FUNCTIONS PRACTICE – MARK SCHEME

1. (a) $2J + 3C = 5.95$ (A2) (C2)

(b) $2 \times 2.15 + 3C = 5.95$ (M1)
$3C = 1.65$ (M1)
$C = 0.55$ (A1)
55 (pence) or £0.55 (C2)

[4]

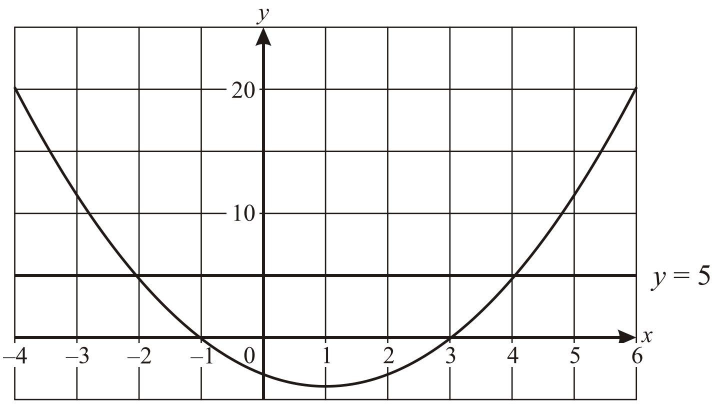
2. (a) $N = 150 \times 2^0 = 150$ (A1) (C1)

(b) $N = 150 \times 2^3 = 1200$ (A1) (C1)

(c) $19200 = 150 \times 2^t$ (M1)
$128 = 2^t$
$7 = t$ (A1) (C2)

[4]

3. (a)



(A1) (C1)

Note: *The equation* y = *5 is not required*

(b) (i) $x = -2$ (A1)
$x = 4$ (A1) (C2)

(ii) $x = 1$ (A1) (C1)

Note: *Allow follow through from candidate's graph*

[4]

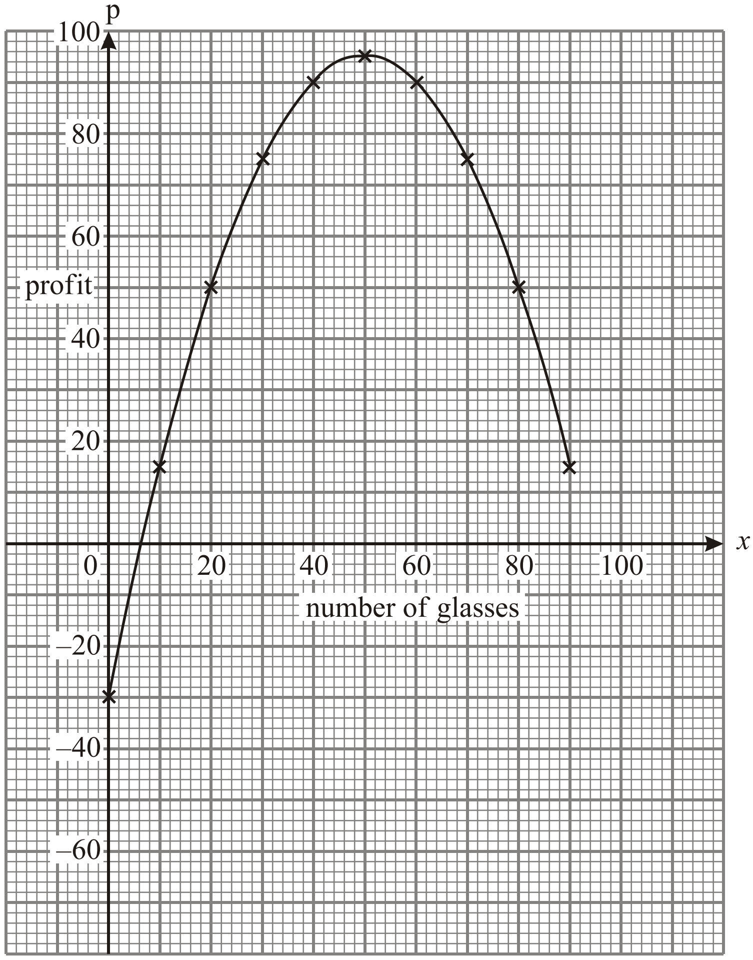
4. (a) (A3)

x	0	10	20	30	40	50	60	70	80	90
P	**–30**	15	**50**	75	90	**95**	**90**	75	50	**15**

Note: *Award ½-mark for each correct bold entry, and round down.*

If a candidate obtains (A0) here but has clearly shown the method of substituting in the values of x into the formula award (M1)

(b)



(A2)(A2)(A1)

Note: *For graph, follow through from candidate's table*

Notes*: Award (A2) for axes, (A2) for plotting points and (A1) for a smooth curve.*

Axes: Award ½-mark for each of the following and then round down:
horizontal axis labelled with "x" or "Numbers of glasses..."
vertical axis labelled with "P" of "Profit"
horizontal scale → consistent and presents values 0→90
vertical scale as for horizontal but represents their range of values for P.

Points: Award (A2) for 0 or 1 error
Award (A1) for 2 or 3 errors
Award (A0) otherwise

(c) (i) maximum profit = 95 swiss francs (A1)

(ii) 50 glasses (A1)

(iii) 67 ± 2 (A1)
33 ± 2 (A1)

(iv) 30 swiss francs (A1)

***Note:** Award no marks for –30 swiss francs*
***Note:** Follow through from candidate's graph*

(d) Fiona's share = $\frac{3}{6}$ (M1)
Profit from 40 glasses = 90 swiss francs
Fiona's profit = $\frac{1}{2} \times 90$
$= 45$ (A1)

[15]

5. (a) $220 = 2(W + x)$ (M1)
Therefore $W = \frac{220 - 2x}{2}$ or $110 - x$ (A1)

(b) Area = $x(110 - x)$ *(allow follow through from part (a))* (A1)

(c) Area = $70(110 - 70) = 2800\ \text{m}^2$ *(allow follow through from part (b))* (A1)

[4]

