

## Integration Past Paper Solutions

1. **Note:** Do not penalize for the omission of C.

(a)  $\int \sin(3x + 7) dx = -\frac{1}{3} \cos(3x + 7) + C$  (A1)(A1) (C2)

**Note:** Award (A1) for  $\frac{1}{3}$ , (A1) for  $-\cos(3x + 7)$ .

(b)  $\int e^{-4x} dx = -\frac{1}{4} e^{-4x} + C$  (A1)(A1) (C2)

**Note:** Award (A1) for  $-\frac{1}{4}$ , (A1) for  $e^{-4x}$ .

[4]

2. (a)  $f'(x) = 5(3x + 4)^4 \times 3 (= 15(3x + 4)^4)$  (A1)(A1)(A1) (C3)

(b)  $\int (3x + 4)^5 dx = \frac{1}{3} \times \frac{1}{6} (3x + 4)^6 + C \left( = \frac{(3x + 4)^6}{18} + C \right)$  (A1)(A1)(A1) (C3)

[6]

3. (a) Using the chain rule

$$f'(x) = (2 \cos(5x - 3)) 5 (= 10 \cos(5x - 3)) \quad \text{A1}$$

$$\begin{aligned} f''(x) &= -(10 \sin(5x - 3)) 5 \\ &= -50 \sin(5x - 3) \end{aligned} \quad \text{A1A1} \quad 4$$

**Note:** Award (A1) for  $\sin(5x - 3)$ , (A1) for  $-50$ .

(b)  $\int f(x) dx = \frac{2}{5} \cos(5x - 3) + C$  A1A1 2

**Note:** Award (A1) for  $\cos(5x - 3)$ , (A1) for  $-\frac{2}{5}$ .

[6]

4. Attempting to integrate.

(M1)

$$y = x^3 - 5x + c \quad (\text{A1})(\text{A1})(\text{A1})$$

$$\text{substitute } (2, 6) \text{ to find } c \left( 6 = 2^3 - 5(2) + c \right) \quad (\text{M1})$$

$$c = 8 \quad (\text{A1})$$

$$y = x^3 - 5x + 8 \quad (\text{Accept } x^3 - 5x + 8) \quad (\text{C6})$$

[6]

5.  $y = \int \frac{dy}{dx} dx$  (M1)  
 $= \frac{x^4}{4} + \frac{2x^2}{2} - x + c$  (A1)(A1)

**Note:** Award (A1) for first 3 terms, (A1) for “+ c”.

$$13 = \frac{16}{4} + 4 - 2 + c \quad (\text{M1})$$

$$c = 7 \quad (\text{A1})$$

$$y = \frac{x^4}{4} + x^2 - x - 7 \quad (\text{A1}) \quad (\text{C6})$$

[6]

6.  $f(x) = -\frac{1}{2}e^{-2x} - \ln(1-x) + c$  (M1)(A1)(A1)

$$\text{Substituting } 4 = -\frac{1}{2}e^{-2(0)} - \ln(1-0) + c \quad \left( \text{or } 4 = -\frac{1}{2} - \ln 1 + c \right) \quad (\text{M1})$$

$$c = 4.5 \quad (\text{A1})$$

$$f(x) = -\frac{1}{2}e^{-2x} - \ln(1-x) + 4.5 \quad (\text{A1})(\text{C2})(\text{C2})(\text{C2})$$

[6]

7. Using  $\int \frac{1}{x} dx = \ln x$  (may be implied) (M1)

$$\int_3^k \frac{1}{x-2} dx = [\ln(x-2)]_3^k \quad (\text{A1})$$

$$= \ln(k-2) - \ln 1 \quad (\text{A1})(\text{A1})$$

$$\ln(k-2) - \ln 1 = \ln 7$$

$$k-2 = 7 \quad (\text{A1})$$

$$k = 9 \quad (\text{A1}) \quad (\text{C6})$$

[6]

