Integration Past Paper Questions

1. Find

(a) $\int \sin(3x+7)\mathrm{d}x;$

(b) $\int e^{-4x}\mathrm{d}x.$

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

2. Let $f(x) = (3x + 4)^5$. Find

(a) $f'(x);$

(b) $\int f(x)\mathrm{d}x.$

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

3. The function f is given by $f(x) = 2\sin(5x - 3)$.

(a) Find $f''(x)$.

(b) Write down $\int f(x)\mathrm{d}x$.

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(Total 6 marks)

4. The curve $y = f(x)$ passes through the point $(2, 6)$.

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 5$, find y in terms of x.

Working:

Answer:

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(Total 6 marks)

5. It is given that $\frac{dy}{dx} = x^3 + 2x - 1$ and that $y = 13$ when $x = 2$.

Find y in terms of x.

Working:

Answer:

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(Total 6 marks)

6. The derivative of the function f is given by $f'(x) = e^{-2x} + \frac{1}{1-x}$, $x < 1$.

The graph of $y = f(x)$ passes through the point (0, 4). Find an expression for $f(x)$.

Working:

Answer:

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(Total 6 marks)

7. Given $\int_3^k \frac{1}{x-2}\,dx = \ln 7$, find the value of k.

Working:

Answers:

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(Total 6 marks)

8. Given that $\int_1^3 g(x)\mathrm{d}x = 10$, deduce the value of

(a) $\int_1^3 \frac{1}{2} g(x)\mathrm{d}x$;

(b) $\int_1^3 (g(x)+4)\mathrm{d}x$.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

9. It is given that $\int_1^3 f(x)\mathrm{d}x = 5$.

(a) Write down $\int_1^3 2f(x)\mathrm{d}x$.

(b) Find the value of $\int_1^3 (3x^2 + f(x))\mathrm{d}x$.

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(Total 6 marks)

10. Let f be a function such that $\int_0^3 f(x)\,dx = 8$.

(a) Deduce the value of

(i) $\int_0^3 2f(x)\,dx$;

(ii) $\int_0^3 \big(f(x)+2\big)dx$.

(b) If $\int_c^d f(x-2)dx = 8$, write down the value of c and of d.

Working:

Answers:

(a) (i) .....

(ii) .....

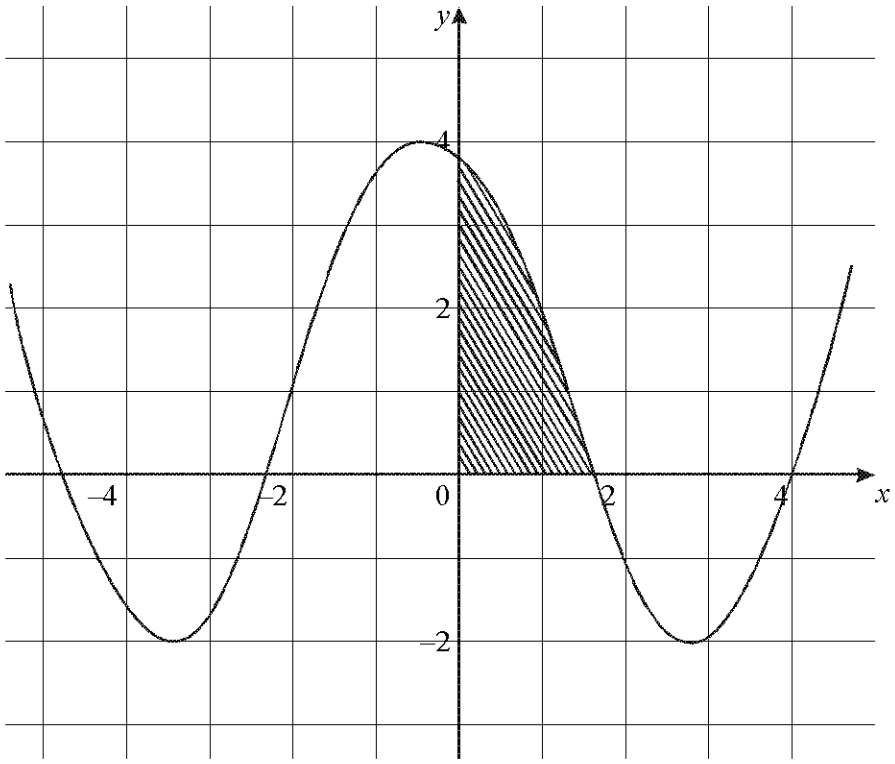
(b) c = ..., d = ...

(Total 6 marks)

11. (a) Find $\int(1 + 3 \sin (x + 2))dx$.

(b) The diagram shows part of the graph of the function $f(x) = 1 + 3 \sin (x + 2)$.

The area of the shaded region is given by $\int_0^a f(x)dx$.



Find the value of a.

