FUNCTIONS PRACTICE

1. At Jumbo's Burger Bar, Jumbo burgers cost £J each and regular cokes cost £C each. Two Jumbo burgers and three regular cokes cost £5.95.

(a) Write an equation to show this.

(b) If one Jumbo Burger costs £2.15, what is the cost, in pence, of one regular coke?

Working:

Answers:

(a) ……..

(b) ……..

(Total 4 marks)

2. In an experiment researchers found that a specific culture of bacteria increases in number according to the formula

$$N = 150 \times 2^t,$$

where N is the number of bacteria present and t is the number of hours since the experiment began.

Use this formula to calculate

(a) the number of bacteria present at the start of the experiment;

(b) the number of bacteria present after 3 hours;

(c) the number of hours it would take for the number of bacteria to reach 19 200.

Working:

Answers:

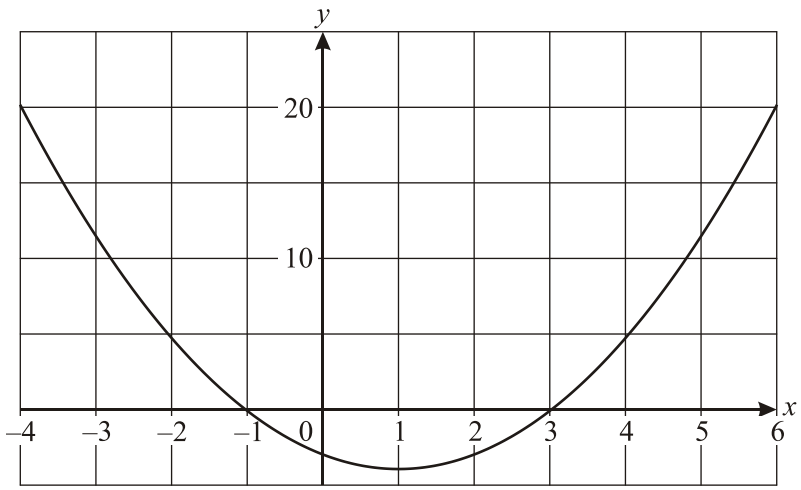
(a) ……..

(b) ……..

(c) ……..

(Total 4 marks)

3. The graph of $y = x^2 - 2x - 3$ is shown on the axes below.



(a) Draw the graph of $y = 5$ on the same axes.

(b) Use your graph to find:

(i) the values of x when $x^2 - 2x - 3 = 5$

(ii) the value of x that gives the minimum value of $x^2 - 2x - 3$

Working:

Answers:

(b) (i) ……...

(ii) ……..

(Total 4 marks)

4. The profit (P) in Swiss Francs made by three students selling homemade lemonade is modelled by the function

$$P = -\frac{1}{20}x^2 + 5x - 30$$

where x is the number of glasses of lemonade sold.

(a) **Copy** and complete the table below

x	0	10	20	30	40	50	60	70	80	90
P		15			90			75	50	

(3)

(b) On graph paper draw axes for x and P, placing x on the horizontal axis and P on the vertical axis. Use suitable scales. Draw the graph of P against x by plotting the points. Label your graph.

(5)

(c) **Use your graph** to find

(i) the maximum possible profit;

(1)

(ii) the number of glasses that need to be sold to make the maximum profit;

(1)

(iii) the number of glasses that need to be sold to make a profit of 80 Swiss Francs;

(2)

(iv) the amount of money initially invested by the three students.

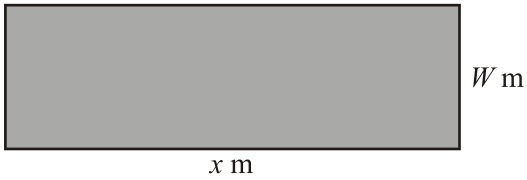
(1)

(d) The three students Baljeet, Jane and Fiona share the profits in the ratio of 1:2:3 respectively. If they sold 40 glasses of lemonade, calculate Fiona's share of the profits.

(2)

(Total 15 marks)

5. The perimeter of this rectangular field is 220 m. One side is x m as shown.



(a) Express the width (W) in terms of x.

(b) Write an expression, in terms of x only, for the area of the field.

