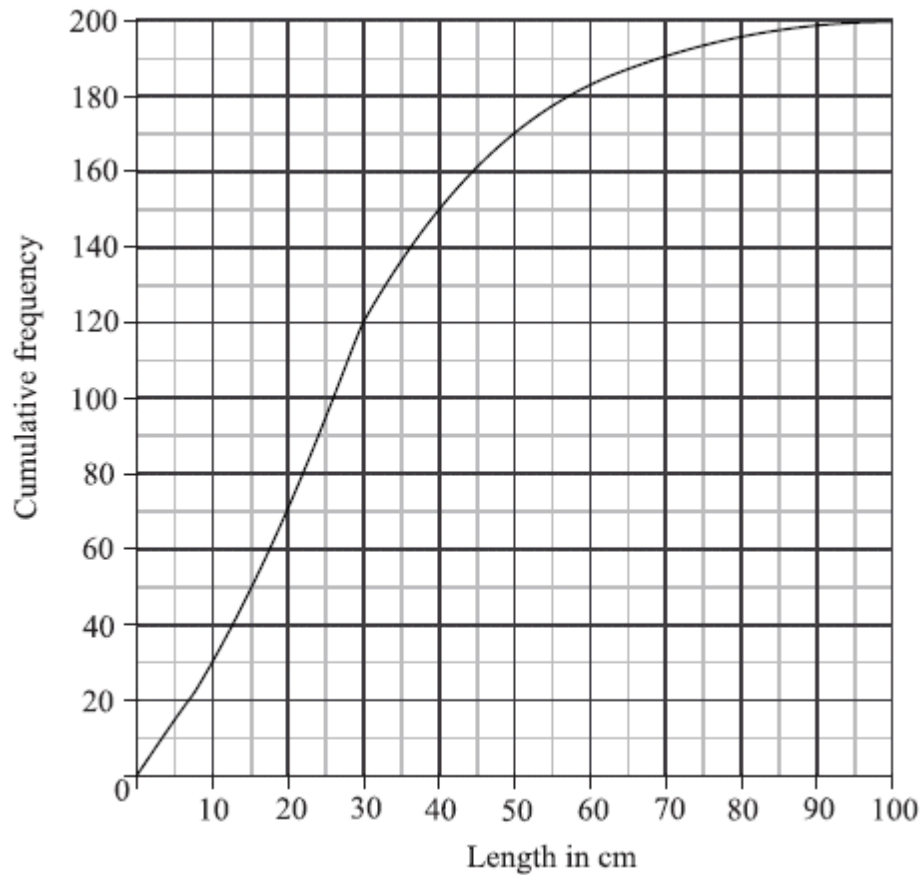
Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 40 % of the fish have a length more than k cm, find the value of k.

(6)

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.
Medium fish have a length greater than or equal to 20 cm but less than 60 cm.
Large fish have a length greater than or equal to 60 cm.

(c) Write down the probability that a fish is small.

(2)

The cost of a small fish is $4, a medium fish $10, and a large fish $12.

(d)   Copy and complete the following table, which gives a probability distribution for the cost X.

Cost $X	4	10	12
P(X = x)		0.565	

(2)

(e)   Find E(X).

(2)

(Total 15 marks)

9.   A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a)   One biscuit is chosen at random from the box. Find the probability that this biscuit

   (i)   weighs less than 8 g;

   (ii)   weighs between 6 g and 8 g.

(4)

(b)   Five percent of the biscuits in the box weigh less than d grams.

   (i)   Copy and complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.

(ii)   Find the value of *d*.

(5)

(c)   The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weigh less than 5 g.

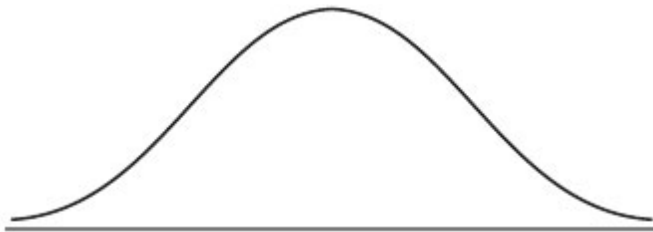Find the value of μ.

(4)

(Total 13 marks)

10.   The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92% are in category A.
The tallest 10.38% are in category C.
All the other plants are in category B with heights between *r* cm and *t* cm.

(a)   Complete the following diagram to represent this information.

(2)

(b)   Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of *r* and of *t*.

(5)

(Total 7 marks)

11.   A multiple choice test consists of ten questions. Each question has five answers.
Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a)   Find the expected number of questions Jose answers correctly.

(1)

(b)   Find the probability that Jose answers exactly three questions correctly.

(2)

(c)   Find the probability that Jose answers more than three questions correctly.

(3)
(Total 6 marks)