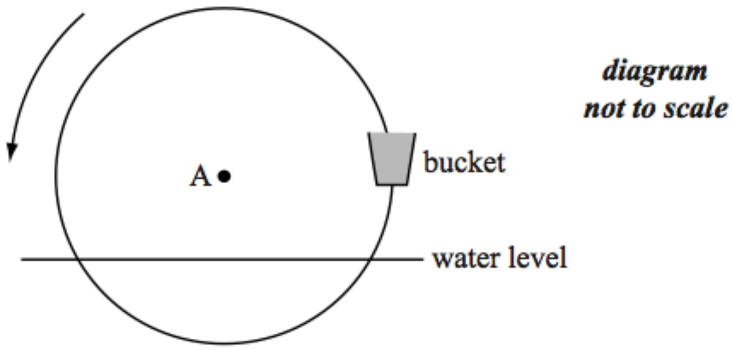
Modelling functions

1)

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counterclockwise) direction.



The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a \sin bt + 2$.

(a) Show that $a = 4$. *[2 marks]*

The wheel turns at a rate of one rotation every 30 seconds.

(b) Show that $b = \frac{\pi}{15}$. *[2 marks]*

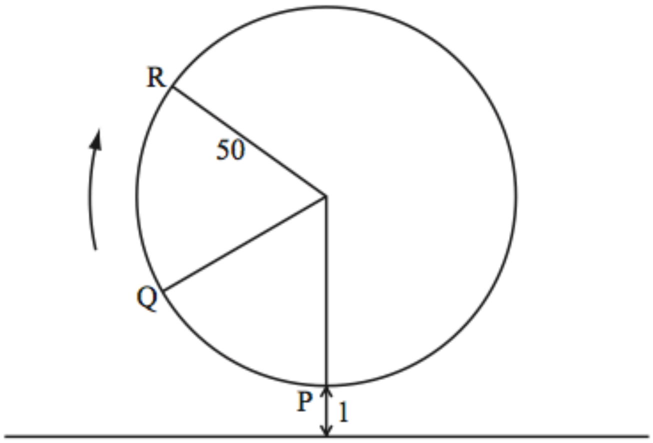
In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 m s^{-1}.

(c) Find these values of t. *[6 marks]*

(d) Determine whether the bucket is underwater at the second value of t. *[4 marks]*

2)

The following diagram represents a large Ferris wheel at an amusement park. The points P, Q and R represent different positions of a seat on the wheel.



The wheel has a radius of 50 metres and rotates clockwise at a rate of one revolution every 30 minutes.

A seat starts at the lowest point P, when its height is one metre above the ground.

(a) Find the height of a seat above the ground after 15 minutes. *[2 marks]*

(b) After six minutes, the seat is at point Q. Find its height above the ground at Q. *[5 marks]*

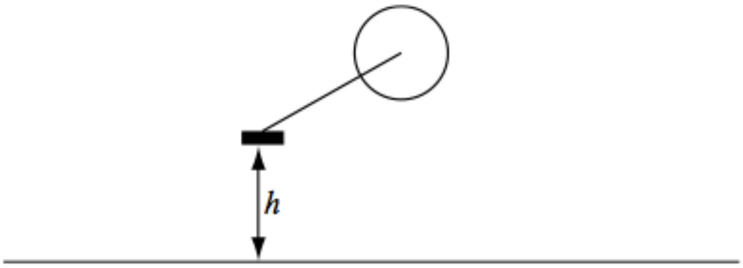
The height of the seat above ground after t minutes can be modelled by the function $h(t) = 50\sin(b(t-c)) + 51$.

(c) Find the value of b and of c. *[6 marks]*

(d) Hence find the value of t the first time the seat is 96 m above the ground. *[3 marks]*

3)

The diagram below shows a bicycle pedal.



The height h cm, of the bicycle pedal above the ground after t seconds is given by $h = 24 - 14\sin 2t$.

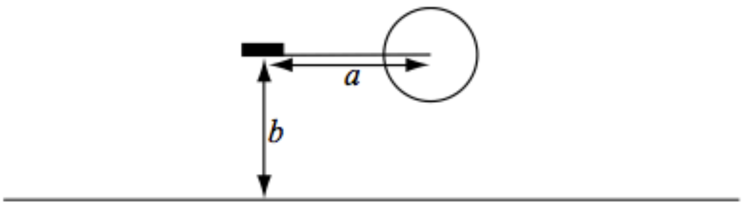
(a) Find the height when $t = 0$. *[2 marks]*

(b) Find the maximum height of the pedal above the ground. *[1 mark]*

(c) Find the first time at which this occurs. *[1 mark]*

(d) How long does one revolution of the pedal take? *[2 marks]*

The diagram shows the pedal in its starting position.



(e) Write down the lengths a and b. *[2 marks]*

(f) Write down a formula for the height of the other pedal above the ground at time t seconds. *[2 marks]*

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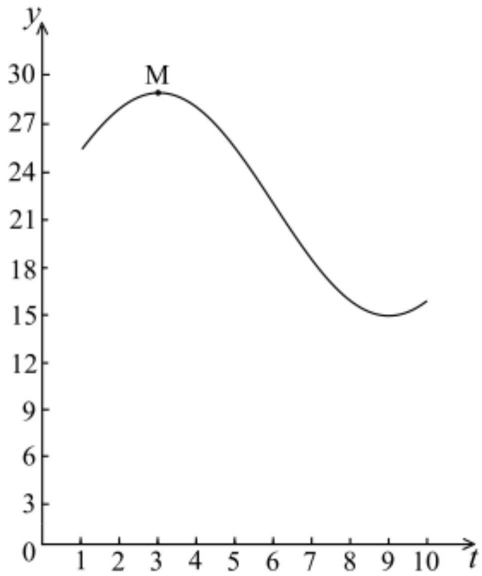
4) No calculator allowed for this one, sorry!