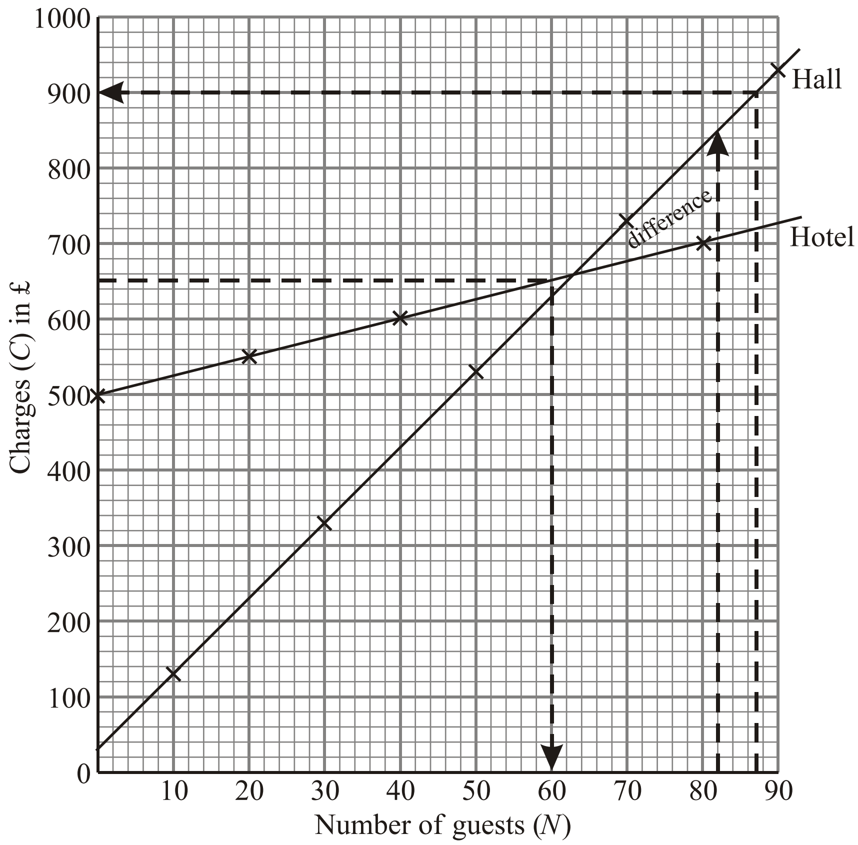
6. (a) (i)

N	10	30	50	70	90
C	**130**	**330**	**530**	**730**	**930**

A2

***Notes:** Award [½ mark] for each correct bold entry and round down to a maximum of [2 marks].*

(ii)



M3

***Notes:** Award (M1) for both axes correctly labelled and with suitable scales.*
Award [½ mark] for each correct point and round up to a maximum of [2 marks].

(iii) $C = 10N + 30$ (A1) 6

(b) (i) "The local hotel will charge £500 plus half of five (or two and a half) times the number of guests" (*or equivalent statement*). (A2)

Note: *Award (Al)(A0) if partly correct.*

(ii)

N	0	20	40	80
C	**500**	**550**	**600**	**700**

(A2)

Note: *Award [½ mark] for each correct bold entry, and round up to a maximum of [2 marks].*

(iii) On same graph as for part (a) (ii). (M2) 6

Note: *Award [½ mark] for each point correctly plotted, and round up to a maximum of [2 marks].*

(c) Explanations abound: Award marks only for any correct explanation arising from candidate's own graph in parts (a) (ii) and (b) (iii). (R2) 2

Notes: *Award (R1) for each correct point given. Maximum [2 marks].*

(d) (i) £900 (M1)(A1)

(ii) 60 guests (M1)(A1)

(iii) £145.00 (M1)(A1) 6

Notes: *For parts (d) (i) to (d) (iii), follow through with candidate's own graphs.*
Answers given here are obtained by calculation and should serve only as a guide.

[20]

7. (a) $(x + 2)(x - 4)$ (A1)

(b) (i) $(-2, 0)$ (A1)

(ii) $(1, -9)$ (A1)(A1)

[4]

8. (a) $c - 0.10k + 1.40$ (A1)

(b) (i) $c = 0.10(7) + 1.40$ *(allow follow through from part (a))*
$= 0.70 + 1.40$ (A1)
$= \$2.10$

(ii) $2.40 = 0.10k + 1.40$ *(allow follow through from part (a))*
$1.00 = 0.10k$ (M1)
$10 = k$
10 km (A1)

[4]

9. (a) III (A1)

(b) I (A1)

(c) II (A1)

(d) IV (A1)

[4]

10. (a) (i) 0.5 (A1)

(ii) 720° (A1)

(b)

x	y	
90°	–1	(A1)
180°	2	(A1)

[4]

11. (a) $\mathbb{R}^+$ (A1)

(b) P(0, 1) (A1)

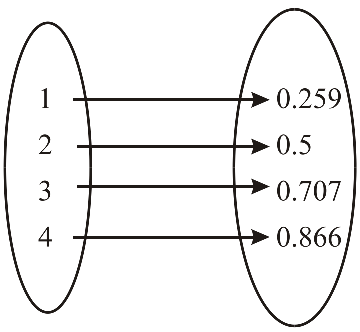
(c) Decreases towards 0 ***or*** $\rightarrow 0$ (A1)(A1)

Note: *Award (A1) for 'Decrease', and (A1) for $\rightarrow 0$.*
Marks awarded at examiner's discretion.

[4]

12. (a) $f : x \mapsto 3x - 2$ (A1)
$x \in \{-1, 0, 1, 2, 3\}$ (A1)

(b)



(A2)

Note: *Award (A1) for the correct domain, (A1) for the correct range.*

[4]

13. (a) $y = x^2 + 3$ (A1)

(b) $y = (x - 2)^2$ (A1)

(c) $y = (x - 2)^2 + 3$ (A2) 4

[4]

14. (a) $a = 2, b = 20, c = 9, d = 8, e = 32$ (A2) 2

Note: *Award (A2) for all 5 correct, (A1) for 3 or 4 correct, (A0) for 2 or less correct.*

(b) $A = 12x - x^2$ (C1) 1

(c) $\frac{dA}{dx} = 12 - 2x$ (A1)

A is maximum when $12 - 2x = 0$ (M1)

$\Rightarrow$ length = 6m and width = 6m (A1)

OR

length = 6m and width = 6m (A2) 3

[6]

15. (a)

Time (seconds)	0	10	20	30
Number of bacteria	1	**2**	**4**	**8**

(A2) (C2)

Note: *Award [½ mark] for each correct entry (round up)*

(b) $N = 2^6$ (M1)

Note: *Award (M1) for any correct method*

$= 64$ (A1) (C2)

[4]

16. (a) $x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$ (A1)

$x = 2$ (A1)

$x = 3$ (A1)

(b) (2, 0)

(3, 0) (A1)

Notes: *Follow through from part (a). Both must be correct and written as coordinates for (A1)*

[4]

17. (a) $c = 1$ (A1) (C1)

(b) amplitude = $\frac{4+2}{2}$ (M1)

$= 3$ (A1)

The graph of $y = \sin x°$ has been reflected in a line parallel to the x-axis therefore $a = -3$ (A1) (C3)

[4]

18. (a) $0° \le x \le 450°$ (A2)

Note: *Award (A1) for $x \ge 0°$, (A1) for $x \le 450°$.*
Award (A1) for 0° and 450° if the inequalities are incorrect.

