DULWICH COLLEGE SHANGHAI

NAME _____

FORM _____ TEACHER _____

YEAR 12 END OF YEAR EXAM 2013

MATHEMATICS STANDARD LEVEL

PAPER 1: CALCULATOR

Instructions to candidates:

- Answer **all** questions.
- You **may** use a calculator.
- Write in blue or black ink. Do not use correction fluid.
- Diagrams and sketches should be drawn in pencil.
- Unless otherwise stated in the question, give answers exactly or to 3 significant figures.
- Answers to Section A should be written on the test paper and Section B on file paper.
- The number of marks allocated to each question is indicated by [ ].
- The total number of marks is 58

_____ 58	%

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 6]*

The following diagram shows triangle ABC.

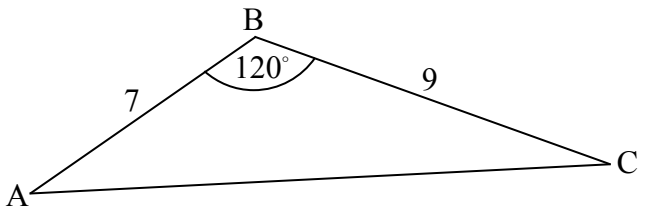

diagram not to scale

$AB = 7\,\text{cm}$, $BC = 9\,\text{cm}$ and $A\hat{B}C = 120^\circ$.

(a) Find AC. *[3 marks]*

(b) Find $B\hat{A}C$. *[3 marks]*

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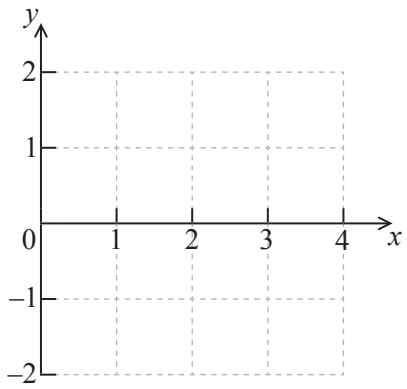
2. *[Maximum mark: 6]*

Let $g(x) = \frac{1}{2} x \sin x$, for $0 \le x \le 4$.

(a) Sketch the graph of g on the following set of axes. *[4 marks]*



(b) Hence find the value of x for which $g(x) = -1$. *[2 marks]*

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3. *[Maximum mark: 6]*

Consider the following circle with centre O and radius 6.8 cm.

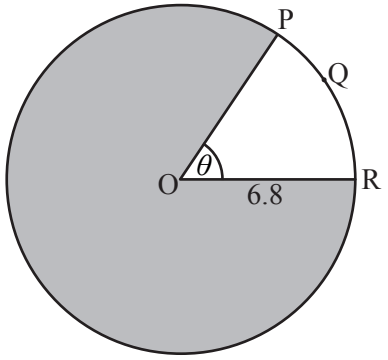

diagram not to scale

The length of the arc PQR is 8.5 cm.

(a) Find the value of θ. *[2 marks]*

(b) Find the area of the shaded region. *[4 marks]*

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4. *[Maximum mark: 5]*

Consider the expansion of $(3x^2 + 2)^9$.

(a) Write down the number of terms in the expansion. *[1 mark]*

(b) Find the term in x^4. *[4 marks]*

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5. *[Maximum mark: 5]*

Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$.

(a) There are two points of inflexion on the graph of f. Write down the x-coordinates of these points. *[3 marks]*

(b) Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion. *[2 marks]*

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

6. *[Maximum mark: 14]*

(a) Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$.

(i) Find u_4.

(ii) Find the sum of the infinite sequence. *[4 marks]*

Consider an arithmetic sequence with n terms, with first term (-36) and eighth term (-8).

(b) (i) Find the common difference.

(ii) Show that $S_n = 2n^2 - 38n$. *[5 marks]*

(c) The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find n. *[5 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

7 *[Maximum mark: 16]*

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$. *[2 marks]*

(b) Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

The function f can also be written in the form $f(x) = \frac{\log ax}{\log b}$.

(c) (i) Write down the value of a and of b.

(ii) Hence on graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote. *[6 marks]*

(d) Write down the value of $f^{-1}(0)$. *[1 mark]*

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A. *[4 marks]*