(c) If the length (x) is 70 m, find the area.

Working:

Answers:

(a) ……..

(b) ……..

(c) ……..

(Total 4 marks)

6. Vanessa wants to rent a place for her wedding reception. She obtains two quotations.

(a) The local council will charge her £30 for the use of the community hall plus £10 per guest.

(i) **Copy** and complete this table for charges made by the local council.

Number of guests (N)	10	30	50	70	90
Charges (C) in £					

(2)

(ii) On graph paper, using suitable scales, draw and label a graph showing the charges. Take the horizontal axis as the number of guests and the vertical axis as the charges.

(3)

(iii) Write a formula for C, in terms N, that can be used by the local council to calculate their charges.

(1)

(b) The local hotel calculates charges for their conference room using the formula:

$$C = \frac{5N}{2} + 500$$

where C is the charge in £ and N is the number of guests.

(i) Describe, **in words only**, what this formula means.

(2)

(ii) complete this table for the charges made by the hotel.

Number of guests (N)	0	20	40	80
Charges (C) in £				

(2)

(iii) On the same axes used in part (a)(ii), draw this graph of C. Label your graph clearly.

(2)

(c) Explain, briefly, what the two graphs tell you about the charges made.

(2)

(d) Using your graphs or otherwise, find

(i) the cost of renting the community hall if there are 87 guests;

(2)

(ii) the number of guests if the hotel charges £650;

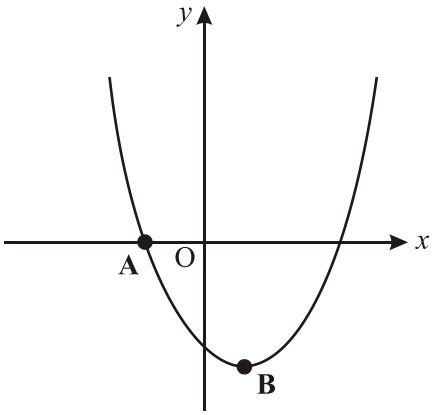
(2)

(iii) the difference in charges between the council and the hotel if there are 82 guests at the reception.

(2)

(Total 20 marks)

7. The diagram shows the graph of $y = x^2 - 2x - 8$. The graph crosses the x-axis at the point A, and has a vertex at B.



(a) Factorize $x^2 - 2x - 8$.

(b) Write down the coordinates of each of these points

(i) A;

(ii) B.

Working:

Answers:

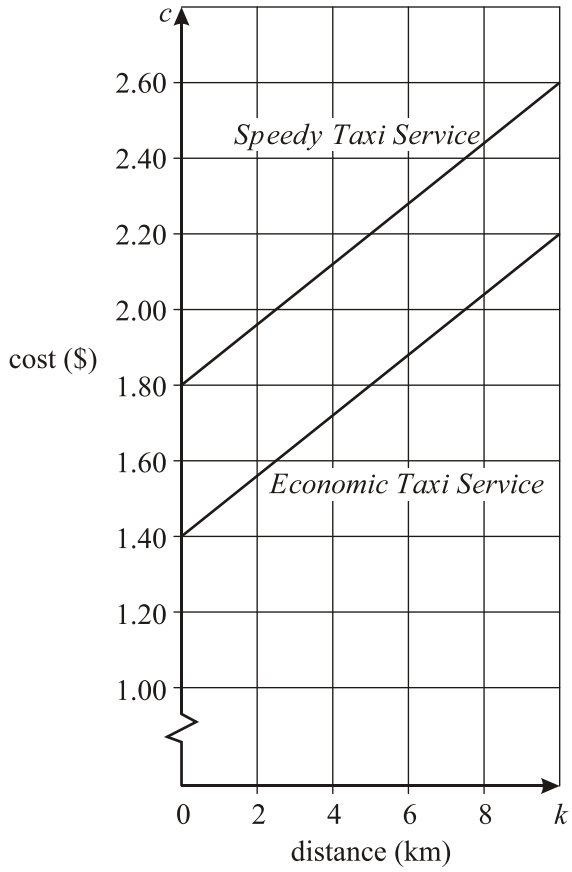
(a) ……..

(b) (i) ……..

(ii) ……..