(b) $1 \le y \le 5$ (A2)

Note: *Award (A1) for $y \ge 1$, (A1) for $y \le 5$.*
Award (A1) for 1 and 5 if the inequalities are incorrect.

Note: *Award (A2) if the candidates have the range and domain reversed, that is,*

(a) $1 \le y \le 5$

(b) $0° < x < 450°$

[4]

19. (a) $x = -\frac{b}{2a}$

$2 = -\frac{4}{2 \times a}$ (M1)

$a = -1$ (A1)

(b) ***Note:*** *Answers to (b) must be written as coordinates.*

(i) M(0, –3) (A1)

(ii) $y = 1 \times 2^2 + 4 \times 2 - 3$
$= 1$
N is (2,1) (A1)

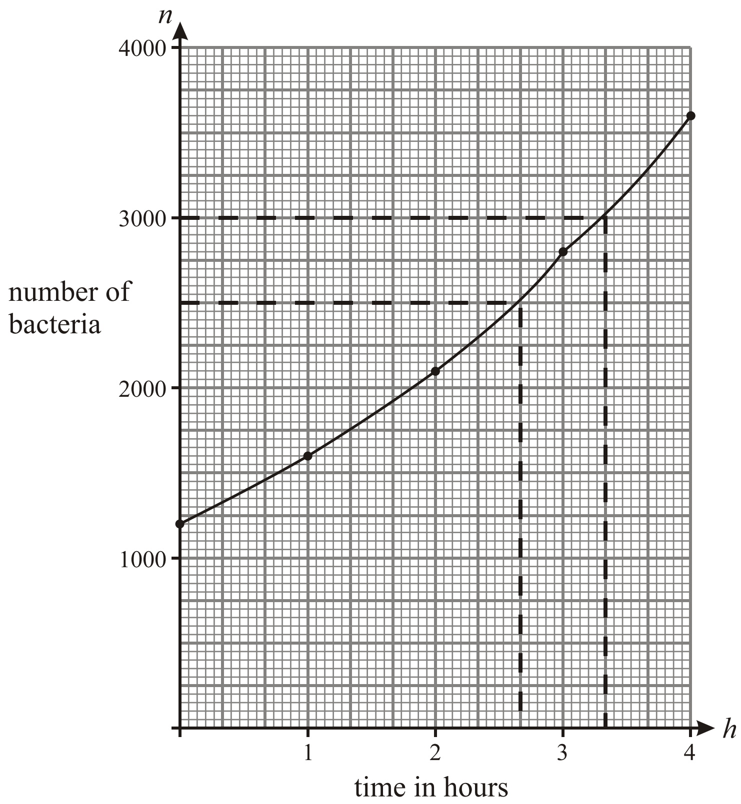
[4]

20. (a)

Time in hours *(h)*	0	1	2	3	4
No. of bacteria *(n)*	1200	**1600**	2100	2700	**3600**

(A1)(A1) 2

(b)



(A2)(A3) 5

Note: *Award (A1) for the axes correctly labelled and (A1) for the correct scales.*

Award (A2) for 4 or 5 points correctly plotted, (A1) for 2 or 3 correct and (A1) for connecting points with a smooth curve.

(c) (i) 2500 (M1)(A1)

(ii) 3hrs 20min (M1)(A1) 4

Note: *Use follow through from graph. If no method is shown from graph give (C1) only for correct answer.*

[11]

21. $c = -10$ (asymptote of graph) (M1)(A1)

$0 = k(2^1) - 10 \Rightarrow 2k = 10$ (M1)

$\Rightarrow k = 5$ (A1)

OR

$k + c = -5$ (M1)

$2k + c = 0$ (M1)

Therefore, $k = 5$ (A1)

$c = -10$ (A1)

[4]

22. (a) $A = (5 + 2x)(7 - 2x)$ (M1)

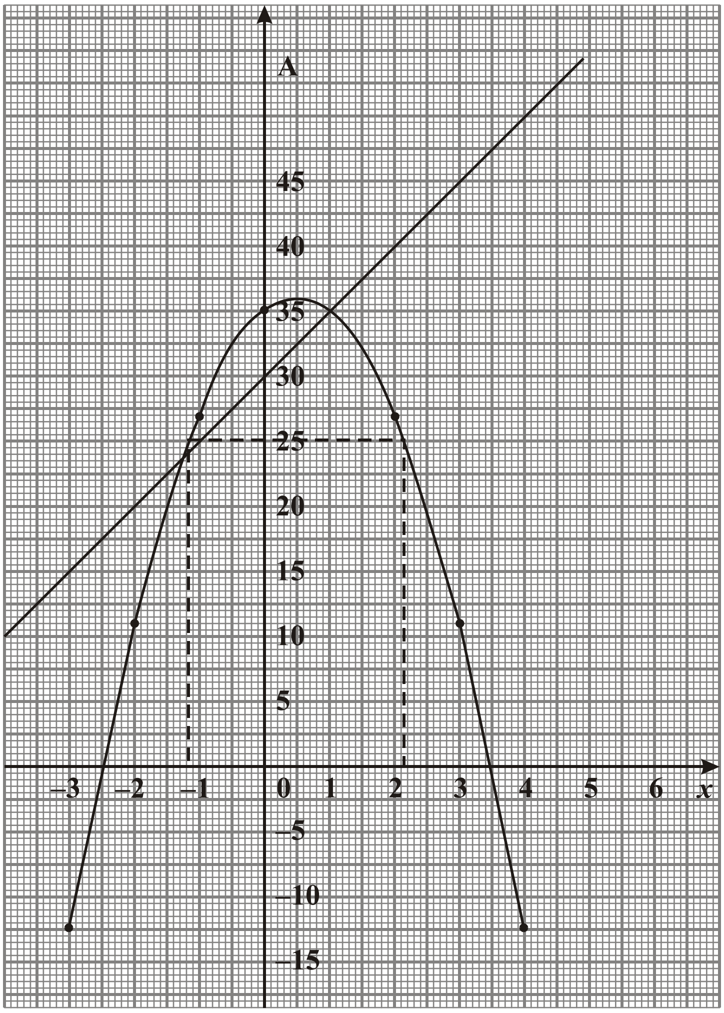
$= 35 - 10x + 14x - 4x^2$

$= 35 + 4x - 4x^2$ (AG) 1

(b) (i) $p = 11$, $q = 35$, $r = 27$, $s = -13$ (A2)

Note: *Award (A2) for all four correct, (A1) for two or three correct.*

(ii)



(A4) 6

Notes: *Award (A1) for axes with correct scales and labelling.*
Award (A2) for 6, 7 or 8 points correctly plotted,
(A1) for 3, 4, or 5 points,
(A0) for 2 or fewer.
Award (A1) for a smooth curve through reasonably correct points.

