prob sl ans

6 min
6 marks

1. (a) correct substitution into formula for E(*X*) (A1)

e.g. 0.05× 240

E(*X*) =12 A1 N2 2

(b) evidence of recognizing binomial probability (may be seen in part (a)) (M1)

e.g. $\binom{240}{15}(0.05)^{15}(0.95)^{225}$, $X \sim B(240,0.05)$

P(*X* =15) = 0.0733 A1 N2 2

(c) P($X \leq 9$) = 0.236 (A1)

evidence of valid approach (M1)

e.g. using complement, summing probabilities

P($X \geq 10$) = 0.764 A1 N3 3

[7]

2. (a) symmetry of normal curve (M1)

e.g. P($X < 25$) = 0.5

P($X > 27$) = 0.2 A1 N2 2

(b) **METHOD 1**

finding standardized value (A1)

e.g. $\frac{27-25}{\sigma}$

evidence of complement (M1)

e.g. 1– *p*, P(X < 27), 0.8

finding *z*-score (A1)

e.g. $z = 0.84\ldots$

attempt to set up equation involving the standardized value M1

e.g. $0.84 = \frac{27-25}{\sigma}, 0.84 = \frac{X-\mu}{\sigma}$

$\sigma = 2.38$ A1 N3 5

METHOD 2

set up using normal CDF function and probability (M1)

e.g. P(25 < X < 27) = 0.3, P(X < 27) = 0.8

correct equation A2

e.g. P(25 < X < 27) = 0.3, P(X > 27) = 0.2

attempt to solve the equation using GDC (M1)

e.g. solver, graph, trial and error (more than two trials must be shown)

$\sigma = 2.38$ A1 N3 5

[7]

3. (a) evidence of recognizing binomial probability (may be seen in (b) or (c)) (M1)

e.g. probability = $\binom{7}{4}(0.9)^4(0.1)^3$, $X \sim$ B(7, 0.9), complementary probabilities

probability = 0.0230 A1 N2

(b) correct expression A1A1 N2

e.g. $\binom{7}{4}p^4(1-p)^3$, $35p^4(1-p)^3$

Note: *Award A1 for binomial coefficient* $\left(\text{accept}\binom{7}{3}\right)$,

A1 for $p^4(1-p)^3$.

(c) evidence of attempting to solve **their** equation (M1)

e.g. $\binom{7}{4} p^4(1-p)^3 = 0.15$, sketch

$p = 0.356, 0.770$ A1A1 N3

[7]

4. (a) evidence of appropriate approach (M1)

e.g. $1 - 0.85$, diagram showing values in a normal curve

$P(w \geq 82) = 0.15$ A1 N2

(b) (i) $z = -1.64$ A1 N1

(ii) evidence of appropriate approach (M1)

e.g. $-1.64 = \frac{x-\mu}{\sigma}, \frac{68-76.6}{\sigma}$

correct substitution A1

e.g. $-1.64 = \frac{68-76.6}{\sigma}$

$\sigma = 5.23$ A1 N1

(c) (i) $68.8 \leq \text{weight} \leq 84.4$ A1A1A1 N3

Note: *Award A1 for 68.8, A1 for 84.4, A1 for giving answer as an interval.*

(ii) evidence of appropriate approach (M1)

e.g. $P(-1.5 \leq z \leq 1.5)$, $P(68.76 < y < 84.44)$

P(qualify) = 0.866 A1 N2

(d) recognizing conditional probability (M1)

e.g. $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

P(woman and qualify) = 0.25×0.7 (A1)

$P(\text{woman} \mid \text{qualify}) = \frac{0.25 \times 0.7}{0.866}$ A1

P(woman | qualify) = 0.202 A1 N3

[15]

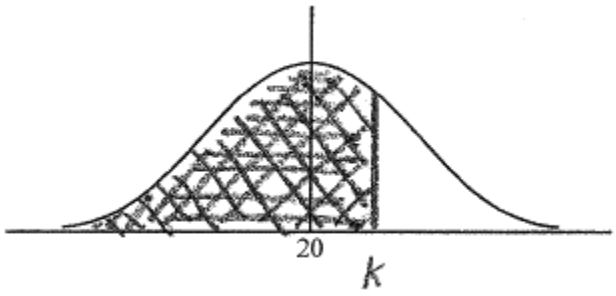
5. (a) $\sigma = 3$ (A1)

evidence of attempt to find $P(X \leq 24.5)$ (M1)

e.g. $z = 1.5$, $\frac{24.5 - 20}{3}$

$P(X \leq 24.5) = 0.933$ A1 N3 3

(b) (i)



A1A1 N2

Note: *Award A1 with shading that clearly extends to right of the mean, A1 for any correct label, either k, area or their value of k*

(ii) $z = 1.03(64338)$ (A1)

attempt to set up an equation (M1)

e.g. $\frac{k - 20}{3} = 1.0364, \frac{k - 20}{3} = 0.85$

$k = 23.1$ A1 N3 5

[8]

6. (a) evidence of attempt to find $P(X \leq 475)$ (M1)

e.g. $P(Z \leq 1.25)$

$P(X \leq 475) = 0.894$ A1 N2

(b) evidence of using the complement (M1)

e.g. 0.73, $1 - p$

$z = 0.6128$ (A1)

setting up equation (M1)

e.g. $\frac{a - 450}{20} = 0.6128$

$a = 462$ A1 N3

[6]

7. $X \sim N(\mu, \sigma^2)$

$P(X > 90) = 0.15$ **and** $P(X < 40) = 0.12$ (M1)