(Total 4 marks)

8. The costs charged by two taxi services are represented by the two parallel lines on the following graph. The *Speedy Taxi Service* charges \$1.80, plus 10 cents for each kilometre.



(a) Write an equation for the cost, c, in \$, of using the *Economic Taxi Service* for any number of kilometres, k.

(b) Bruce uses the *Economic Taxi Service*.

(i) How much will he pay for travelling 7 km?

(ii) How far can he travel for \$2.40?

Working:

Answers:

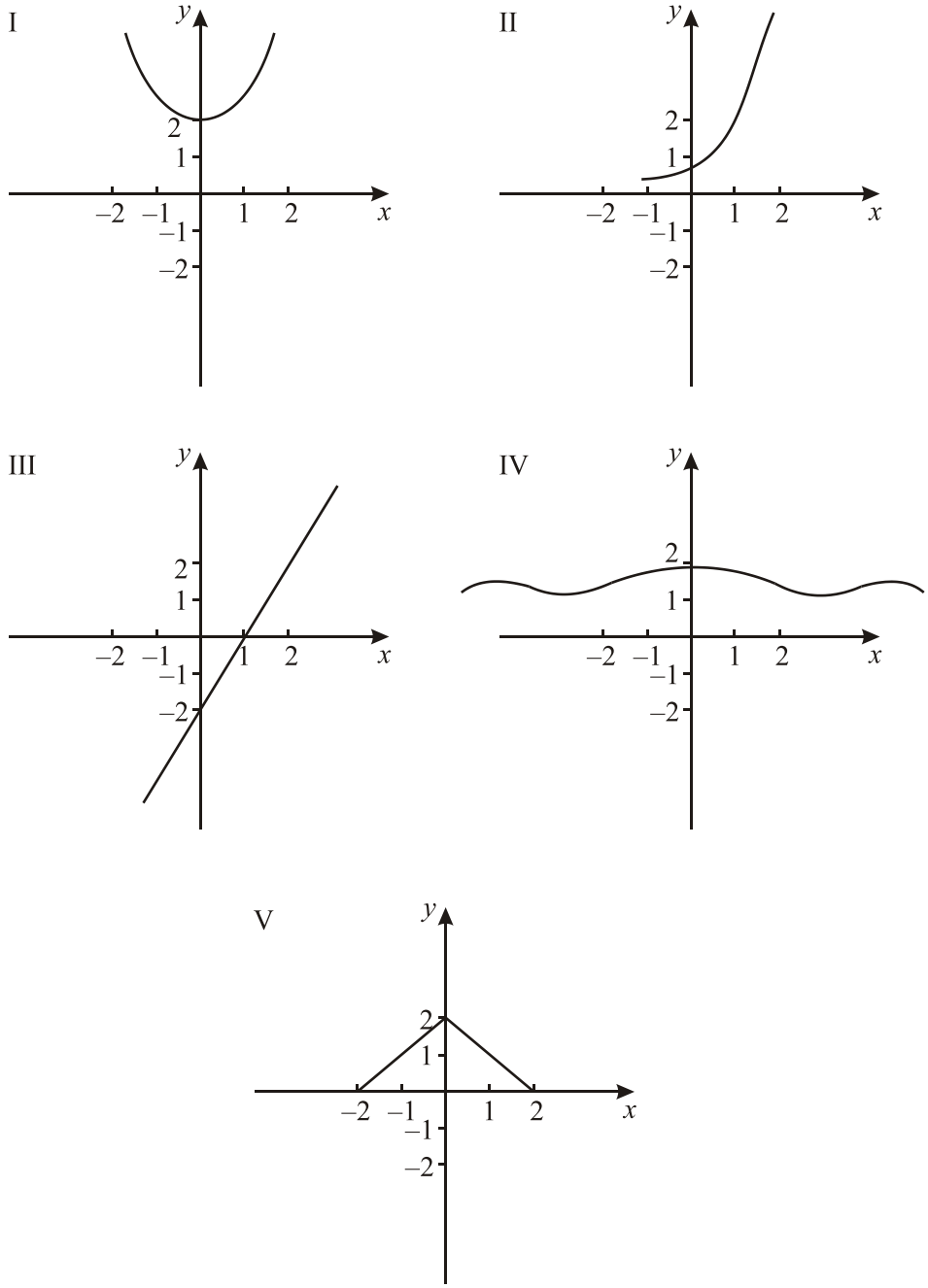
(a) ……..

