Integration 2 Area Answers

1)

OR		
(i) $\frac{dy}{dx} = 8x - 6x^2$	M1	M1 for differentiation
Grad at A = 2, perp grad = $-\frac{1}{2}$	M1	M1 for use of $m_1 m_2 = -1$
At A, $y = 2$	B1	B1 for y coordinate
Equation of normal: $y - 2 = -\frac{1}{2}(x - 1)$	DM1	DM1 for finding equation of normal
C (0, 2.5)	A1 [5]	A1 **answer given**
(ii) B (2,0)	B1	B1 for coords of B
$A = \frac{1}{2}(2.5 + 2)1 + \int_1^2 4x^2 - 2x^3 dx$	M1	M1 for area of trapezium
$= 2.25 + \left[\frac{4x^3}{3} - \frac{x^4}{2}\right]_1^2$	M1 A1 DM1	M1 for attempt to integrate A1 all integration correct DM1 for correct use of limits
$= \frac{49}{12}$ or 4.08	A1 [6]	

2)

12 OR (i) $y = x + \cos 2x$	M1	M1 for attempt to differentiate
$\frac{dy}{dx} = 1 - 2\sin 2x$	A1	
when $\frac{dy}{dx} = 0$, $\sin 2x = \frac{1}{2}$	M1	M1 for setting to 0 and attempt to solve
leading to $x = \frac{\pi}{12}, \frac{5\pi}{12}$	M1 A1,A1 [6]	M1 for correct order of operations
(ii) Area = $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} x + \cos 2x dx$	M1	M1 for attempt to integrate
$= \left[\frac{x^2}{2} + \frac{1}{2}\sin 2x\right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$	A1,A1 DM1	A1for each term correct DM1 for correct use of limits (Trig terms cancel out)
$= \frac{\pi^2}{12}$	A1 [5]	

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3)

(i)	$\frac{dy}{dx} = 3x^2 - 16x + 16$	B1
	equate to 0 and solve 3 term quadratic	M1
	$x = 4, y = 0$	A1 AG
	$x = \frac{4}{3}\ y = 9\frac{13}{27}$ or $\frac{256}{27}$ or 9.48 or 9.5	A1
(ii)	integrate	M1
	$\frac{x^4}{4} - \frac{8x^3}{3} + 8x^2$	A1
	use limits of 4 (and 0)	DM1
	$21\frac{1}{3}$ or 21.3	A1

[8]

4)

(i) At X, $y = 0$, $x = 16$	B1 cso
$dy / dx = x^{-1/2}$ ×4×½ – 1	M1 A1
(dy/dx) = 0 ⇒ at M, $x = 4$, $y = 4$	M1 A1
(ii) $\int 4\sqrt{x}\,dx = x^{3/2}$ ×4×⅔ or ×4 ÷ 3/2 $\int(-x)dx = -x^2/2$	M1 A1 B1
$\left[\frac{8x^{3/2}}{3} - \frac{x^2}{2}\right]_0^{16}$ = 42⅔	A1

5)

(i) $y = 4\sin 2x + c$	M1 M1	M1 for attempt to integrate M1 for attempt to get c provided a function of sin $2x$ is used
passes through $\left(\frac{\pi}{4}, 7\right)$, $c = 3$	A1 [3]	
(ii) $5 = 4\sin 2x + 3$	M1	M1 for attempt to equate to 5 and solve
$0.5 = \sin 2x$	M1	M1 for a correct method to find x
$x = \frac{\pi}{12}, \frac{5\pi}{12}$	A1 √A1 [4]	 √A1 on first solution
(iii) $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 4\sin 2x + 3dx$	M1	M1 for attempt to integrate
$\left[-2\cos 2x + 3x\right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$	A1 DM1	 DM1 for correct use of limits
$= \pi + 2\sqrt{3}$		
Shaded area $= \pi + 2\sqrt{3} - \frac{5\pi}{3}$	M1	M1 for area of rectangle
(= 1.37)	A1 [5]	

6)

12 EITHER

(i) amplitude = 1	B1	[1]	
(ii) period = 6π, 18.8	B1	[1]	
(iii) $\sin\left(\frac{x}{3}\right)=\frac{1}{2},\ x=\frac{\pi}{2},\frac{5\pi}{2}$	M1 A1, A1	[3]	M1 for attempt to solve correctly A1 for each (allow degrees here)
(iv) Area under curve			
$\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}}\left(1+\sin\frac{x}{3}\right)\mathrm{d}x = \left[x-3\cos\frac{x}{3}\right]_{\frac{\pi}{2}}^{\frac{5\pi}{2}}$	M1 B1, B1		M1 for attempt to integrate B1 for x, B1 for $-3\cos\frac{x}{3}$
leading to $2\pi+3\sqrt{3}$	DM1		DM1 for **correct** use of limits
Area of rectangle $=\left(\frac{5\pi}{2}-\frac{\pi}{2}\right)\times\frac{3}{2}$ $=3\pi$	M1		M1 for attempt at rectangle plus subtraction – must be working in radians
Shaded area = $3\sqrt{3}-\pi$ (2.05)	A1	[6]	
Alternative solution: Shaded area	M1 M1		M1 for subtraction (must be using radians) M1 for attempt to integrate
$\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}}\left(\sin\frac{x}{3}-0.5\right)\mathrm{d}x=\left[-0.5x-3\cos\frac{x}{3}\right]_{\frac{\pi}{2}}^{\frac{5\pi}{2}}$	B1, B1 DM1, A1		B1 for $-0.5x$, B1 for $-3\cos\frac{x}{3}$ DM1 for correct use of limits

7)

(i) A (0, 6)	B1
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}e^{\frac{1}{2}x}$	B1
uses $m_1m_2=-1$	M1
B (3, 0)	B1√
(ii) Integrates for area below curve	M1
$2e^{\frac{1}{2}x}+5x$	A1
uses limits of 0 and x_B	M1
$13+2e^{1.5}$ or 21.96 or 22	A1
Area rectangle = $3(e^{1.5}+5)$ or 28.4(4..)	M1
Area = $e^{1.5}+2$ or 6.45 to 6.5	A1
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