Modelling functions Answers

1)

8. (a) **METHOD 1**

evidence of recognizing the amplitude is the radius *(M1)*
e.g. amplitude is half the diameter

$a = \frac{8}{2}$ *A1*

$a = 4$ *AG* *N0*
[2 marks]

METHOD 2

evidence of recognizing the maximum height *(M1)*
e.g. $h = 6$, $a \sin bt + 2 = 6$

correct reasoning
e.g. $a \sin bt = 4$ and $\sin bt$ has amplitude of 1 *A1*

$a = 4$ *AG* *N0*
[2 marks]

(b) **METHOD 1**

period $= 30$ *(A1)*

$b = \frac{2\pi}{30}$ *A1*

$b = \frac{\pi}{15}$ *AG* *N0*
[2 marks]

METHOD 2

correct equation *(A1)*
e.g. $2 = 4 \sin 30b + 2$, $\sin 30b = 0$

$30b = 2\pi$ *A1*

$b = \frac{\pi}{15}$ *AG* *N0*
[2 marks]

continued …

Modelling functions Answers

1)

Question 8 continued

(c) recognizing $h'(t) = -0.5$ (seen anywhere) ***R1***

attempting to solve ***(M1)***
e.g. sketch of h', finding h'

correct work involving h' ***A2***

e.g. sketch of h' showing intersection, $-0.5 = \frac{4\pi}{15}\cos\left(\frac{\pi}{15}t\right)$

$t = 10.6$, $t = 19.4$ ***A1A1*** ***N3***

[6 marks]

(d) **METHOD 1**

valid reasoning for **their** conclusion (seen anywhere) ***R1***
e.g. $h(t) < 0$ so underwater; $h(t) > 0$ so not underwater

evidence of substituting into h ***(M1)***

e.g. $h(19.4)$, $4\sin\frac{19.4\pi}{15} + 2$

correct calculation ***A1***
e.g. $h(19.4) = -1.19$

correct statement ***A1*** ***N0***
e.g. the bucket is underwater, yes

[4 marks]

METHOD 2

valid reasoning for **their** conclusion (seen anywhere) ***R1***
e.g. $h(t) < 0$ so underwater; $h(t) > 0$ so not underwater

evidence of valid approach ***(M1)***
e.g. solving $h(t) = 0$, graph showing region below x-axis

correct roots ***A1***
e.g. 17.5, 27.5

correct statement ***A1*** ***N0***
e.g. the bucket is underwater, yes

[4 marks]

Total [14 marks]

M11/5/MATME/SP2/ENG/TZ1/XX/M

2)

QUESTION 10

(a)	valid approach *e.g.* 15 mins is half way, top of the wheel, $d+1$ height $=101$ (metres)	***(M1)*** ***A1***	 ***N2***
			[2 marks]
(b)	evidence of identifying rotation angle after 6 minutes *e.g.* $\frac{2\pi}{5}$, $\frac{1}{5}$ of a rotation, 72°	***A1***	
	evidence of appropriate approach *e.g.* drawing a right triangle and using cosine ratio	***(M1)***	
	correct working (seen anywhere) *e.g.* $\cos\frac{2\pi}{5}=\frac{x}{50}$, 15.4(508…)	***A1***	
	evidence of appropriate method *e.g.* height $=$ radius $+1-15.45\ldots$	***M1***	
	height $=35.5$ (metres) (accept 35.6)	***A1***	***N2***
			[5 marks]

continued …

2)

Question 10 continued

(c) **METHOD 1**

evidence of substituting into $b = \frac{2\pi}{\text{period}}$ ***(M1)***

correct substitution

e.g. period $= 30$ minutes, $b = \frac{2\pi}{30}$ ***A1***

$b = 0.209\left(\frac{\pi}{15}\right)$ ***A1*** ***N2***

substituting into $h(t)$ ***(M1)***

e.g. $h(0) = 1$, $h(15) = 101$

correct substitution ***A1***

$1 = 50\sin\left(-\frac{\pi}{15}c\right) + 51$

$c = 7.5$ ***A1*** ***N2***

[6 marks]

METHOD 2

evidence of setting up a system of equations ***(M1)***

two correct equations

e.g. $1 = 50\sin b(0 - c) + 51$, $101 = 50\sin b(15 - c) + 51$ ***A1A1***

attempt to solve simultaneously ***(M1)***

e.g. evidence of combining two equations

$b = 0.209\left(\frac{\pi}{15}\right)$, $c = 7.5$ ***A1A1*** ***N2N2***

[6 marks]

(d) evidence of solving $h(t) = 96$ ***(M1)***

e.g. equation, graph

$t = 12.8$ (minutes) ***A2*** ***N3***

[3 marks]

Total [16 marks]

3)

QUESTION 3

(a) $h = 24 - 14\sin 0$ *(M1)*
$= 24$ (cm) *(A1)* *(N2)*

[2 marks]