(c)	(i)	Axis of symmetry is $x = \frac{1}{2}$	(A1)		
	(ii)	$A = 27 \Rightarrow x = -1$ or $x = 2$	(A1)		
		Note: *Award (A1) for one correct value of x.*			
	(iii)	$x = -1$, rectangle is $(5 - 2) \times (7 + 2)$	(M1)		
		i.e. 3×9	(A1)		
		OR			
		$x = 2$, rectangle is $(5 + 4) \times (7 - 4)$	(M1)		
		i.e. 9×3	(A1)	4	
		Notes: *Award (A2) for the correct answer.* *Follow through with answers for x from the candidate's graph.*			
(d)	(i)	Line on graph.	(A1)		
	(ii)	From graph solutions are $x = 1$ and $x = -1.3$ (± 0.1) (Follow through with candidate's graph of parabola and straight line.)	(A2)		
		OR			
		Factorizing gives $(x - 1)(4x + 5) = 0$	(M1)		
		$\Rightarrow x = 1$ or $x = -1.25$	(A1)	3	
					[14]

23.	(a)	(ii)	(A2)	(C2)	
	(b)	(i)	(A2)	(C2)	
	(c)	(iii)	(A2)	(C2)	
	(d)	(iv)	(A2)	(C2)	
					[8]

24.	(a)	$2925 = 12r + s$	(M2)		
		$4525 = 20r + s$	(M2)		
		$1600 = 8r$			
		$200 = r$	(A2)	(C6)	
	(b)	$2925 = 12(200) + s$			
		$525 = s$	(A2)	(C2)	
		Note: *Award (C2)(C2) if the candidate correctly solves an incorrect system of equations.*			
					[8]

25. (a) (i) $y = 3^{-0} + 2$ (M1)

$y = 1 + 2$ (A1)

$a = 3$ (A1) (C3)

(ii) $y = 3^{-1} + 2$ (M1)

$y = \frac{1}{3} + 2$ (A1)

$b = 2\frac{1}{3}$ (A1) (C3)

(b) $y = 2$ (A2) (C2)

Note: *Award (A1) for y = any constant.*

[8]

26. (a) (i) 1.75 m ± 0.1 m (A1)

(ii) 01:34 ± 10 minutes (A1)

06:26 ± 10 minutes (A1) 3

(b) $2 < t < 6$ (A3) 3

Notes: *Award (A1) each for times ± 5 minutes, (A1) for both strict inequalities. Award (A2) for the answer "between 02:00 and 06:00".*

(c) $a = \frac{4.5 - 1.5}{2}$ (M1)

$= 1.5$ (A1)

$b = \frac{360°}{8}$ (M1)

$= 45°$ (A1) 4

(d) $h(t) = 1.5 \cos(45t°) + 3$

13:00 $\Rightarrow$ 13 hours after midnight (M1)

$h(13) = 1.5 \cos(45 \times 13) + 3$ (M1)

$= 1.94$ m (A1)

OR

13:00 $\Rightarrow$ 5 hours into the next period (M1)

$h(5) = 1.5 \cos(45 \times 5) + 3$ (M1)

$= 1.94$ m (A1) 3

(e) next low $\Rightarrow$ 8 hours after first (M1)

time = 04:00 + 8 hours

= 12 noon (or 12:00 or 12 hours after midnight) (A1) 2

[15]

27. (a) (0,1) (A2)(A2) (C4)

(b) $16 = a^4$ (M2)

$a = 2$ (A2) (C4)

[8]

28. (a) (i) Domain: $\mathbb{R}$ (A2)

(ii) Range: $\{y \mid y \geq 2\}$ accept $y \geq 2$ (A2) (C4)

(b) (i) Domain: $\{x \mid -360° \leq x \leq 360°\}$ (A2)

Accept $-360 \leq x \leq 360$

(ii) Range: $\{y \mid -1.5 \leq y \leq 1.5\}$ (A2) (C4)

Accept $-1.5 \leq y \leq 1.5$

[8]

29. (a) $C = \dfrac{5(50-32)}{9}$ (M1)

$= 10°C.$ (A2) (C3)

(b) Put $C = -273$ (A1)

so $-273 = \dfrac{5(F-32)}{9}$ (M1)

Hence $9 \times -273 = 5(F - 32)$ (M1)

$F = -491.4 + 32 = -459.4$ (accept -459). (M1)(A1) (C5)

Note: *(M1) is for adding 32, even if the other number is incorrect.*

[8]

30.

Equation	Diagram number	
$y = c$	2	(A2)
$y = -x + c$	3	(A2)
$y = 3x + c$	4	(A2)
$y = \frac{1}{3}x + c$	1	(A2) (C8)

[8]

31. (a) Point P is at $x = 0$. Find $y(0)$. (M1)

$y(0) = 2^0 + 2^{-0}$ (A1)

$= 1 + 1 = 2$ (A1) (C3)

(b) (0, 2) is the lowest point.

Highest points are at $x = 2$ or $x = -2$.

At $x = 2$ or -2, $y = 2^2 + 2^{-2} = 4\frac{1}{4}$ or 4.25 (M1)(A1)

So the range is $[2, 4\frac{1}{4}]$ or $2 \leq y \leq 4.25$. (A1)(A1)(A1) (C5)

Note: *Award (A1) for each inequality (or bracket).*
If both inequalities are strict (or parentheses) award (A0)(A1)(ft).
Award (A1) for both numbers in order.

[8]

32. (a) At $x = 0$ we have $y = 6 = c$, (M1)
so $c = 6$. (A1) (C2)

(b) At $x = 3$ we have $9a + 12 + c = 0$ (M2)
$a = -2$ (A1)

OR

at $x = -1$ we have $a - 4 + c = 0$ (M2)
$a = -2$ (A1) (C3)

(c) Factorisation is $y = -2(x - 3)(x + 1)$ (A1)(A1)(A1) (C3)