Working:

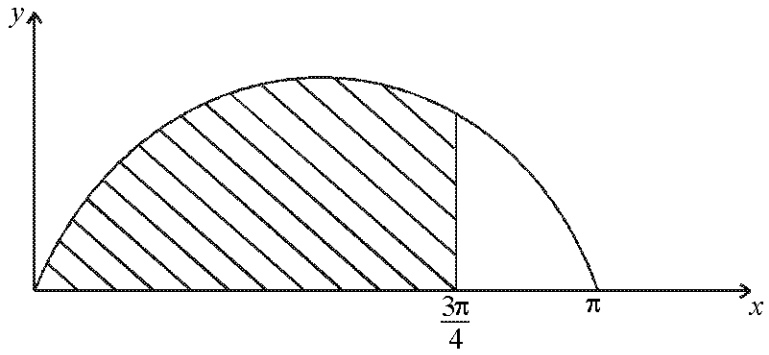
Answers:

(a) .....

(b) .....

(Total 6 marks)

12. The diagram shows part of the curve $y = \sin x$. The shaded region is bounded by the curve and the lines $y = 0$ and $x = \frac{3\pi}{4}$.



Given that $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, calculate the **exact** area of the shaded region.

Working:

Answer:

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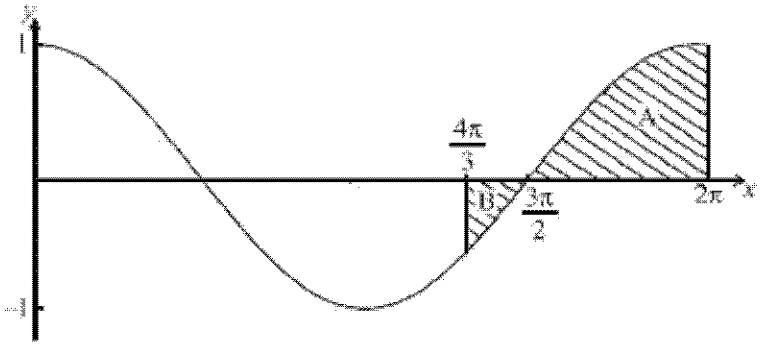
(Total 6 marks)

13. The following diagram shows part of the graph of $y = \cos x$ for $0 \leq x \leq 2\pi$. Regions A and B are shaded.



(a) Write down an expression for the area of A.

(1)

(b) Calculate the area of A.

(1)

(c) Find the total area of the shaded regions.

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(4)
(Total 6 marks)

14. The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}.

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

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(Total 6 marks)

15. A particle moves with a velocity v m s^{-1} given by $v = 25 - 4t^2$ where $t \geq 0$.

(a) The displacement, s metres, is 10 when t is 3. Find an expression for s in terms of t.

(6)

(b) Find t when s reaches its maximum value.

(3)

(c) The particle has a positive displacement for $m \leq t \leq n$. Find the value of m and the value of n.

(3)

(Total 12 marks)

16. The velocity v in m s^{-1} of a moving body at time t seconds is given by $v = e^{2t-1}$. When $t = 0\ 5$. the displacement of the body is 10 m. Find the displacement when $t = 1$.

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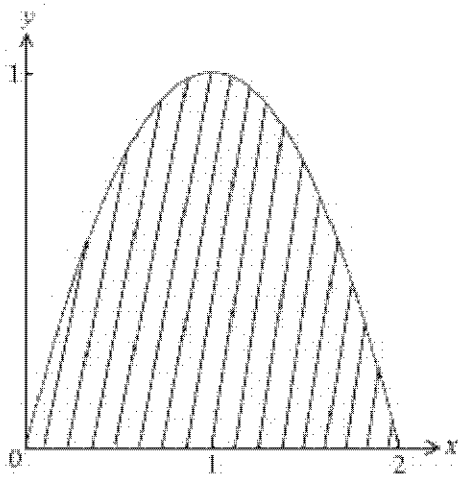
(Total 6 marks)

17. A part of the graph of $y = 2x - x^2$ is given in the diagram below.



The shaded region is revolved through 360° about the x-axis.

(a) Write down an expression for this volume of revolution.

(b) Calculate this volume.

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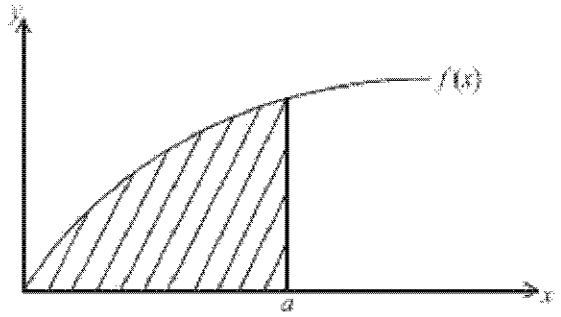
(Total 6 marks)

18. The shaded region in the diagram below is bounded by $f(x) = \sqrt{x}$, $x = a$, and the x-axis. The shaded region is revolved around the x-axis through 360°. The volume of the solid formed is 0.845π.



Find the value of a.

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(Total 6 marks)

19. Find the area between the curve $y = x^2 + x - 6$, the x – axis and the lines $x = 0$ and $x = 3$