(b) 38 (cm) *(A1)*

[1 mark]

(c) 2.36 (secs) $\left(\text{or } \frac{3\pi}{4}\right)$ (Accept 135 (working in degrees)) *(A1)*

[1 mark]

(d) Use of period of $\sin bt = \frac{2\pi}{b}$ *(M1)*

$\frac{2\pi}{2} = \pi$ (Accept 180) *(A1)* *(N2)*

[2 marks]

(e) $a = 14$ *(A1)*
$b = 24$ *(A1)*

[2 marks]

(f) $h = 24 + 14\sin 2t$ *(A2)*

OR

$h = 24 - 14\sin(2t + \pi)$ *(A2)*

OR

$h = 24 - 14\cos\left(2t + \frac{\pi}{2}\right)$ *(A2)*

[2 marks]

Total [10 marks]

M04/522/S(2)

4)

QUESTION 10

(a)	(i)	attempt to substitute *e.g.* $a=\frac{29-15}{2}$	*(M1)*	
		$a=7$ (accept $a=-7$)	*A1*	*N2*
	(ii)	period $=12$	*(A1)*	
		$b=\frac{2\pi}{12}$	*A1*	
		$b=\frac{\pi}{6}$	*AG*	*N0*
	(iii)	attempt to substitute *e.g.* $d=\frac{29+15}{2}$	*(M1)*	
		$d=22$	*A1*	*N2*
	(iv)	$c=3$ (accept $c=9$ from $a=-7$)	*A1*	*N1*

Note: Other correct values for c can be found, $c=3\pm 12k$, $k\in\mathbb{Z}$.

[7 marks]

(b)	stretch takes 3 to 1.5	*(A1)*	
	translation maps $(1.5, 29)$ to $(4.5, 19)$ $\left(\text{so M}' \text{ is } (4.5, 19)\right)$	*A1*	*N2*

[2 marks]

(c)	$g(t)=7\cos\frac{\pi}{3}(t-4.5)+12$	*A1A2A1*	*N4*

Note: Award *A1* for $\frac{\pi}{3}$, *A2* for 4.5, *A1* for 12.
Other correct values for c can be found $c=4.5\pm 6k$, $k\in\mathbb{Z}$.

[4 marks]

(d)	translation $\begin{pmatrix}-3\\10\end{pmatrix}$	*(A1)*	
	horizontal stretch of a scale factor of 2	*(A1)*	
	completely correct description, in correct order *e.g.* translation $\begin{pmatrix}-3\\10\end{pmatrix}$ then horizontal stretch of a scale factor of 2	*A1*	*N3*

[3 marks]

Total [16 marks]

5)

QUESTION 8

(a)	(i)	7	*A1*	*N1*
	(ii)	1	*A1*	*N1*
	(iii)	10	*A1*	*N1*
				[3 marks]
(b)	(i)	evidence of appropriate approach *e.g.* $A=\frac{18-2}{2}$	*M1*	
		$A=8$	*AG*	*N0*
	(ii)	$C=10$	*A2*	*N2*
	(iii)	**METHOD 1**		
		period $=12$	*(A1)*	
		evidence of using $B\times$ period $=2\pi$ (accept 360°) *e.g.* $12=\frac{2\pi}{B}$	*(M1)*	
		$B=\frac{\pi}{6}$ (accept 0.524 or 30)	*A1*	*N3*
		METHOD 2		
		evidence of substituting *e.g.* $10=8\cos 3B+10$	*(M1)*	
		simplifying *e.g.* $\cos 3B=0\ \left(3B=\frac{\pi}{2}\right)$	*(A1)*	
		$B=\frac{\pi}{6}$ (accept 0.524 or 30)	*A1*	*N3*
				[6 marks]
(c)		correct answers *e.g.* $t=3.52$, $t=10.5$, between 03:31 and 10:29 (accept 10:30)	*A1A1*	*N2*
				[2 marks]

Total [11 marks]

M08/5/MATME/SP2/ENG/TZ2/XX+

6)

QUESTION 5

Note: Accept **exact** answers given in terms of π.

(a) Evidence of using $l = r\theta$ *(M1)*

arc AB $= 7.85$ (m) *A1* *N2*

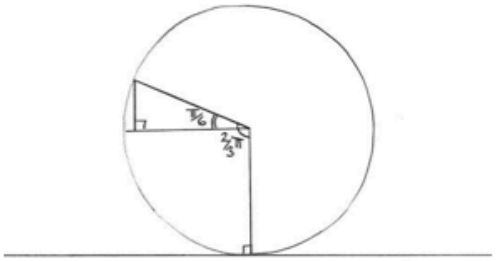
[2 marks]

(b) Evidence of using $A = \frac{1}{2}r^2\theta$ *(M1)*

Area of sector AOB $= 58.9$ (m^2) *A1* *N2*

[2 marks]