8. (a)  $\frac{1}{2} \times 10 = 5$  (M1)(A1) (C2)

$$(b) \int_1^3 g(x)dx + \int_1^3 4dx \quad (M1)$$

$$\int_1^3 4dx [4x]_1^3 \quad (A1)$$

$$= 4 \times 2 = 8 \quad (A1)$$

$$\int_1^3 (g(x) + 4)dx = 10 + 8 = 18 \quad (A1) \quad (C4)$$

[6]

9. (a) 10 A1 N1

$$(b) \int_1^3 3x^2 + f(x)dx = \int_1^3 3x^2 dx + \int_1^3 f(x)dx$$

$$\int_1^3 3x^2 dx = [x^3]_1^3 = 27 - 1 \quad (A1)$$

$$= 26 \text{ (may be seen later)} \quad A1$$

Splitting the integral (seen anywhere) M1

$$e.g. \int 3x^2 dx + \int f(x)dx$$

$$\text{Using } \int_1^3 f(x)dx = 5 \quad (M1)$$

$$eg \int_1^3 3x^2 + f(x)dx = 26 + 5$$

$$\int_1^3 3x^2 + f(x)dx = 31 \quad A1 \quad N3$$

[6]

10. (a) (i) 16 (A2) (C2)

$$(ii) \int_0^3 f(x)dx + \int_0^3 2dx \quad (\text{or appropriate sketch}) \quad (M1)$$

$$= 14 \quad (A1) \quad (C2)$$

$$(b) \int_c^d f(x-2)dx = 8$$

$$c = 2, d = 5 \quad (A2) \quad (C2)$$

[6]

11. (a)  $\int (1 + 3 \sin(x+2))dx = x - 3 \cos(x+2) + c \quad (A1)(A1)(A1) \quad (C3)$

**Notes:** Award A1 for  $x$ , A1 for  $-\cos(x+2)$  A1 for coefficient 3,  
ie A1 A1 for the second term, which may be written as

$+3(-\cos(x+2))$   
*Do not penalize the omission of c.*

$$(b) \quad 1 + 3 \sin(x+2) = 0 \quad (\text{M1})$$

$$\sin(x+2) = -\frac{1}{3}$$

$$x+2 = -0.3398, \pi + 0.3398, \dots \quad (\text{A1})$$

$$x = -2.3398, 1.4814, \dots$$

Required value of  $x = 1.48$  (A1) (C3)

[6]

$$12. \quad \text{Area} = \int_a^b \sin x \, dx \quad (\text{M1})$$

$$a = 0, b = \frac{3\pi}{4} \quad (\text{A1})$$

$$\text{Area} = \int_0^{\frac{3\pi}{4}} \sin x \, dx = [-\cos x]_0^{\frac{3\pi}{4}} \quad (\text{A1})$$

$$= \left( -\cos \frac{3\pi}{4} \right) - (-\cos 0) \quad (\text{A1})$$

$$= -\left( -\frac{\sqrt{2}}{2} \right) - (-1) \quad (\text{A1})$$

$$= 1 + \frac{\sqrt{2}}{2} \quad (\text{A1}) \quad (\text{C6})$$

**Note:** Award (G3) for a gcd answer of 1.71 or 1.707.

[6]

$$13. \quad (a) \quad \int_{\frac{3\pi}{2}}^{2\pi} \cos x \, dx \quad \text{A1} \quad \text{N1}$$

$$(b) \quad \text{Area of A} = 1 \quad \text{A1} \quad \text{N1}$$

(c) Evidence of attempting to find the area of B (M1)

$$eg \int_{\frac{3\pi}{4}}^{\frac{3\pi}{2}} y \, dx, -0.134$$

Evidence of recognising that area B is under the curve/integral is negative (M1)

$$eg - \int_{\frac{3\pi}{4}}^{\frac{3\pi}{2}} y \, dx, \int_{\frac{3\pi}{2}}^{\frac{4\pi}{3}} \cos x \, dx, \left| \int_{\frac{3\pi}{4}}^{\frac{3\pi}{2}} \cos x \, dx \right|$$

$$\text{Area of B} = 0.134 \left( \text{accept } \frac{2-\sqrt{3}}{2} \right) \quad (\text{A1})$$

$$\text{Total Area} = 1 + 0.134$$

$$= 1.13 \left( \text{accept } \frac{4-\sqrt{3}}{2} \right) \quad \text{A1 N4}$$

[6]

