NORMAL DISTRIBUTIONS 2

1) In a large school, the heights of all fourteen-year-old students are measured.

The heights of the girls are normally distributed with mean $155\,\text{cm}$ and standard deviation $10\,\text{cm}$.

The heights of the boys are normally distributed with mean $160\,\text{cm}$ and standard deviation $12\,\text{cm}$.

(a) Find the probability that a girl is taller than $170\,\text{cm}$. *[3 marks]*

(b) Given that 10% of the girls are shorter than $x\,\text{cm}$, find x. *[3 marks]*

(c) Given that 90% of the boys have heights between $q\,\text{cm}$ and $r\,\text{cm}$ where q and r are symmetrical about $160\,\text{cm}$, and $q < r$, find the value of q and of r. *[4 marks]*

2) The heights of a group of students are normally distributed with a mean of 160 cm and a standard deviation of 20 cm.

(a) A student is chosen at random. Find the probability that the student's height is greater than 180 cm.

(b) In this group of students, 11.9 % have heights less than d cm. Find the value of d.

3) The heights of boys at a particular school follow a normal distribution with a standard deviation of $5\,\text{cm}$. The probability of a boy being shorter than 153 cm is 0.705.

(a) Calculate the mean height of the boys.

(b) Find the probability of a boy being taller than $156\,\text{cm}$.

4) The weights of a group of children are normally distributed with a mean of 22.5 kg and a standard deviation of $2.2\,\text{kg}$.

(a) Write down the probability that a child selected at random has a weight more than 25.8 kg.

(b) Of the group 95 % weigh less than k kilograms. Find the value of k.

4c)

(c)   The diagram below shows a normal curve.
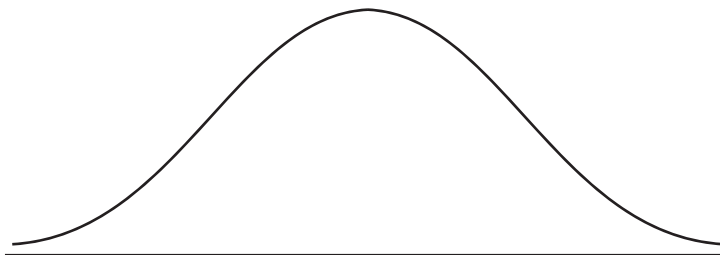

On the diagram, shade the region that represents the following information:

87 % of the children weigh less than 25 kg

5)

(i)   Reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds.

(a)   The graph below is that of the standard normal curve. The shaded area represents the probability that the reaction time of a person chosen at random is between 0.70 and 0.79 seconds.

a 0 b
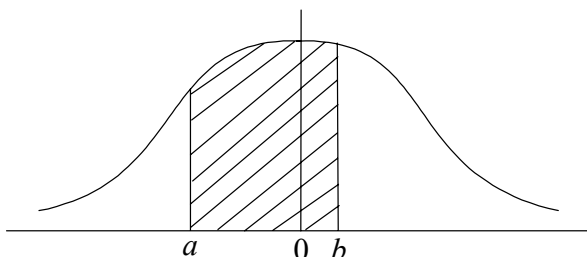
(i)   Write down the value of a and of b.

(ii)   Calculate the probability that the reaction time of a person chosen at random is

(a)   greater than 0.70 seconds;

(b)   between 0.70 and 0.79 seconds.   *[6 marks]*

Three percent (3 %) of the population have a reaction time less than c seconds.

(b)   (i)   Represent this information on a diagram similar to the one above. Indicate clearly the area representing 3 %.

(ii)   Find c.   *[4 marks]*

6) The scores of a test given to students are normally distributed with a mean of 21. 80 % of the students have scores less than 23.7.

(a) Find the standard deviation of the scores. *[3 marks]*

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than *b*.

(b) (i) Find the probability the student has a score less than 25.4.

(ii) Find the value of *b*. *[4 marks]*

7) A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $P(X \le 475)$. *[2 marks]*

(b) Given that $P(X > a) = 0.27$, find *a*. *[4 marks]*

8) The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg. *[2 marks]*

(b) (i) Write down the standardized value, *z*, for 68 kg.

(ii) Hence, find the standard deviation of weights. *[4 marks]*

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament. *[5 marks]*

9)

The scores of a test given to students are normally distributed with a mean of 21. 80 % of the students have scores less than 23.7.

(a)   Find the standard deviation of the scores.   *[3 marks]*

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than b.

(b)   (i)   Find the probability the student has a score less than 25.4.

(ii)   Find the value of b.   *[4 marks]*