(c) **METHOD 1**



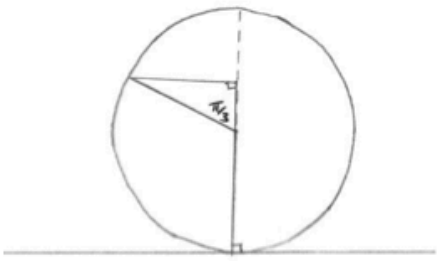
angle $= \frac{\pi}{6}$ (30°) *(A1)*

attempt to find $15\sin\frac{\pi}{6}$ *M1*

height $= 15 + 15\sin\frac{\pi}{6}$

$= 22.5$ (m) *A1* *N2*

METHOD 2



angle $= \frac{\pi}{3}$ (60°) *(A1)*

attempt to find $15\cos\frac{\pi}{3}$ *M1*

height $= 15 + 15\cos\frac{\pi}{3}$

$= 22.5$ (m) *A1* *N2*

[3 marks]

continued ...

6)

Question 5 continued

(d) (i) $h\left(\frac{\pi}{4}\right) = 15 - 15\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$ ***(M1)***

$= 25.6$ (m) ***A1*** ***N2***

(ii) $h(0) = 15 - 15\cos\left(0 + \frac{\pi}{4}\right)$ ***(M1)***

$= 4.39$ (m) ***A1*** ***N2***

(iii) **METHOD 1**

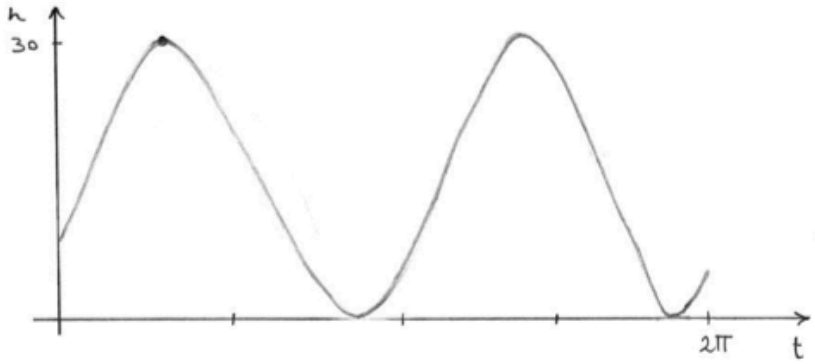
Highest point when $h = 30$ ***R1***

$30 = 15 - 15\cos\left(2t + \frac{\pi}{4}\right)$ ***M1***

$\cos\left(2t + \frac{\pi}{4}\right) = -1$ ***(A1)***

$t = 1.18 \left(\text{accept } \frac{3\pi}{8}\right)$ ***A1*** ***N2***

METHOD 2



Sketch of graph of h ***M2***

Correct maximum indicated ***(A1)***

$t = 1.18$ ***A1*** ***N2***

METHOD 3

Evidence of setting $h'(t) = 0$ ***M1***

$\sin\left(2t + \frac{\pi}{4}\right) = 0$ ***(A1)***

Justification of maximum ***R1***

e.g. reasoning from diagram, first derivative test, second derivative test

$t = 1.18 \left(\text{accept } \frac{3\pi}{8}\right)$ ***A1*** ***N2***

[8 marks]

continued ...

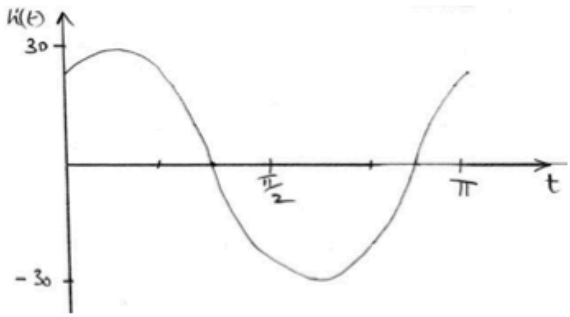
6)

Question 5 continued

(e) $h'(t) = 30\sin\left(2t + \frac{\pi}{4}\right)$ (may be seen in part (d)) ***A1A1*** ***N2***

[2 marks]

(f) (i)



A1A1A1 ***N3***

Note:	Award ***A1*** for range –30 to 30, ***A1*** for two zeros. Award ***A1*** for approximate correct **sinusoidal** shape.

(ii) **METHOD 1**

Maximum on graph of h' ***(M1)***
$t = 0.393$ ***A1*** ***N2***

METHOD 2

Minimum on graph of h' ***(M1)***
$t = 1.96$ ***A1*** ***N2***

METHOD 3

Solving $h''(t) = 0$ ***(M1)***
One or both correct answers ***A1***
$t = 0.393$, $t = 1.96$ ***N2***
[5 marks]

Total [22 marks]