Geometric Series Answers

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0 marks

1. (a) $r = \frac{16}{32}\left(= \frac{1}{2}\right)$ A1 N1

(b) correct calculation or listing terms (A1)

e.g. $32 \times \left(\frac{1}{2}\right)^{6-1}, 8 \times \left(\frac{1}{2}\right)^{3}$, 32, ... 4, 2, 1

$u_6 = 1$ A1 N2

(c) evidence of correct substitution in S_∞ A1

e.g. $\frac{32}{1-\frac{1}{2}}, \frac{32}{\frac{1}{2}}$

$S_\infty = 64$ A1 N1

[5]

2. (a) $u_{10} = 3(0.9)^9$ A1 N1

(b) recognizing $r = 0.9$ (A1)
correct substitution A1

e.g. $S = \dfrac{3}{1-0.9}$

$S = \dfrac{3}{0.1}$ (A1)

$S = 30$ A1 N3

[5]

3. $S = \dfrac{u_1}{1-r} = \dfrac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)}$ (M1)(A1)

$= \dfrac{2}{3} \times \dfrac{3}{5}$ (A1)

$= \dfrac{2}{5}$ (A1) (C4)

[4]

4. (a) evidence of substituting into formula for *n*th term of GP (M1)

e.g. $u_4 = \dfrac{1}{81} r^3$

setting up correct equation $\dfrac{1}{81} r^3 = \dfrac{1}{3}$ A1

$r = 3$ A1 N2

(b) **METHOD 1**

setting up an inequality (accept an equation) M1

e.g. $\dfrac{\frac{1}{81}(3^n - 1)}{2} > 40; \dfrac{\frac{1}{81}(1-3^n)}{-2} > 40; 3^n > 6481$

evidence of solving M1
e.g. graph, taking logs

$n > 7.9888...$ (A1)
$n = 8$ A1 N2

METHOD 2

if $n = 7$, sum = 13.49...; if $n = 8$, sum = 40.49... A2

$n = 8$ (is the smallest value) A2 N2

[7]

5. (a) evidence of dividing two terms (M1)

e.g. $-\frac{1800}{3000}, -\frac{1800}{1080}$

$r = -0.6$ A1 N2

(b) evidence of substituting into the formula for the 10th term (M1)

e.g. $u_{10} = 3000(-0.6)^9$

$u_{10} = -30.2$ (accept the exact value -30.233088) A1 N2

(c) evidence of substituting into the formula for the infinite sum (M1)

e.g. $S = \frac{3000}{1.6}$

$S = 1875$ A1 N2

[6]

6. (a) $5000(1.063)^n$ A1 N1

(b) Value = \$ $5000(1.063)^5$ (= \$ 6786.3511...)
= \$ 6790 to 3 s.f. (accept \$ 6786, or \$ 6786.35) A1 N1

(c) (i) $5000(1.063)^n > 10\,000$ or $(1.063)^n > 2$ A1 N1

(ii) Attempting to solve the inequality $n\log(1.063) > \log 2$ (M1)
$n > 11.345$ (A1)
12 years A1 N3

Note: *Candidates are likely to use TABLE or LIST on a GDC to find n.*
A good way of communicating this is suggested below.

Let $y = 1.063^x$ (M1)
When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ (A1)
$x = 12$ *i.e.* 12 years A1 N3

[6]

7. (a) $\frac{1}{5}$ (0.2) A1 N1

(b) (i) $u_{10} = 25\left(\frac{1}{5}\right)^9$ (M1)

$= 0.0000128 \left(\left(\frac{1}{5}\right)^7, 1.28\times10^{-5}, \frac{1}{78125}\right)$ A1 N2

(ii) $u_n = 25\left(\frac{1}{5}\right)^{n-1}$ A1 N1

(c) For attempting to use infinite sum formula for a GP $\left(\frac{25}{1-\left(\frac{1}{5}\right)}\right)$ (M1)

$S = \frac{125}{4} = 31.25 \ (=31.3 \text{ to } 3\,s\,f)$ A1 N2

[6]

8. (a) For taking three ratios of consecutive terms (M1)

$\frac{54}{18} = \frac{162}{54} = \frac{486}{162} \ (=3)$ A1

hence geometric AG N0

(b)	(i)	$r = 3$	(A1)		
		$u_n = 18 \times 3^{n-1}$	A1	N2	
	(ii)	For a valid attempt to solve $18 \times 3^{n-1} = 1062882$	(M1)		
		eg trial and error, logs			
		$n = 11$		A1	N2

[6]