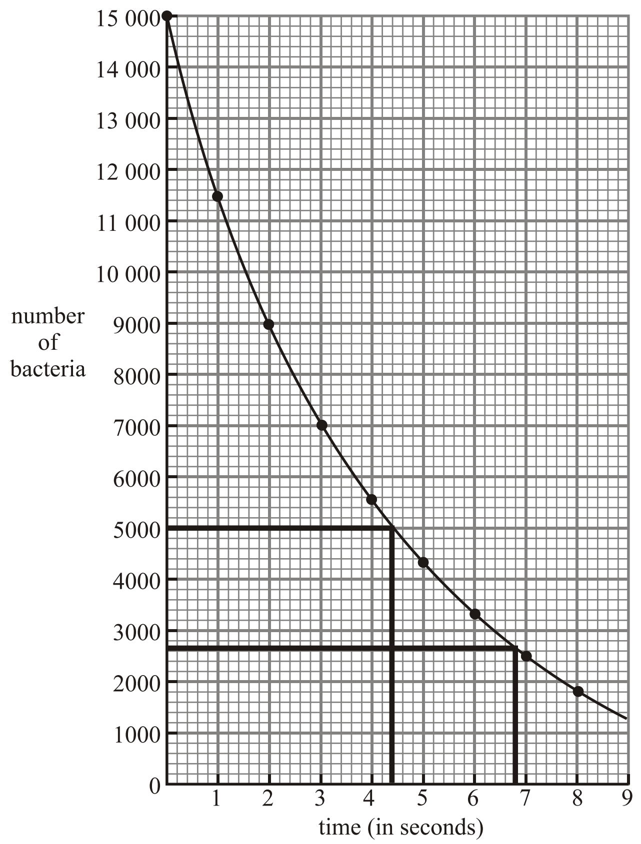
OR

can include 2 and/or sign in a factor. (A1)(A2) (C3)

[8]

33. (a) $a = 15000$ (A1)
$b = 5500$ (A1)
$c = 2000$ (A1) 3

(b)



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Note: *Award (A1) for axes correctly labelled, (A1) for correct scales, (A1) for smooth curve, (A2) for all points correctly plotted, (A1) for at least 4 points correct.*

(c) (i) 4.4 secs (M1)(A1)

Note: *Award (M1)(A1)(ft) from graph (see (b)) or (A1) if correct and no line seen.*

(ii) 2700 bacteria (±200 bacteria) (M1)(A1)

Note: *Award (M1)(A1)(ft) from graph (see (b)) or (A1) if correct and no line seen.*

(iii) No — theoretically, the curve never touches the horizontal axis (or any answer to suggest that the horizontal axis is an asymptote). (A1)(R1) 6

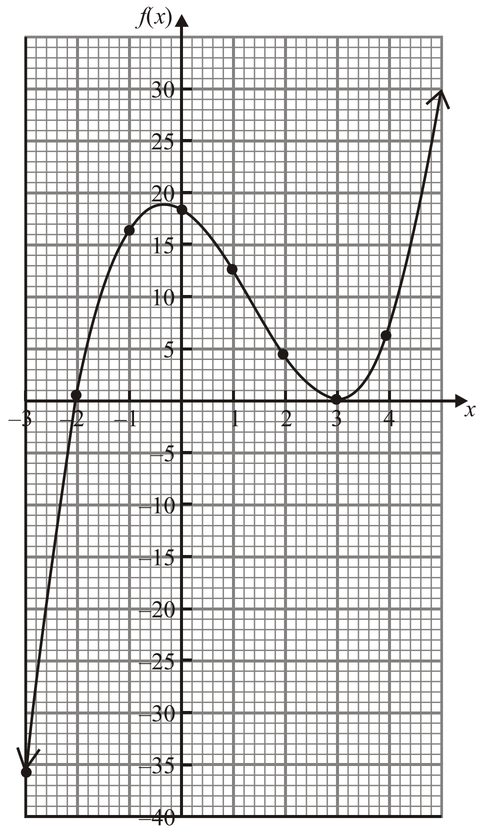
Note: *Award (A1)(R1) for any time over 39 seconds with a reasonable explanation (when the number of bacteria is less than one).*
Award (A0)(R0) for a yes or no with no explanation. Do not award (A1) if (R1) is not awarded.

[14]

34.

(b) $a = 0$ (A1)
$b = 18$ (A1) 2

(c)



(A5) 5

Note: *Award (A1) for scales and axes labelled correctly, (A1)(A1) for maximum and minimum placed correctly, (A1) for smooth curve, (A1) for all points plotted correctly.*

35. (a) $5x(6 - x)$ (A1)(A1)(A1) (C3)

Note: *Award (A1) for each factor. Therefore x(30 – 5x) would be awarded (A0)(A1)(A1).*

(b) $5x(6 - x) = 0$ (M1)

$x = 0$ or $x = 6$

$A = (6, 0)$ (A1)(A1) (C3)

(c) $x = 3$ (A2) (C2)

OR

$x = \frac{-b}{2a} = \frac{-30}{2 \times -5}$ (M1)

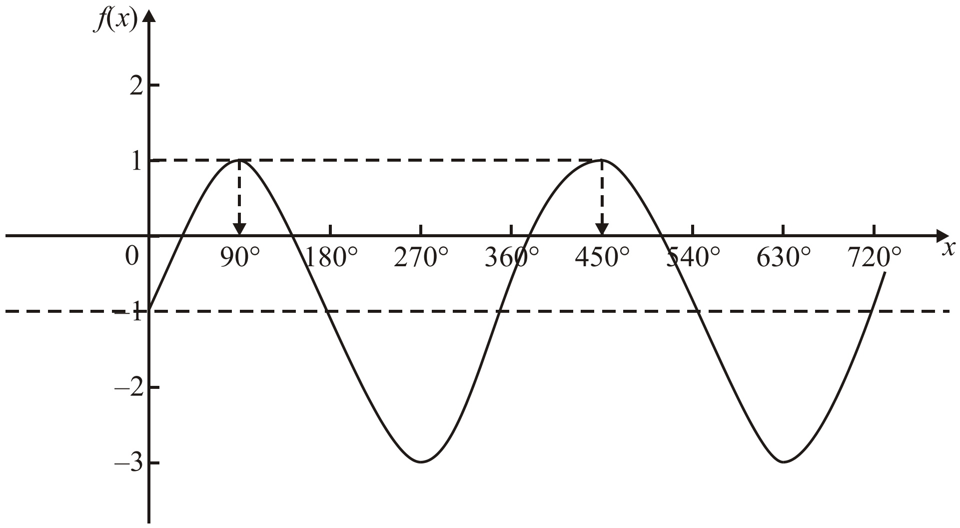
$= 3$ (A1) (C2)

[8]

36. (a) 360° (A1) (C1)

(b) $-2 - 1 = -3$ (M1)(A1)(A1) (C3)

(c)



$x = 90°$, $x = 450°$ (G2)(G2)(C2)(C2)

OR

$2 \sin x - 1 = 1$ (M1)

$2 \sin x = 2$

$\sin x = 1$ (A1)

$x = 90°, 450°$ (A1)(A1)(C2)(C2)

Notes: *Award (C2) for x = 90° with no working shown.*

Answers given in radians

Award (A0) for (a) 2π.

Award (M1)(A1)(A1)(A1)ft for (c) $\frac{\pi}{2}, \frac{5\pi}{2}$.

