Integration 1

1) **(i)** Find $\int \frac{1}{\sqrt{1+x}}\,\mathrm{d}x$. [2]

(ii) Given that $y = \frac{2x}{\sqrt{1+x}}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{\sqrt{1+x}} + \frac{Bx}{\left(\sqrt{1+x}\right)^3}$, where A and B are to be found. [4]

(iii) Hence find $\int \frac{x}{\left(\sqrt{1+x}\right)^3}\,\mathrm{d}x$ and evaluate $\int_0^3 \frac{x}{\left(\sqrt{1+x}\right)^3}\,\mathrm{d}x$. [4]

2) **(a)** Find $\int \left(x^{\frac{1}{3}} - 3\right)^2\,\mathrm{d}x$. [3]

(b) **(i)** Given that $y = x\sqrt{x^2+6}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(ii) Hence find $\int \frac{x^2+3}{\sqrt{x^2+6}}\,\mathrm{d}x$. [2]

3) **(i)** Given that $y = x\sqrt{4x+12}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k(x+2)}{\sqrt{4x+12}}$, where k is a constant to be found. [4]

(ii) Hence evaluate $\int_{-2}^{6} \frac{3x+6}{\sqrt{4x+12}}\,\mathrm{d}x$. [3]

4) **(i)** Differentiate $x\ln x$ with respect to x. [2]

(ii) Hence find $\int \ln x\,\mathrm{d}x$. [3]

5) Evaluate

$$\int_0^{\frac{\pi}{6}} \sin\left(2x + \frac{\pi}{6}\right)\mathrm{d}x.$$ [4]

6) Find $\int \left(2 + 5x - \frac{1}{(x-2)^2}\right)\mathrm{d}x$. [3]

7) **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x\mathrm{e}^{3x} - \frac{\mathrm{e}^{3x}}{3}\right)$. [3]

(ii) Hence find $\int x\mathrm{e}^{3x}\mathrm{d}x$. [3]

8) **(a)** Find

(i) $\int \frac{12}{(2x-1)^4}\,\mathrm{d}x$, [2]

(ii) $\int x(x-1)^2\mathrm{d}x$. [3]

(b) **(i)** Given that $y = 2(x-5)\sqrt{x+4}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(x+1)}{\sqrt{x+4}}$. [3]

(ii) Hence find $\int \frac{(x+1)}{\sqrt{x+4}}\,\mathrm{d}x$. [2]