Logs Revision and Answers worksheet (not past papers)

1. Rewrite each of these statements as a logarithm.

(i) $10^3 = 1000$ (ii) $2^7 = 128$

(iii) $10^{\frac{1}{3}} = \sqrt[3]{10}$ (iv) $2^{-3} = \frac{1}{8}$

(v) $5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}}$ (vi) $3^{\frac{3}{2}} = \sqrt{27}$

2. Find the values of the following:

(i) $\log_2 16$ (ii) $\log_{10} 1000000$

(iii) $\log_6 1$ (iv) $\log_4\left(\frac{1}{4}\right)$

(v) $\log_5 \sqrt{5}$ (vi) $\log_3\left(\frac{1}{27}\right)$

(vii) $\log_8 4$ (viii) $\log_2\left(\frac{1}{\sqrt{32}}\right)$

3. Find the value of x in each of the following:

(i) $\log_2 x = -5$ (ii) $\log_3 x = \frac{3}{2}$

(iii) $\log_x 64 = 2$ (iv) $\log_x\left(\frac{1}{\sqrt{5}}\right) = \frac{1}{2}$

4. Write the following as a single logarithm:

(i) $\log 2 + \log 3$ (ii) $\log 10 - \log 2$

(iii) $3 \log 5$ (iv) $2 \log 3 - 4 \log 2$

(v) $\frac{1}{2}\log 3 - \frac{1}{4}\log 4$ (vi) $2 \log a + 5 \log b - 3 \log c$

5. Write the following in terms of log 2 and log 3:

(i) $\log 12$ (ii) $\log\left(\frac{16}{27}\right)$

(iii) $\log\sqrt{54}$ (iv) $\log\frac{\sqrt{3}}{16}$

6. Solve the following equations:

(i) $2^x = 18$ (ii) $5^x = 100$

(iii) $1.5^x = 0.001$ (iv) $10^x = 2$

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1. (i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(ii) $2^7 = 128 \Rightarrow \log_2 128 = 7$

(iii) $10^{\frac{1}{3}} = \sqrt[3]{10} \Rightarrow \log_{10} \sqrt[3]{10} = \frac{1}{3}$

(iv) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$

(v) $5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}} \Rightarrow \log_5 \frac{1}{\sqrt{5}} = -\frac{1}{2}$

(vi) $3^{\frac{3}{2}} = \sqrt{27} \Rightarrow \log_3 \sqrt{27} = \frac{3}{2}$

2. (i) $x = \log_2 16 \Rightarrow 2^x = 16 \Rightarrow x = 4$
so $\log_2 16 = 4$

(ii) $x = \log_{10} 1000000 \Rightarrow 10^x = 1000000 \Rightarrow x = 6$
so $\log_{10} 1000000 = 6$

(iii) $x = \log_6 1 \Rightarrow 6^x = 1 \Rightarrow x = 0$
so $\log_6 1 = 0$

(iv) $x = \log_4 \left(\frac{1}{4}\right) \Rightarrow 4^x = \frac{1}{4} \Rightarrow x = -1$
so $\log_4 \left(\frac{1}{4}\right) = -1$

(v) $x = \log_5 \sqrt{5} \Rightarrow 5^x = \sqrt{5} \Rightarrow x = \frac{1}{2}$
so $\log_5 \sqrt{5} = \frac{1}{2}$

(vi) $x = \log_3 \left(\frac{1}{27}\right) \Rightarrow 3^x = \frac{1}{27} \Rightarrow x = -3$
so $\log_3 \left(\frac{1}{27}\right) = -3$

(vii) $x = \log_8 4 \Rightarrow 8^x = 4 \Rightarrow 2^{3x} = 2^2 \Rightarrow x = \frac{2}{3}$
so $\log_8 4 = \frac{2}{3}$

(viii) $x = \log_2\left(\frac{1}{\sqrt{32}}\right) \Rightarrow 2^x = \frac{1}{\sqrt{32}} = \frac{1}{\sqrt{2^5}} = 2^{-\frac{5}{2}} \Rightarrow x = -\frac{5}{2}$

so $\log_2\left(\frac{1}{\sqrt{32}}\right) = \frac{5}{2}$

3. (i) $\log_2 x = -5 \Rightarrow 2^{-5} = x \Rightarrow x = \frac{1}{32}$

(ii) $\log_3 x = \frac{3}{2} \Rightarrow 3^{\frac{3}{2}} = x \Rightarrow x = \sqrt{27}$

(iii) $\log_x 64 = 2 \Rightarrow x^2 = 64 \Rightarrow x = 8$

(iv) $\log_x\left(\frac{1}{\sqrt{5}}\right) = \frac{1}{2} \Rightarrow x^{\frac{1}{2}} = \frac{1}{\sqrt{5}} \Rightarrow x = \frac{1}{5}$

4. (i) $\log 2 + \log 3 = \log(2 \times 3) = \log 6$

(ii) $\log 10 - \log 2 = \log\frac{10}{2} = \log 5$

(iii) $3\log 5 = \log 5^3 = \log 125$

(iv) $2\log 3 - 4\log 2 = \log 3^2 - \log 2^4 = \log\frac{3^2}{2^4} = \log\frac{9}{16}$

(v) $\frac{1}{2}\log 3 - \frac{1}{4}\log 4 = \log 3^{\frac{1}{2}} - \log 4^{\frac{1}{4}} = \log\frac{\sqrt{3}}{\sqrt{2}} = \log\sqrt{\frac{3}{2}}$

(vi) $2\log a + 5\log b - 3\log c = \log a^2 + \log b^5 - \log c^3 = \log\frac{a^2 b^5}{c^3}$

5. (i) $\log 12 = \log(2^2 \times 3) = \log 2^2 + \log 3 = 2\log 2 + \log 3$

(ii) $\log\left(\frac{16}{27}\right) = \log\left(\frac{2^4}{3^3}\right) = \log 2^4 - \log 3^3 = 4\log 2 - 3\log 3$

(ii) $\log\sqrt{54} = \log\left(2 \times 3^3\right)^{\frac{1}{2}} = \log 2^{\frac{1}{2}} + \log 3^{\frac{3}{2}} = \frac{1}{2}\log 2 + \frac{3}{2}\log 3$

(iv) $\log\frac{\sqrt{3}}{16} = \log\frac{3^{\frac{1}{2}}}{2^4} = \log 3^{\frac{1}{2}} - \log 2^4 = \frac{1}{2}\log 3 - 4\log 2$

6. (i) $2^x = 18$

$\log 2^x = \log 18$

$x \log 2 = \log 18$

$x = \dfrac{\log 18}{\log 2} = 4.17$ (3 s.f.)

(ii) $5^x = 100$

$\log 5^x = \log 100$

$x \log 5 = \log 100$

$x = \dfrac{\log 100}{\log 5} = 2.86$ (3 s.f.)

(iii) $1.5^x = 0.001$

$\log 1.5^x = \log 0.001$

$x \log 1.5 = \log 0.001$

$x = \dfrac{\log 0.001}{\log 1.5} = -17.0$ (3 s.f.)

(iv) $10^x = 2$

$\log 10^x = \log 2$

$x \log 10 = \log 2$

$x = \dfrac{\log 2}{\log 10} = 0.301$ (3 s.f.)