[8]

37.	(a)	(iv)	(A2)	(C2)
	(b)	(i)	(A2)	(C2)
	(c)	(ii)	(A2)	(C2)
	(d)	(v)	(A2)	(C2)

[8]

38. (a) The area covered before 7 July (R2) 2

Note: *Award (R1) for "area", (R1) for "before" 7 July.*

(b) $t = 8 \pm 0.4$ (A1) 1

(c) $100(1.075)^{21}$ (M1)(A1)
$= 457 \text{ m}^2$ (A1) 3

[6]

Note: *Award (M1) for correct formula, (A1) for correct power and (A1) for correct answer.*

39. (a) Put $x = 0$ to find $y = -2$ (M1)
Coordinates are $(0, -2)$ (A1) (C2)

Note: *Award (M1)(A0) for –2 if working is shown. If not, award (M0)(A0).*

(b) Factorise fully, $y = (x - 2)(x + 1)$. (A1)(A1)
$y = 0$ when $x = -1, 2$. (A1)(A1)
Coordinates are A(–1, 0), B(2, 0). (A1)(A1) (C6)

Note: *Award (C2) for each correct x value if no method shown and full coordinates not given. If the quadratic formula is used correctly award (M1)(A1)(A1)(A1)(A1)(A1). If the formula is incorrect award only the last (A1)(A1) as ft.*

[8]

40. (a) The range is [1, 3] or $1 \le y \le 3$ (A1)(A1)(A1) (C3)

Note: *Award (A1) for both closed interval brackets or for both correct inequality signs with y or f(x), and (A1) for each correct end value.*
Award (A0) if the domain is given.

(b) The amplitude is 1. (A1) (C1)

(c) The period is $\frac{2\times180}{2} = 180°$. (M1)(A1) (C2)

Note: *Or award (R1)(A1) for just observing the period on the graph.*

(d) Halve the period, 90° (marks for either). (A2) (C2)

Note: *Award (M1)(A1) for a correct graph sketch as long as the new period is indicated.*

[8]

41. (a) 0.40 (A2) (C2)

(b) $0.55 + 0.50 = 1.05$ (A1)(A1)(A1) (C3)

Note: *Award (A1) for 0.55, (A1) for 0.50, (A1) for correct total of amounts given.*

(c) $0.80 + 1.40 = 2.20$ (A1)(A1)(A1) (C3)

Note: *Award (A1) for 0.80, (A1) for 1.40, (A1) for correct total of amounts given.*

[8]

42. (a) Domain $x < 3$ (accept $-4 \le x < 3$) Range $y \le 2$ (accept $-2 \le y \le 2$) (A2)(A2) (C4)

Note: *Award (A1) for $x \le 3$ and (A1) for $y < 2$. If the domain and range are reversed award [0 marks] in this part of the question. Allow for other notation such as $[-\infty, 3]$ or $[\infty, 3]$ for domain and $[-\infty, 2]$ for range.*

(b) Domain $\{-3, -2, -1, 0, 1, 2, 3\}$ Range $\{1, 2, 3, 4\}$ (A2)(A2) (C4)

Note: *Award (A2) ft, (A2) ft if domain and range are reversed. Award (A1) if 1 number is omitted from the domain and (A1) if 1 number is omitted from the range.*
Award (A0) if more than 1 number is omitted from the domain and (A0) if more than 1 number is omitted from the range.
Award (A0) for $-3 \le x \le 3$ and $1 \le y \le 4$.

[8]

43. (a) $(x - 3)(x + 1)$ (A1)(A1) (C2)

Note: *Award (A0)(A1) if the signs are reversed.*

(b) A(1, 0), B(3, 0) (A1)(A1) (C2)

(c) $x = 1$ or $x = \dfrac{(-1+3)}{2} = 1$ or $x = \dfrac{-(-2)}{2(1)} = 1$ (A1)(A1) (C2)

Note: *Award (A1) for $x =$ and (A1) for 1.*

(d) C(1, –4) (A1)(A1) (C2)

[8]

44. (a) (i) 120° (A2) (C2)

(ii) 1 (A2) (C2)

(iii) 1 (A2) (C2)

(b) $\dfrac{360}{a} = 120 \Rightarrow a = 3$ (A2) (C2)

[8]

45. (a) 85 ± 1 (M1)(A1) (C2)

(b) 21.5 ± 0.5 (M1)(A1) (C2)

(c) $y = 100 \times (5^{-0.02\times 80})$
$= 7.61$ (M1)(A1) (C2)

(d) $y = 0$ (A1)(A1) (C2)

Note: *Award (A1) for y = and (A1) for 0.*

[8]

(d) Gradient $= \dfrac{4-0}{8-6} = 2$ (A1)

$y = mx + c$
$0 = 2 \times 6 + c$ (M1)
$c = -12$ (A1)
Equation is $y = 2x - 12$ (or correct alternatives).
Ft from candidate's previous work. (A1) 4

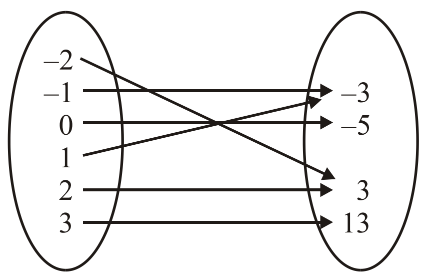
[14]

47. (a) For using tan (M1)
$h = 12.3 \times \tan 63$ For using tan something (A1)
$h = 24.1$ (A1) (G3)

(b) $24.1 = 4.9t^2$ For substituting for h in the formula and attempting to solve (M1)
For taking a square root (can be implied) (M1)
2.22 sec (A1) (C3)

[6]

48. (a)



For six single lines going to correct y (y-value can be repeated) (M1)
Correct diagram (y-values not repeated) (A1) (C2)

(b) $x \in \{-2, -1, 0, 1, 2, 3\}$ (A2) (C2)

Note: *Award (A1) if one value omitted.*

(c) $y \in \{-3, -5, 3, 13\}$ (A2) (C2)

[6]

49. (a) 4 (A1) (C1)

(b) For raising to a power of 6.4 (M1)
28 (A1) (C2)

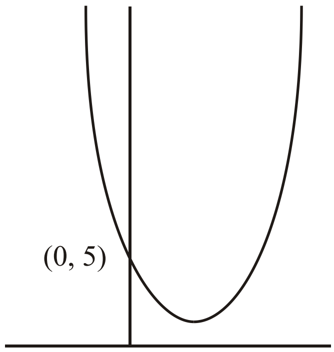
(c) $1200 = 4 \times (1.356)^{0.4t}$ (for substituting in the formula) (M1)
$300 = (1.356)^{0.4t}$ (A1)
$t = 46.8$ (by trial and error) (A1)

OR

$t = 46.8$ (G3) (C3)

[6]

50. (a)



