Geometric Sequences Ans

0 min
0 marks

1. (a) $\frac{a}{8}=\frac{1}{2}$

$a = 4$ (A1)

OR

$\frac{2}{a}=\frac{1}{2}$

$a = 4$ (A1) (C1)

(b) $8\left(\frac{1}{2}\right)^{7}=0.0625$ (M1)(A1)(ft)

OR

$2\left(\frac{1}{2}\right)^{5}=0.0625$ (M1)(A1)(ft) (C2)

(c) $\frac{8\left(\left(\frac{1}{2}\right)^{12}-1\right)}{\frac{1}{2}-1}=16.0\,(3\,s.f.)\ \ (=4095/256)$ (M1)(A1)(ft)

(A1)(ft) (C3)

Note: *Award (M1) for using correct formula and correct substitution, (A1) for correct answer (15.99...). (A1) for correct answer to 3 s.f.*

[6]

2. (a) 10000×1.05
$= 10\,500$ (A1) (C1)

(b) 10000×1.05^6 (M1)

Note: *Award (M1) for correct substitution into correct formula.*

$= 13400$ (A1) (C2)

(c) $50\,000 = 10000 \times 1.05^n$ (M1)(A1)

Note: *Award (M1) for 10000×1.05^n or equivalent, (A1) for $= 50000$*

$n = 33.0$ (Accept 33) (A1) (C3)

[6]

3. (a) For obtaining an equation in r^2, can be implied (M1)
$28 = 7r^2$ (A1)
$r = 2$ (A1) (C3)

(b) For using their value of r in the GP sum formula (M1)
For obtaining 114681 (accept fewer s.f. up to 115000) (M1) (A1) (C3)

[6]

4. (a) $r = \dfrac{36}{108}\left(\dfrac{1}{3}\right)$ (A1) (C1)

Note: *Accept 0.333.*

(b) $u_1\left(\dfrac{1}{3}\right)^7 = 36$ (M1)

Note: *Award (M1) for correct substitution in formula for n^{th} term of a GP. Accept equivalent forms.*

$u_1 = 78732$ (A1)(ft) (C2)

Notes: *Accept 78700. Follow through from their common ratio found in part (a). If 0.333 used from part (a) award (M1)(A1)(ft) for an answer of 79285 or 79300 irrespective of whether working is shown.*

(c) $118096 = \dfrac{78732\left(1-\left(\frac{1}{3}\right)^{k}\right)}{\left(1-\frac{1}{3}\right)}$ (M1)(M1)

Notes: *Award (M1) for correct substitution in the sum of a GP formula, (M1) for equating their sum to 118096. Follow through from parts (a) and (b).*

OR

Sketch of the function $y = 78732\dfrac{\left(1-\left(\frac{1}{3}\right)^{k}\right)}{\left(1-\frac{1}{3}\right)}$ (M1)

Indication of point where $y = 118\ 096$ (M1)

OR

78 732 + 26 244 + 8748 + 2916 + 972 + 324 + 108 + 36 + 12 + 4
= 118 096 (M1)(M1)

Note: *Award (M1) for a list of at least 8 correct terms, (M1) for the sum of the terms equated to 118096.*

$k = 10$ (A1)(ft) (C3)

Notes: *Follow through from parts (a) and (b). If k is not an integer, do not award final (A1). Accept alternative methods. If 0.333 and 79285 used award (M1)(M1)(A1)(ft) for k = 5. If 0.333 and 79300 used award (M1)(M1)(A0).*

[6]

5. (a) $0.5\left(\frac{1}{2}\right)$ (A1) (C1)

(b) (i) 4 (A1)

(ii) 1 (A1) (C2)

(c) $\frac{16(1-0.5^n)}{(1-0.5)} = 31.9375$ (M1)(M1)

Note: *Award (M1) for correct substitution in the GP formula, (M1) for equating their sum to 31.9375*

OR

sketch of the function $y = \frac{16(1-0.5^n)}{(1-0.5)}$ (M1)

indication of point where $y = 31.9375$ (M1)

