# SL - Exponents and Logs Answers 

0 min<br>0 marks

1. $9^{x-1}=\left(\frac{1}{3} \stackrel{.}{\dot{j}}\right)^{2 x}$

$$
\begin{align*}
& 3^{2 x-2}=3^{-2 x}  \tag{M1}\\
& 2 x-2=-2 x  \tag{A1}\\
& x=\frac{1}{2} \tag{A1}
\end{align*}
$$

2. $4^{3 x-1}=1.5625 \times 10^{-2}$
$(3 x-1) \log _{10} 4=\log _{10} 1.5625-2$
$\Rightarrow 3 x-1=\frac{\log _{10} 1.5625-2}{\log _{10} 4}$
$\Rightarrow 3 x-1=-3$
$\Rightarrow x=-\frac{2}{3}$
3. (a) $\log _{2} 5=\frac{\log _{a} 5}{\log _{a} 2}$

$$
\begin{equation*}
=\frac{y}{x} \tag{M1}
\end{equation*}
$$

(b) $\log _{a} 20=\log _{a} 4+\log _{a} 5$ or $\log _{a} 2+\log _{a} 10$

$$
\begin{equation*}
=2 \log _{a} 2+\log _{a} 5 \tag{M1}
\end{equation*}
$$

$$
=2 x+y
$$

(A1) (C2)


$$
\begin{align*}
2 \log _{10}\left(\frac{P}{Q R^{3}} \frac{)}{\stackrel{+}{\mathrm{j}}}\right. & =2\left(\log _{10} P-\log _{10}\left(Q R^{3}\right)\right)  \tag{M1}\\
& =2\left(\log _{10} P-\log _{10} Q-3 \log _{10} R\right)  \tag{M1}\\
& =2(x-y-3 z) \\
& =2 x-2 y-6 z \text { or } 2(x-y-3 z) \tag{A1}
\end{align*}
$$

5. METHOD 1

$$
\begin{align*}
& \log _{81}+\log _{9}\left(\frac{1}{9} \frac{9}{)}+\log _{9} 3=2-1+\frac{1}{2}\right.  \tag{M1}\\
& \Rightarrow \frac{3}{2}=\log _{9} x  \tag{A1}\\
& \Rightarrow x=9^{\frac{3}{2}}  \tag{M1}\\
& \Rightarrow x=27
\end{align*}
$$

METHOD 2

$$
\begin{align*}
& \log _{9} 81+\log _{9}\left(\frac{1}{9} \frac{)}{)}+\log _{9} 3=\log _{9}\left[81\left(\frac{1}{9} \frac{-}{3}\right]\right.\right.  \tag{M2}\\
& =\log _{9} 27  \tag{A1}\\
& \Rightarrow x=27 \tag{A1}
\end{align*}
$$

6. $\quad \log _{27}(x(x-0.4))=1$

$$
\begin{align*}
& x^{2}-0.4 x=27  \tag{M1}\\
& x=5.4 \text { or } x=-5  \tag{G2}\\
& x=5.4
\end{align*}
$$

Note: Award (C5) for giving both roots.
7.

| Statement | (a) Is the statement true for all <br> real numbers $x$ ? (YesfNo) | (b) If not true, example |
| :---: | :---: | :---: |
| A | No | $x=-1\left(\log _{10} 0.1=-1\right)$ |
| B | No | $x=0(\cos 0=1)$ |
| C | (b) (A3) (C3) (C3) |  |

## Notes:

(a) Award (A1) for each correct answer.
(b) Award (A) marks for statements $A$ and $B$ only if NO in column (a).
Award (A2) for a correct counter example to statement A, (A1)
for a correct counter example to statement B (ignore other incorrect examples).
Special Case for statement $C$ :
Award (A1) if candidates write NO, and give a valid reason (e.g.
$\left.\arctan 1=\frac{5 \text { 〕 }}{4}\right)$.
8. (a) $\log _{5} x^{2}=2 \log _{5} x$