(A3) (C3)

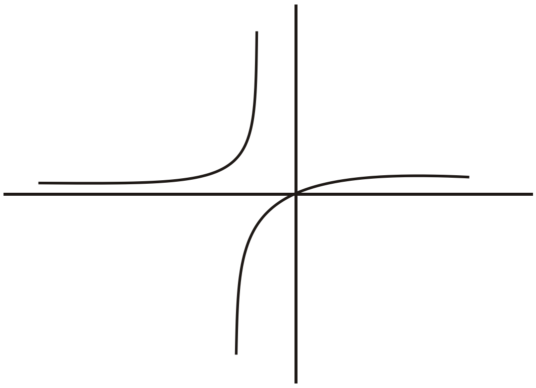
Notes: *Award (A1) for point (0,5) indicated. Award (A2) for correct shape.*

(b) (1.5, 0.5) (A1)(A1) (C2)

(c) $x = 1.5$ (A1) (C1)

[6]

51. (a)



(A4) (C4)

Notes: *Award (A1) for correct scales.*
Award (A1)(A1) for two correct parts to the graph.
Award (A1) if asymptotes are shown.

(b) Horizontal asymptote $y = 1$. (A1) (C1)

[6]

52. (a) (i) $\{-3, -2, -1, 0, 1, 2, 3\}$ (A1)(A1)

Notes: *Award (A1) for set brackets.*
Award (A1) for all and only correct numbers.

(ii) $\{0, 1, 4, 9\}$ (A1)

Notes: *Award (A1) for all and only correct numbers.*
If domain and range reversed, can follow through in (ii).

(iii) $f(x) = x^2$ (A2) (C5)

Note: *Allow any other rule that works.*

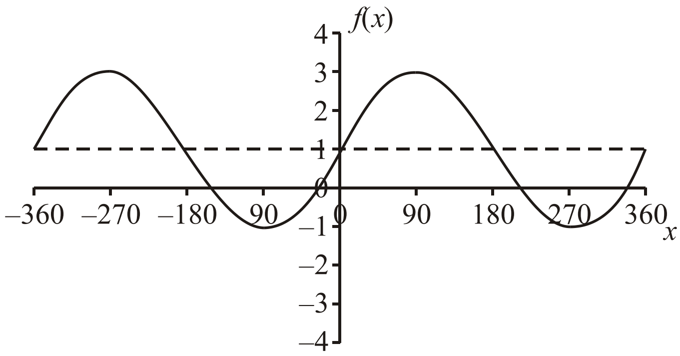
(b) $[1, \infty]$ or $\{x \in \mathbb{R} \mid x \geq 1\}$ (A1) (C1)

[6]

53. $a = 5$ (A2)
$b = 2$ (A2)
$c = 3$ (A2)

[6]

54. (a)



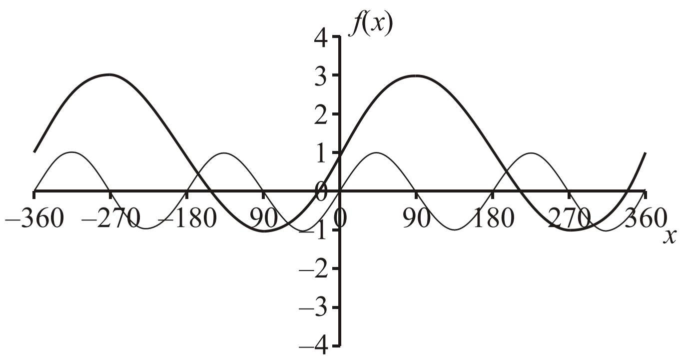
x-axis from –360° to 360°	(A1)	
2 maxima at $y = 3$	(A1)	
2 minima at $y = –1$	(A1)	
Correct shape of graph with reasonable axes intercepts.	(A1)	4

(b) Range $-1 \le y \le 3$ or $[-1, 3]$ (A2) 2

Note: *Award (A1) for –1 to 3.*

(c) Amplitude = 2 (A1) 1

(d)



Correct maximum	(A1)	
Correct minimum	(A1)	
Correct period	(A1)	
Correct shape with reasonable axes intercepts.	(A1)	4

(e) Period 180° (A1) 1

(f) 4 solutions (A1) 1

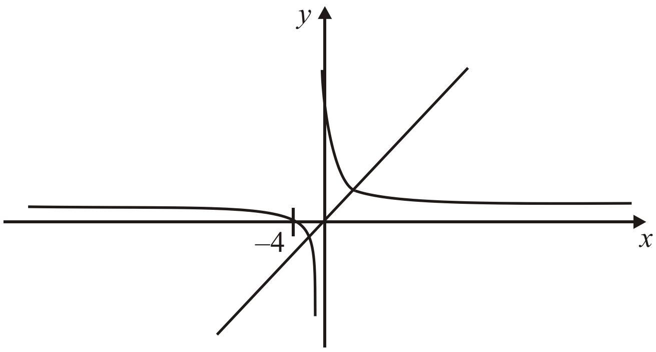
(g) $x = 195°$ (G2)

$x = 296°$ (G2) 4

Note: *If more than two solutions given award (A2).*

[17]

55. (a)



For x-axis from –10 to 10. (A1)
For –4 marked. (A1)
For correct shape of graph. (A1)(A1) 4

(b) Horizontal asymptote (A1)
$y = 1$ (A1)
Vertical asymptote (A1)
$x = 0$ (A1) 4

(c) Line drawn on sketch (A2) 2

(d) (2.56, 2.56) (–1.56, –1.56) (A1)(A1)(A1)(A1) 4

(e) Range $y \in \mathbb{R}, y \neq 1$ (A1)(A1) 2

[16]

56. (a) A = (0, 1)
For parentheses (A1)
For numbers (A1) 2

(b) B = (2, 4), C = (4, 16)
For 2,4 (A1)
For 4,16 (A1) 2

(c) At D, $x = –0.767$ (A1) 1

(d) $x \leq –0.767$ (A1)
$2 \leq x \leq 4$
For inequalities (A1)
For numbers (A1)

OR

$[–\infty, –0.767] \cup [2, 4]$ (A3) 3

[8]

57. (a) $M = 90 \times 2^{\frac{0}{20}} = 90$ (grams) (M1)(A1) (C2)

Note: *Award (M1) for t = 0.*

(b) $t = 80$…… $M = 90 \times 2^{\frac{-80}{20}}$ (M1)

Therefore, $v = 5.625$ (grams) (5.63 3 s.f.) (accept either) (A1) (C2)

(c) $45 = 90 \times 2^{\frac{-t}{20}}$ (M1)

$2^{\frac{-t}{20}} = 0.5$ (M1)

$t = 20$ years (A2) (C4)

[8]

