Integration revision IB HL

1. Show that $\int \left(e^x \cos x\right) dx = \dfrac{e^x(\sin x + \cos x)}{2} + c$. [5 marks]

2. Integrate $\int \dfrac{x}{(x-2)^3} dx$ using the substitution $u = x - 2$. [4 marks]

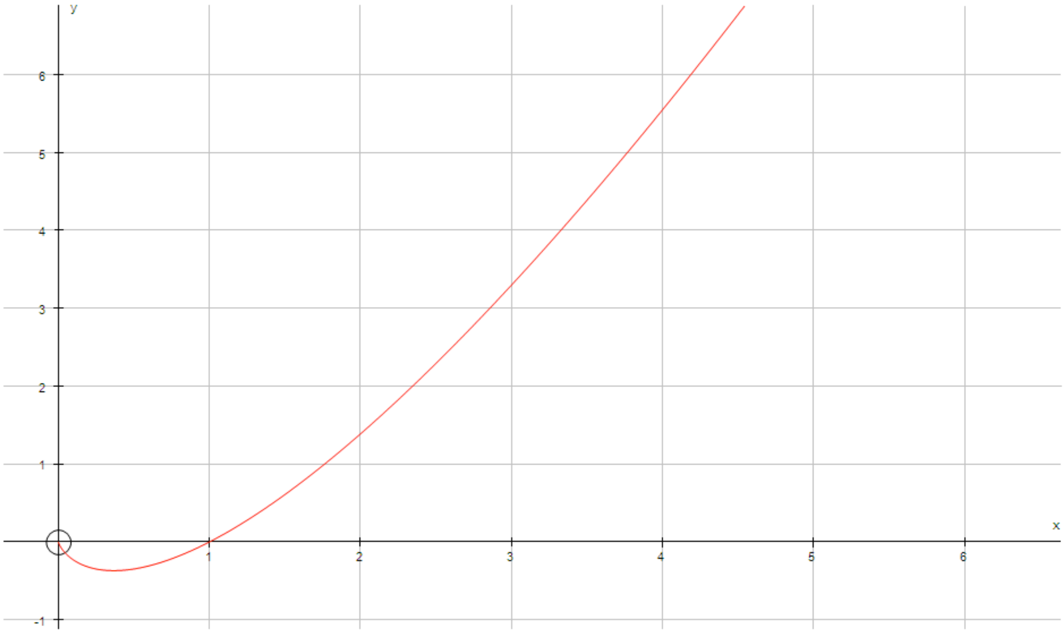
3. Evaluate $\int_1^3 \dfrac{4x}{(2x-1)} dx$. [5 marks]

4. a) Find $f(x)$ when $f'(x) = \sqrt{3x-4}$. [3 marks]

   b) Find the volume of revolution created when $f(x)$ is rotated through 2π radians about the x-axis and the lines $x=2$ and $x=0$. [3 marks]

5. Let $f(x) = x \sin x$. Integrate the function by parts. [4 marks]

6. The diagram below shows the curve $y = x\ln(x)$.



   a) Integrate $y = x\ln(x)$ with respect to x. [5 marks]

   b) Find the area between the x-axis, the lines $x=2$ and $x=4$ and the graph of $y = x\ln(x)$. [2 marks]

7. Integrate $\int \frac{dx}{4+x^2}$ by using the substitution $x = 2\tan u$.

Show all your working. *[5 marks]*

8. A particle is moving in a straight line with velocity given by,

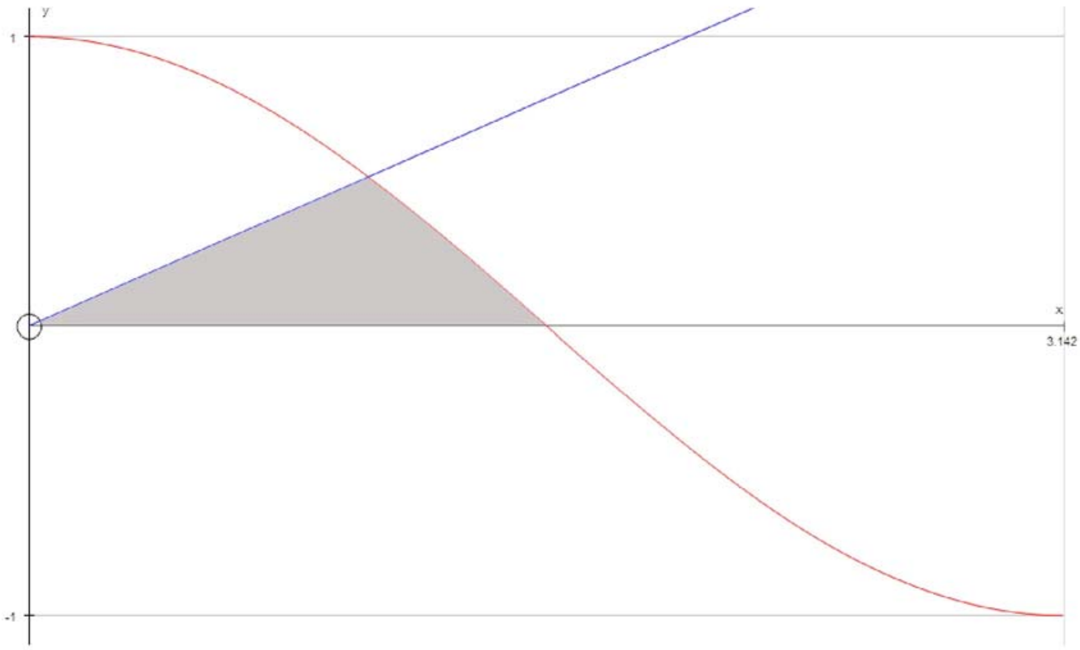
$v(t) = 5t^2 - 9t + 1$, where t is time in seconds and v is metres per second.

a) Find the distance traveled in the first 4 seconds. *[4 marks]*

b) Find the acceleration at 5 seconds. *[3 marks]*

c) Find an expression for the distance traveled, if the initial displacement is 4 metres. *[2 marks]*

9. The diagram below show two graphs of $y = \frac{1}{2}x$ and $y = \cos x$.



Find the shaded region shown in the diagram. *[5 marks]*

Answers

2. $-\dfrac{1}{(x-2)}-\dfrac{1}{(x-2)^2}+c$

3. 5.61 units^2

4. a) $\dfrac{2(3x-4)^{\frac{3}{2}}}{9}+c$

   b) $\dfrac{240\pi}{243}$ units^2

5. $\sin x - x\cos x + c$

6. a) $\dfrac{x^2\ln x}{2}-\dfrac{x^2}{4}+c$

   b) 5.71 units^2

7. $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right)+c$

8. a) $\dfrac{232}{6}$ metres

   b) 41 m/s^{-2}

   c) $S=\dfrac{5t^3}{3}-\dfrac{9t^2}{2}+t+4$

9. 0.408 units^2