$$
\begin{equation*}
=2 y \tag{A1}
\end{equation*}
$$

(b) $\log _{5} \frac{1}{x}=-\log _{5} x$

$$
\begin{equation*}
=-y \tag{A1}
\end{equation*}
$$

(c) $\log _{25} x=\frac{\log _{5} x}{\log _{5} 25}$

$$
=\frac{1}{2} y
$$

9. List of frequencies with $p$ in the middle
e.g. $5+10, p, 6+2 \Rightarrow 15,8$, or $15<\frac{23+p}{2}$, or $p>7$.

Consideration that $p<10$ because 2 is the mode or discretionary for further processing.
Possible values of $p$ are 8 and 9
(A2)(A2) (C6)
10. METHOD 1

$$
\begin{equation*}
9=3^{2}, 27=3^{3} \tag{A1}
\end{equation*}
$$

expressing as a power of $3,\left(3^{2}\right)^{2 x}=\left(3^{3}\right)^{1-x}$

$$
\begin{align*}
3^{4 x} & =3^{3-3 x}  \tag{A1}\\
4 x & =3-3 x  \tag{A1}\\
7 x & =3 \\
\Rightarrow x & =\frac{3}{7} \tag{A1}
\end{align*}
$$

## METHOD 2

$$
\begin{align*}
& 2 x \log 9=(1-x) \log 27 \\
& \frac{2 x}{1-x}=\frac{\log 27}{\log 9}\left(=\frac{3}{2}\right)  \tag{A1}\\
& 4 x=3-3 x  \tag{A1}\\
& 7 x=3 \\
& \Rightarrow x=\frac{3}{7} \tag{A1}
\end{align*}
$$

$$
(\mathrm{M} 1)(\mathrm{A} 1)(\mathrm{A} 1)
$$

Notes: Candidates may use a graphical method. Award (M1)(A1) (A1) for a sketch, (A1) for showing the point of intersection, (A1) for $0.4285 \ldots$, and (A1) for $\frac{3}{7}$.

## 11. METHOD 1

$$
\begin{equation*}
\log x^{2}=2 \log x \tag{A1}
\end{equation*}
$$

$\log \sqrt{y}=\frac{1}{2} \log y$
$\log z^{3}=3 \log z$
$2 \log x+\frac{1}{2} \log y-3 \log z$
(A1)(A1)
$2 a+\frac{1}{2} b-3 c$

## METHOD 2

$$
\begin{align*}
& x^{2}=10^{2 a}, \quad \sqrt{y}=10^{\frac{b}{2}}, \quad z^{3}=10^{3 c}  \tag{A1}\\
& \log _{10}\left(\frac{x^{2} \sqrt{y}}{z^{3}}\right)=\log _{10}\left(\frac{10^{2 a} \times 10^{\frac{b}{2}}}{10^{3 c}}\right)  \tag{A1}\\
& =\log _{10}\left(10^{2 a+\frac{b}{2}-3 c}\right)\left(=2 a+\frac{b}{2}-3 c\right) \tag{A2}
\end{align*}
$$

12. (a) $\log _{3} x-\log _{3}(x-5)=\log _{3}\left(\frac{x}{x-5}\right)$

$$
\begin{equation*}
A=\frac{x}{x-5} \tag{A1}
\end{equation*}
$$

Note: If candidates have an incorrect or no answer to part (a) award (A1)(A0)
if $\log \left(\frac{x}{x-5}\right)$ seen in part (b).
(b) EITHER

$$
\begin{align*}
\log _{3}\left(\frac{x}{x-5}\right) & =1 \\
\frac{x}{x-5} & =3^{1}(=3)  \tag{M1}\\
x & =3 x-15 \\
-2 x & =-15 \\
x & =\frac{15}{2} \tag{A1}
\end{align*}
$$

OR

$$
\begin{align*}
& \frac{\log _{10}\left(\frac{x}{x-5}\right)}{\log _{10} 3}=1  \tag{M1}\\
& \log _{10}\left(\frac{x}{x-5}\right)=\log _{10} 3  \tag{A1}\\
& x=7.5
\end{align*}
$$

(A1) (C4)