14. (a)  $a = \frac{dv}{dt}$  (M1)

$$= -10 \quad \text{A1} \quad 3$$

(b)  $s = \int v dt$  (M1)

$$= 50t - 5t^2 + c \quad \text{A1}$$

$$40 = 50(0) - 5(0) + c \Rightarrow c = 40 \quad \text{A1}$$

$$s = 50t - 5t^2 + 40 \quad \text{A1} \quad 3$$

*Note: Award (M1) and the first (A1) in part (b) if c is missing, but do not award the final 2 marks.*

[6]

15. (a)  $s = 25t - \frac{4}{3}t^3 + c$  (M1)(A1)(A1)

*Note:* Award no further marks if “c” is missing.

Substituting  $s = 10$  and  $t = 3$  (M1)

$$10 = 25 \times 3 - \frac{4}{3}(3)^3 + c$$

$$10 = 75 - 36 + c$$

$$c = -29 \quad (\text{A1})$$

$$s = 25t - \frac{4}{3}t^3 - 29 \quad (\text{A1}) \quad (\text{N3})$$

(b) **METHOD 1**

$$s \text{ is a maximum when } v = \frac{ds}{dt} = 0 \text{ (may be implied)} \quad (\text{M1})$$

$$25 - 4t^2 = 0 \quad (\text{A1})$$

$$t^2 = \frac{25}{4}$$

$$t = \frac{5}{2} \quad (\text{A1}) \quad (\text{N2})$$

**METHOD 2**

$$\text{Using maximum of } s \left( 12\frac{2}{3}, \text{ may be implied} \right) \quad (\text{M1})$$

$$25t - \frac{4}{3}t^3 - 29 = 12\frac{2}{3} \quad (\text{A1})$$

$$t = 2.5 \quad (\text{A1}) \quad (\text{N2})$$

(c)  $25t - \frac{4}{3}t^3 - 29 > 0$  (accept equation) (M1)

$$m = 1.27, n = 3.55 \quad (\text{A1})(\text{A1}) \quad (\text{N3})$$

[12]

16.  $s = \int v dt$  (M1)

$$s = \frac{1}{2} e^{2t-1} + c \quad \text{A1A1}$$

Substituting  $t = 0.5$

$$\frac{1}{2} + c = 10 \quad c = 9.5 \quad (\text{A1})$$

Substituting  $t = 1$  M1

$$s = \frac{1}{2} e + 9.5 (= 10.9 \text{ to } 3s.f.) \quad \text{A1 N3}$$

[6]

17. (a) Attempting to use the formula  $V = \int_a^b \pi y^2 dx$  (M1)

$$\text{Volume} = \pi \int_0^2 (2x - x^2)^2 dx \quad \text{A2 N3}$$

(b) Volume =  $\pi \int_0^2 (4x^2 - 4x^3 + x^4) dx$  (A1)

$$= \pi \left[ 4 \frac{x^3}{3} - 4 \frac{x^4}{4} + \frac{x^5}{5} \right]_0^2 \quad (\text{A1})$$

$$= \frac{16\pi}{15} \text{ or } 3.35 \quad (\text{accept } 1.07\pi) \quad \text{A1 N3}$$

[6]

18. Using  $V = \int \pi y^2 dx$  (M1)

Correctly integrating  $\int \left( x^{\frac{1}{2}} \right)^2 dx = \frac{x^2}{2}$  A1

$$V = \pi \left[ \frac{x^2}{2} \right]_0^a \quad \text{A1}$$

$$= \frac{\pi a^2}{2} \quad (\text{A1})$$

Setting up their equation  $\left( \frac{1}{2} \pi a^2 = 0.845\pi \right)$  M1

$$a^2 = 1.69$$

$$a = 1.3 \quad \text{A1 N2}$$

[6]