DULWICH COLLEGE SHANGHAI

NAME _____

FORM _____ TEACHER _____

YEAR 12 END OF YEAR EXAM 2013

MATHEMATICS STANDARD LEVEL

PAPER 2: NON-CALCULATOR

Instructions to candidates:

- Answer **all** questions.
- You may **not** use a calculator.
- Write in blue or black ink. Do not use correction fluid.
- Diagrams and sketches should be drawn in pencil.
- Unless otherwise stated in the question, give answers exactly or to 3 significant figures.
- Answers to Section A should be written on the test paper and Section B on file paper.
- The number of marks allocated to each question is indicated by [ ].
- The total number of marks is 60.

_____ 60	%

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 6]*

In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

(a) Find d. *[2 marks]*

(b) Find u_{20}. *[2 marks]*

(c) Find S_{20}. *[2 marks]*

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2. *[Maximum mark: 5]*

Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$. *[2 marks]*

(b) Write down $g^{-1}(x)$. *[1 mark]*

(c) Find $(f \circ g^{-1})(5)$. *[2 marks]*

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3. *[Maximum mark: 4]*

Let $f(x) = \frac{1}{4}x^2 + 2$. The line L is the tangent to the curve of f at $(4, 6)$.

Find the equation of L. *[4 marks]*

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4. *[Maximum mark: 8]*

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

(a) Find $\cos\theta$. *[3 marks]*

(b) Find $\tan 2\theta$. *[5 marks]*

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5. *[Maximum mark: 7]*

Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k. *[5 marks]*

(b) The line $y = p$ intersects the graph of f. Find all possible values of p. *[2 marks]*

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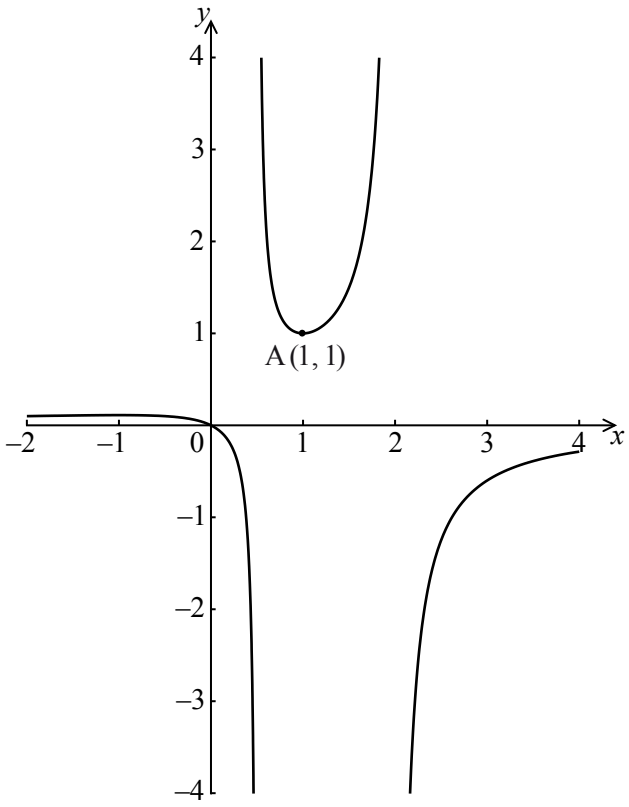
SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

*Do **NOT** write solutions on this page.*

6. *[Maximum mark: 16]*

Let $f(x) = \dfrac{x}{-2x^2 + 5x - 2}$ for $-2 \le x \le 4$, $x \ne \dfrac{1}{2}$, $x \ne 2$. The graph of f is given below.



The graph of f has a local minimum at $\mathrm{A}(1, 1)$ and a local maximum at B.

(a) Use the quotient rule to show that $f'(x) = \dfrac{2x^2 - 2}{(-2x^2 + 5x - 2)^2}$. *[6 marks]*

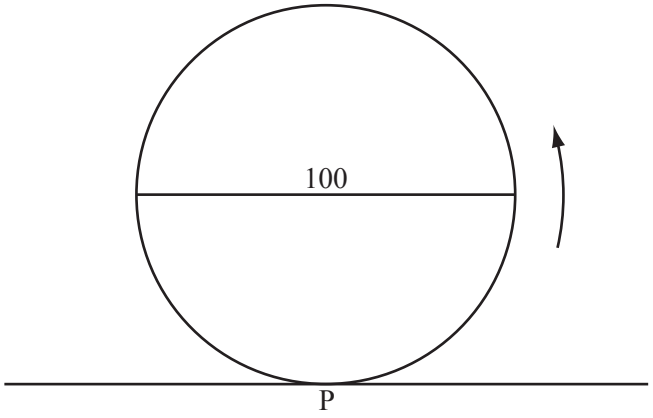
(b) Hence find the coordinates of B. *[7 marks]*

(c) Given that the line $y = k$ does not meet the graph of f, find the possible values of k. *[3 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

7. *[Maximum mark: 14]*

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction. One revolution takes 20 minutes.

(a) Write down the height of P above ground level after

(i) 10 minutes;

(ii) 15 minutes. *[2 marks]*

Let $h(t)$ metres be the height of P above ground level after t minutes. Some values of $h(t)$ are given in the table below.

t	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

(b) (i) Show that $h(8) = 90.5$.

(ii) Find $h(21)$. *[4 marks]*

(c) **Sketch** the graph of h, for $0 \le t \le 40$. *[3 marks]*

(d) Given that h can be expressed in the form $h(t) = a\cos bt + c$, find a, b and c. *[5 marks]*