# **SL - Exponents and Logs Answers**

0 min 0 marks

1. 
$$9^{x-1} = \left(\frac{1}{3}\right)^{2x}$$
  
 $3^{2x-2} = 3^{-2x}$  (M1) (A1)  
 $2x - 2 = -2x$  (A1)  
 $x = \frac{1}{2}$  (A1)

2. 
$$4^{3x-1} = 1.5625 \times 10^{-2}$$
  
 $(3x-1)\log_{10}4 = \log_{10}1.5625 - 2$  (M1)  
 $\Rightarrow 3x - 1 = \frac{\log_{10} 1.5625 - 2}{\log_{10} 4}$  (A1)  
 $\Rightarrow 3x - 1 = -3$  (A1)  
 $\Rightarrow x = -\frac{2}{3}$  (A1)

**3.** (a) 
$$\log_2 5 = \frac{\log_a 5}{\log_a 2}$$

=

$$\frac{y}{x}$$
 (A1) (C2)

(b) 
$$\log_a 20 = \log_a 4 + \log_a 5 \text{ or } \log_a 2 + \log_a 10$$
 (M1)  
=  $2 \log_a 2 + \log_a 5$   
=  $2x + y$  (A1) (C2)

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(M1)

4. 
$$\log_{10}\left(\frac{P}{QR^3}\right)^2 = 2\log_{10}\left(\frac{P}{QR^3}\right)^2$$
 (M1)

$$2\log_{10}\left(\frac{P}{QR^{3}}\frac{1}{\dot{J}}=2(\log_{10}P-\log_{10}(QR^{3}))$$
(M1)

$$= 2(\log_{10}P - \log_{10}Q - 3\log_{10}R)$$
(M1)  
= 2(x - y - 3z)

$$= 2x - 2y - 6z \text{ or } 2(x - y - 3z)$$
(A1)

# **5. METHOD 1**

 $\log 81 + \log_9 \left(\frac{1}{9}\right) + \log_9 3 = 2 - 1 + \frac{1}{2}$ (M1)

$$\Rightarrow \frac{3}{2} = \log_9 x \tag{A1}$$
$$\Rightarrow x = 9^{\frac{3}{2}} \tag{M1}$$

$$\Rightarrow x = 27 \tag{M1}$$

$$\Rightarrow x = 27 \tag{A1} (C4)$$

#### METHOD 2

$$\log_{9} 81 + \log_{9} \left(\frac{1}{9}\right) + \log_{9} 3 = \log_{9} \left[81 \left(\frac{1}{9}\right)^{3}\right]$$

$$= \log_{9} 27$$

$$\Rightarrow x = 27$$
(M2)
(M2)
(A1)
(A1)
(C4)

6. 
$$\log_{27}(x(x-0.4)) = 1$$
 (M1)(A1)  
 $x^2 - 0.4x = 27$  (M1)  
 $x = 5.4 \text{ or } x = -5$  (G2)  
 $x = 5.4$  (A1) (C6)  
*Note:* Award (C5) for giving both roots.

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Statement	(a) Is the statement true for all	(b) If not true, example	
	real numbers x? (YesfNo)		
А	No	$x = -1 (\log_{10} 0.1 = -1)$	(a) (A3) (C3)
В	No	$x = 0 \ (\cos 0 = 1)$	(b) (A3) (C3)
С	Yes	N/A	

Notes:

(a) Award (A1) for each correct answer.

(b) Award (A) marks for statements A and B only if NO in column (a).

Award (A2) for a correct counter example to statement A, (A1) for a correct counter example to statement B (ignore other incorrect examples).

#### Special Case for statement C:

Award (A1) if candidates write NO, and give a valid reason (e.g.

arctan 
$$l = \frac{5\delta}{4}$$
).