OR

$16 + 8 + 4 + 2 + 1 + 0.5 + 0.25 + 0.125 + 0.0625 = 31.9375$ (M1)(M1)

Note: *Award (M1) for a list of at least 7 correct terms, (A1) for the sum of the terms equated to 31.9375.*

$n = 9$ (A1)(ft) (C3)

Note: *Follow through from their answer to part (a) but answer mark is lost if n is not a whole number.*

[6]

6. (a) $u_1r^4 = 324$ (A1)

$u_1r = 12$ (A1)

$r^3 = 27$ (M1)

$r = 3$ (A1)(G3)

Note: *Award at most (G3) for trial and error.*

(b) $4 \times 3^9 = 78732$ **or** $12 \times 3^8 = 78732$ (A1)(M1)(A1)(ft)(G3)

Note: *Award (A1) for $u_1 = 4$ if $n = 9$, **or** $u_1 = 12$ if $n = 8$, (M1) for correctly substituted formula. (ft) from their (a).*

(c) $4 \times 3^{k-1} > 2000$ (M1)

Note: *Award (M1) for correct substitution in correct formula. Accept an equation.*

$k > 6$ (A1)

$k = 7$ (A1)(ft)(G2)

Notes: *If second line not seen award (A2) for correct answer. (ft) from their (a).*

*Accept a list, must see at least **3 terms** including the 6^{th} and 7^{th}.*

Note: *If arithmetic sequence formula is used consistently in parts (a), (b) and (c), award (A0)(A0)(M0)(A0) for (a) and (ft) for parts (b) and (c).*

[10]

7. (a) $100 + 15 \times 10$ (M1)
$= 250$ (A1)
OR
250 (using table function of the GDC) (G2) (C2)

(b) $100(1.08)^{10}$ (M1)
$= 215.89$ (A1)
OR
215.89 (using table function of the GDC) (G2) (C2)

(c) $100 + 15x = 100(1.08)^x$ (M1)
After 16 years (A1)

Note: *Candidate can use trial and error so not necessary to see the first line to award (A2).*

OR
16 years (using table function of the GDC). (G2) (C2)

[6]

8. (a) Let the population at the end of 1999 be x.

$$\frac{44100}{x} = \frac{x}{40000}$$

$x = 42\ 000$ (A1)

(b) $r = \dfrac{44100}{42000}$ (M1)

$r = 1.05$

$u_n = u_1 r^{n-1}$ (M1)

$44\ 100 = u_1(1.05)^6$

$u_1 = 32\ 908$ (or 32 900 to 3 s.f.) (A1)

[4]

9. (a) (i) $a = \$250$ (A1)
$d = \$200$ (A1)
$T_{10} = 250 + 9 \times 200$
$= 2050$ (A1)

Note: *Award the marks for the values of 'a' and 'd' if they are correctly substituted into the formula without being explicitly stated.*

(ii) $a = \$10$ (A1)
$r = 2$ (A1)
$T_0 = 10 \times 2^9$
$= 5120$ (A1) 6

Note: *Award the marks for the values of 'a' and 'r' if they are correctly substituted into the formula without being explicitly stated.*

(b) $S_{10} = \frac{10}{2}(250 + 2050)$ (M1)
$= 11500$ (A1)

OR

$S_{10} = \frac{10}{2}\{2 \times 250 + (10 - 1) \times 200\}$ (M1)
$= 11500$ (A1) 2

(c) Option One: \$10000 (A1)
Option Two: \$11500
Option Three: $S_{10} = \frac{10(2^{10} - 1)}{2 - 1}$ (M1)
$= 10\ 230$ (A1)
Therefore, Option Two would be best. (R1) 4

[12]

10. (a) $u_n = 2(0.9)^7 = 0.957$ m (M1)(A1) 2

Note: *Award (M1) for substitution into formula, list or suitable diagram.*

(b) $S_n = \frac{2(1 - (0.9)^5)}{1 - (0.9)} = 8.19$ m (M1)(M1)(A1)

Note: *Award (M1) for substitution into formula, list or suitable diagram.*

Total distance travelled = $2 \times 8.19 = 16.4$ m. (A1) 4

[6]