Finding standardized values 1.036, –1.175 A1A1

Setting up the equations $1.036 = \frac{90-\mu}{\sigma}, -1.175 = \frac{40-\mu}{\sigma}$ (M1)

$\mu = 66.6$, $\sigma = 22.6$ A1A1 N2N2

[6]

8. (a) evidence of using mid-interval values (5, 15, 25, 35, 50, 67.5, 87.5) (M1)

$\sigma = 19.8$ (cm) A2 N3

(b) (i) $Q_1 = 15$, $Q_3 = 40$ (A1)(A1)

$IQR = 25$ (accept any notation that suggests the interval 15 to 40) A1 N3

(ii) **METHOD 1**

60 % have a length less than k (A1)

$0.6 \times 200 = 120$ (A1)

k 30 (cm) A1 N2

METHOD 2

$0.4 \times 200 = 80$ (A1)

$200 - 80 = 120$ (A1)

$k = 30$ (cm) A1 N2

(c) $l < 20$ cm $\Rightarrow$ 70 fish (M1)

P(small) = $\frac{70}{200}$ (= 0.35) A1 N2

(d)

Cost \$$X$	4	10	12
$P(X = x)$	**0.35**	0.565	**0.085**

A1A1 N2

(e) correct substitution (of their p values) into formula for $E(X)$ (A1)

e.g. $4 \times 0.35 + 10 \times 0.565 + 12 \times 0.085$

$E(X) = 8.07$ (accept \$8.07) A1 N2

[15]

9. $X \sim N(7, 0.5^2)$

(a) (i) $z = 2$ (M1)

$P(X < 8) = P(Z < 2) = 0.977$ A1 N2

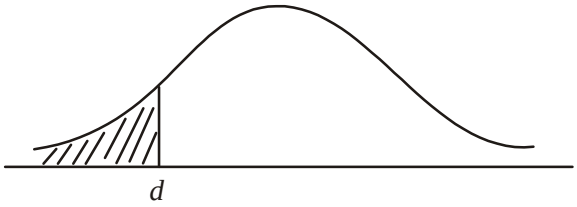
(ii) evidence of appropriate approach (M1)

e.g. symmetry, $z = -2$

$P(6 < X < 8) = 0.954$ (tables 0.955) A1 N2

Note: *Award M1A1(AP) if candidates refer to 2 standard deviations from the mean, leading to 0.95.*

(b) (i)



A1A1 N2

Note: *Award A1 for d to the left of the mean, A1 for area to the left of d shaded.*

(ii) $z = -1.645$ (A1)

$\frac{d-7}{0.5} = -1.645$ (M1)

$d = 6.18$ A1 N3

(c) $Y \sim N(\mu, 0.5^2)$

$P(Y < 5) = 0.2$ (M1)

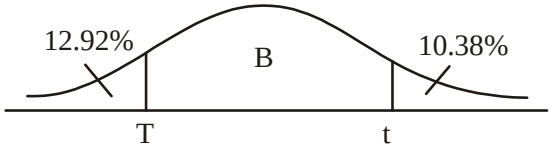
$z = -0.84162...$ A1

$\frac{5-\mu}{0.5} = -0.8416$ (M1)

$\mu = 5.42$ A1 N3

[13]

10. (a)



A1A1 N2

Notes: *Award A1 for three re.g.ions, (may be shown by lines or shading) A1 for clear labelling of two re.g.ions (may be shown by percentages or cate.g.ories).*

r and t need not be labelled, but if they are, they may be interchanged.

(b) **METHOD 1**

$P(X < r) = 0.1292$ (A1)

$r = 6.56$ A1 N2

$1 - 0.1038$ (= 0.8962) (may be seen later) A1

$P(X < t) = 0.8962$ (A1)

$t = 7.16$ A1 N2

METHOD 2

finding z-values $-1.130...$, $1.260...$ A1A1

evidence of setting up one standardized equation (M1)

e.g. $\frac{r-6.84}{0.25} = -1.13\text{K}$, $t = 1.260 \times 0.25 + 6.84$

$r = 6.56$, $t = 7.16$ A1A1 N2N2

[7]

11. (a) $E(X) = 2$ A1 N1

(b) evidence of appropriate approach involving binomial (M1)

e.g. $\binom{10}{3}(0.2)^3$, $(0.2)^3(0.8)^7$, $X \sim B(10, 0.2)$

$P(X = 3) = 0.201$ A1 N2

(c) **METHOD 1**

$P(X \leq 3) = 0.10737 + 0.26844 + 0.30199 + 0.20133$ (= 0.87912...) (A1)

evidence of using the complement (seen anywhere) (M1)

e.g. 1 – any probability, $P(X > 3) = 1 - P(X \leq 3)$

$P(X > 3) = 0.121$ A1 N2

METHOD 2

recognizing that $P(X > 3) = P(X \geq 4)$ (M1)
e.g. summing probabilities from $X = 4$ to $X = 10$

correct expression or values (A1)
e.g. $\sum_{r=4}^{10} \binom{10}{r} (0.2)^{10-r}(0.8)^r$

$0.08808 + 0.02642 + 0.005505 + 0.000786 + 0.0000737 + 0.000004 + 0.0000001$

$P(X > 3) = 0.121$ A1 N2

[6]

12. The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than 90 km h^{-1} and 12 % of them at speeds less than 40 km h^{-1}. Find μ and σ.

(Total 6 marks)