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8. (a) 
$$\log_5 x^2 = 2\log_5 x$$
 (M1)  
= 2y (A1) (C2)

(b) 
$$\log_5 \frac{1}{x} = -\log_5 x$$
 (M1)  
= -v (A1) (C2)

(c) 
$$\log_{25} x = \frac{\log_5 x}{\log_5 25}$$
 (M1)

$$=\frac{1}{2}y$$
 (A1) (C2)

9. List of frequencies with *p* in the middle

*e.g.* 
$$5 + 10, p, 6 + 2 \Rightarrow 15, 8, \text{ or } 15 < \frac{23 + p}{2}, \text{ or } p > 7.$$
 (M1)

Consideration that $p < 10$ because 2 is the mode or discretionary for		
further processing.	(M1)	
Possible values of <i>p</i> are 8 and 9	(A2)(A2) (C6)	

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#### **10. METHOD 1**

$$9 = 3^2, \ 27 = 3^3$$
 (A1)(A1)

expressing as a power of 3,  $(3^2)^{2x} = (3^3)^{1-x}$  (M1)

$$3^{4x} = 3^{3-3x} \tag{A1}$$

$$4x = 3 - 3x \tag{A1}$$

$$7x = 3$$

$$\Rightarrow x = \frac{3}{7} \tag{A1} \tag{C6}$$

### **METHOD 2**

 $2x \log 9 = (1 - x) \log 27$  (M1)(A1)(A1)

$$\frac{2x}{1-x} = \frac{\log 27}{\log 9} \left( = \frac{3}{2} \right)$$
(A1)

$$4x = 3 - 3x$$
 (A1)  
 $7x = 3$ 

$$\Rightarrow x = \frac{3}{7} \tag{A1} \tag{C6}$$

**Notes:** Candidates may use a graphical method. Award (M1)(A1) (A1) for a sketch, (A1) for showing the point of intersection,

(A1) for 0.4285...., and (A1) for 
$$\frac{3}{7}$$
.

#### **11. METHOD 1**

 $\log x^2 = 2\log x$ 

$$\log\sqrt{y} = \frac{1}{2}\log y \tag{A1}$$

$$\log z^3 = 3\log z \tag{A1}$$

$$2\log x + \frac{1}{2}\log y - 3\log z$$
 (A1)(A1)

$$2a + \frac{1}{2}b - 3c$$
 (A1) (C6)

(A1)

# METHOD 2

$$x^{2} = 10^{2a}, \quad \sqrt{y} = 10^{\frac{b}{2}}, \quad z^{3} = 10^{3c}$$
 (A1)(A1)(A1)

$$\log_{10}\left(\frac{x^2\sqrt{y}}{z^3}\right) = \log_{10}\left(\frac{10^{2a} \times 10^{\frac{b}{2}}}{10^{3c}}\right)$$
(A1)

$$= \log_{10} \left( 10^{2a + \frac{b}{2} - 3c} \right) \left( = 2a + \frac{b}{2} - 3c \right)$$
(A2)

12. (a) 
$$\log_3 x - \log_3 (x-5) = \log_3 \left(\frac{x}{x-5}\right)$$
 (A1)

$$A = \frac{x}{x-5}$$
(A1) (C2)
  
*Note:* If candidates have an incorrect or no answer to part (a)
  
*award* (A1)(A0)

if 
$$\log\left(\frac{x}{x-5}\right)$$
 seen in part (b).

# (b) **EITHER**

$$\log_{3}\left(\frac{x}{x-5}\right) = 1$$

$$\frac{x}{x-5} = 3^{1} (=3)$$

$$x = 3x - 15$$

$$-2x = -15$$

$$x = \frac{15}{2}$$
(A1) (C4)

OR

$$\frac{\log_{10}\left(\frac{x}{x-5}\right)}{\log_{10}3} = 1$$
(M1)(A1)

$$\log_{10}\left(\frac{x}{x-5}\right) = \log_{10}3\tag{A1}$$

$$x = 7.5$$
 (A1) (C4)

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