Let $f(t) = a\cos b(t-c)+d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c. *[7 marks]*

The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′. *[2 marks]*

The graph of g is the image of the graph of f under P.

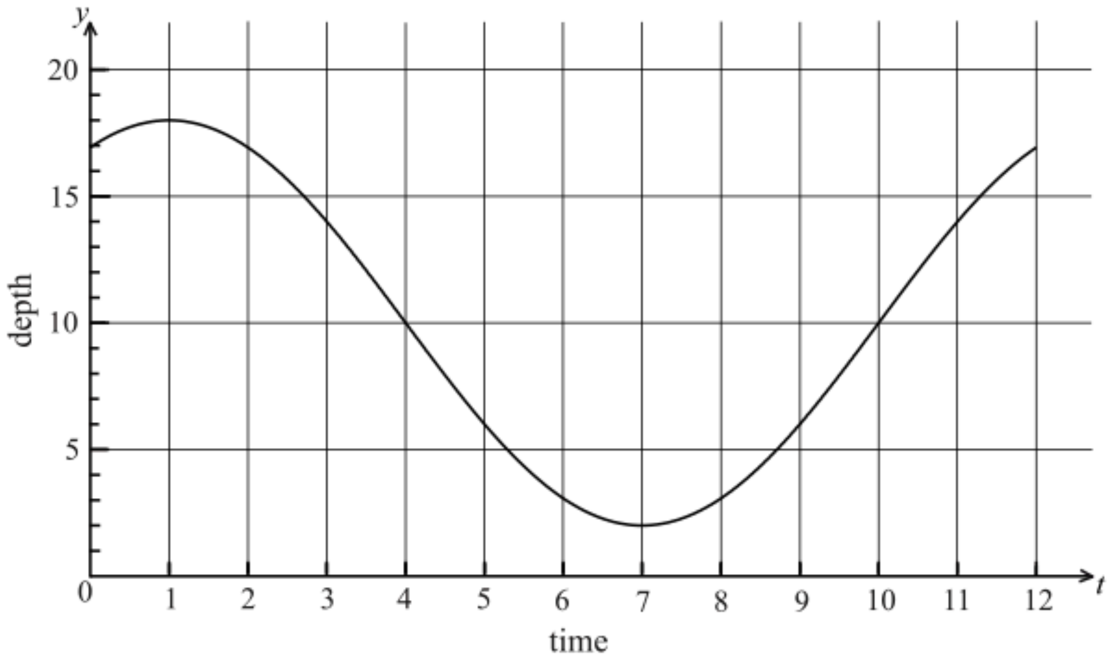
(c) Find $g(t)$ in the form $g(t) = 7\cos B(t-C)+D$. *[4 marks]*

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f. *[3 marks]*

Modelling functions

5)

The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly. *[3 marks]*

(b) The depth of water can be modelled by the function $y = A\cos\left(B(t-1)\right)+C$.

(i) Show that $A=8$.

(ii) Write down the value of C.

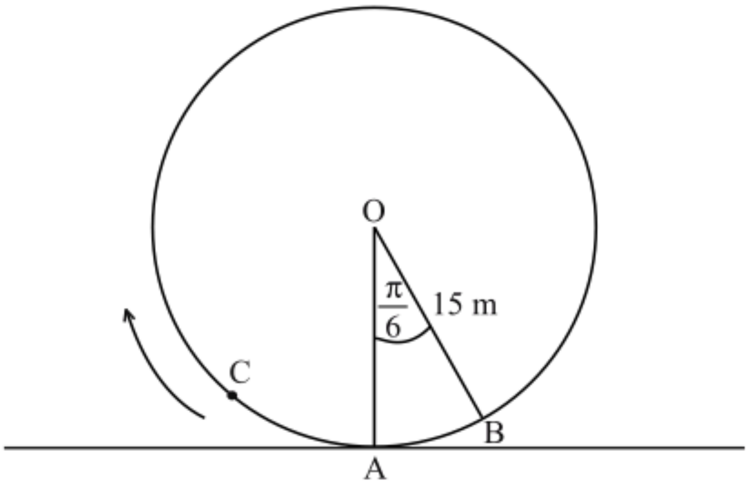
(iii) Find the value of B. *[6 marks]*

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P. *[2 marks]*

Modelling functions

6)

A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram below. Initially seat A is at ground level. The next seat is B, where $A\hat{O}B = \frac{\pi}{6}$.



(a) Find the length of the arc AB. *[2 marks]*

(b) Find the area of the sector AOB. *[2 marks]*

(c) The wheel turns clockwise through an angle of $\frac{2\pi}{3}$. Find the height of A above the ground. *[3 marks]*

The height, h metres, of seat C above the ground after t minutes, can be modelled by the function

$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right).$$

(d) (i) Find the height of seat C when $t = \frac{\pi}{4}$.

(ii) Find the initial height of seat C.

(iii) Find the time at which seat C first reaches its highest point. *[8 marks]*

(e) Find $h'(t)$. *[2 marks]*

(f) For $0 \le t \le \pi$,

(i) sketch the graph of h';

(ii) find the time at which the height is changing most rapidly. *[5 marks]*