58. (a) $y = x(5 - x)$ or $y = 5x - x^2$ or $25 = c + 5k$ (M1)

$c = 0, k = 5$ (A1)(A1) (C3)

Note: *Award (A1) if no method is indicated but c = 0 or k = 5 is given alone.*

(b) Vertex at $x = \dfrac{-b}{2a} = \dfrac{-5}{-2} = 2.5$ (M1)(A1)

$y = 5(2.5) - 2.5^2 = 6.25$ (M1)(A1)

Note: *The substitutions must be attempted to receive the method marks.*

Q(2.5, 6.25) (A1) (C5)

Notes: *Coordinate pair is required for (A1) but Q is not essential. If no working shown and answer not fully correct, award (G2) for each correct value and (A1) for coordinate brackets. However, if values are close but not exactly correct (eg (2.49, 6.25)) award only (G1) for each less precise value. In this case AP might also apply if number of digits is inappropriate.*
If differentiation is used, award (M1) for correct process, (A1) for x = 2.5, (M1)(A1) or (G2) for 6.25 and (A1) for coordinate brackets.

[8]

59. (a)

FUNCTION	GRAPH LABEL
$y = a \cos (x)$	C
$y = a \sin (2x)$	B
$y = 2 + a \sin (x)$	A

(A1)(A1)(A1) (C3)

(b) $P = \dfrac{360}{2} = 180$ (M1)(A1)

or the period is 180 degrees. (C2)

Note: *Award (A1) only for 0 – 180 or π.*

(c) The range is $[2 - a, 2 + a]$ **or** $2 - a \le y \le 2 + a$ (A1)(A1)(A1) (C3)

Note: *Award (A1) for each value seen and (A1) for correct [ ] or ≤ y ≤.*
[–a, a] (or equivalent) can receive (A2) and (–a, a) or equivalent can receive (A1).

[8]

60. (a) Profit = Income – Cost

$P(x) = 150x - 0.6x^2 - (2600 + 0.4x^2)$ (M1)

$= 150\,x - 0.6x^2 - 2600 - 0.4x^2$ (M1)

Note: *Award (M2) for either line seen without the other, but award only (M1) if omission of brackets results in* $+ 0.4x^2$.

$= -x^2 + 150x - 2600$ (AG) 2

(b) maximum profit when $x = -\dfrac{-150}{2\times -1}$ or $x = -\dfrac{20+130}{2}$ (M1)

= 75 machines (A1)

or (G2) 2

Note: *Sketch or table of values from GDC can receive (M1) as long as the values are appropriate. Table must include at least evaluation for 74, 75, 76, and sketch must show 75 beneath the maximum, however, any non-integer answer must receive (A0). If differentiation is used, award (M1) for* $-2x + 150 = 0$.

(c) $I\,(75) = 150(75) - 0.6(75)^2$ (M1)

= \$7875 (A1)

Selling price per machine = $\dfrac{7875}{75}$ (M1)

= \$105 (A1) 4

Notes: *If P(75) or C(75) used, award M0 A0 but ft with candidate's value to the selling price.*

(d) $P(x) = 0$ or $(x - 20)(130 - x) = 0$. (M1)

$x = 20$ (130 need not be mentioned) (A2)

Smallest number must be 21. (A1) 4

Notes: *If no working shown:*
Award (G2) if answer is 20, $x > 20$ *or* $x = 20$,
Award (G3) if answer is 21 or $x = 21$.
*A sketch of the function showing the intercepts receives (M1) with (A2) or (A3) for **separate** indication of answer 20 or 21 respectively. If brackets are expanded and quadratic formula is used, the (M1) should be awarded only for correct expansion and correct substitution into the formula.*

[12]

61. (a) (i) 120° (A2)

(ii) 4 (A2) (C4)

(b) correct line on graph (A2) (C2)

(c) 10° (±3°) (A1)

50° (±3°) (A1) (C2)

Note: *Answers by calculation are 10° and 50° exactly.*

[8]

62.

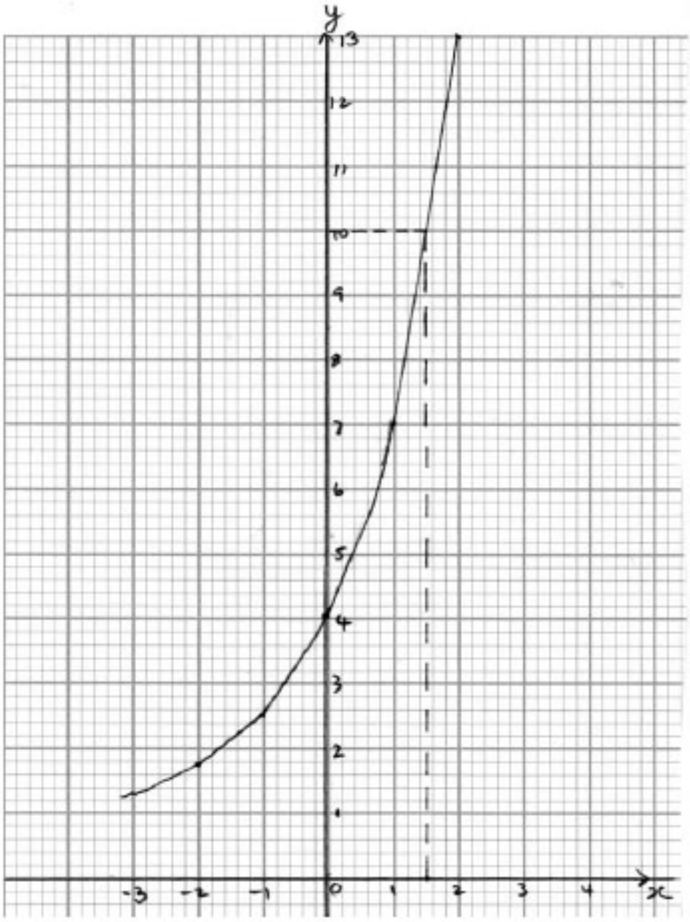
equation	sketch	
(i)	2	(A2)
(ii)	4	(A2)
(iii)	3	(A2)
(iv)	1	(A2) (C8)

Note: *Award (A2) for each correct sketch.*

[8]

63. (a) $a = 2.5$, $b = 13$ (A1)(A1) 2

(b) (A4) 4



Note: *Award (A1) for scales and labels, (A2) for all points accurate ((A1) for 5 correct), (A1) for smooth curve.*

(c) Range $f(x) > 1$

($y > 1$) (A2) 2

Note: *Award (A1) for* $f(x) >$*, (A1) for 1.*

(d) $x = 1.6$ ($\pm$ 0.1) (M1)(A1) (or (G2)) 2

Note: *Answer by calculation is 1.58.*

[10]