(b) (i) ……..

(ii) ……..

(Total 4 marks)

9. The following diagrams show the graphs of five functions.



Each of the following sets represents the range of one of the functions of the graphs. Write down which diagram is linked to each set.

(a) $\{y \mid y \in \quad \}$

(b) $\{y \mid y \geq 2\}$

(c) $\{y \mid y > 0\}$

(d) $\{y \mid 1 \leq y \leq 2\}$

(Total 4 marks)

10. (a) For $y = 0.5 \cos 0.5x$, find

(i) the amplitude;

(ii) the period.

(b) Let $y = -3 \sin x + 2$, where $90° \leq x \leq 270°$.

By drawing the graph of y or otherwise, complete the table below for the given values of y.

x	y
	-1
	2

Working:

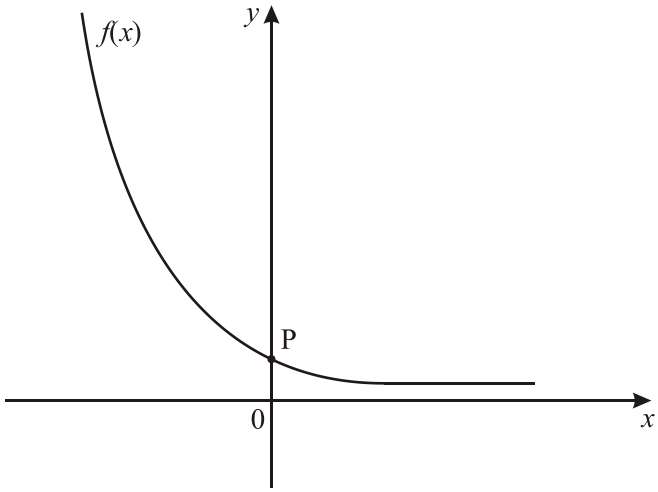
Answers:

(a) (i) ……..

(ii) ……..

(Total 4 marks)

11. The following diagram shows part of the graph of an exponential function $f(x) = a^{-x}$, where $x \in \mathbb{R}$.



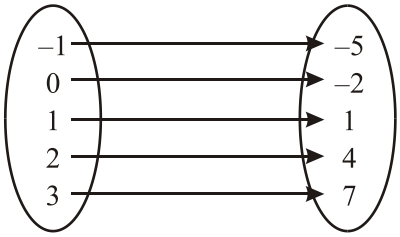
(a) What is the range of f?

(b) Write down the coordinates of the point P.

(c) What happens to the values of $f(x)$ as elements in its domain increase in value?

(Total 4 marks)

12. (a) A function f is represented by the following mapping diagram.



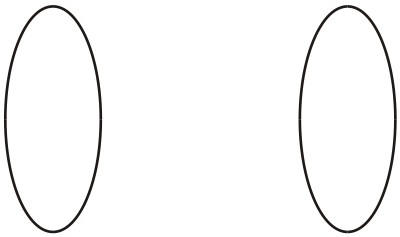
Write down the function f in the form

$f : x \mapsto y, \quad x \in \{\text{the domain of } f\}$.

(b) The function g is defined as follows

$g : x \mapsto \sin 15x°, \quad \{x \in \mathbb{N} \text{ and } 0 < x \leq 4\}$.

Complete the following mapping diagram to represent the function g.



Working:

Answer:

(a) ……..

(Total 4 marks)

13. **Diagram 1** shows a part of the graph of $y = x^2$.

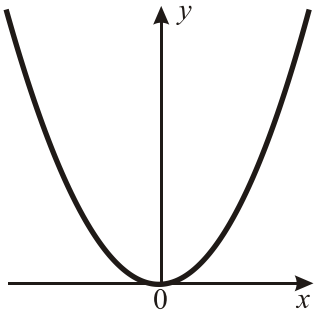


Diagram 1

Diagrams 2, 3 and **4** show a part of the graph of $y = x^2$ after it has been moved parallel to the x-axis, or parallel to the y-axis, or parallel to one axis, then the other.

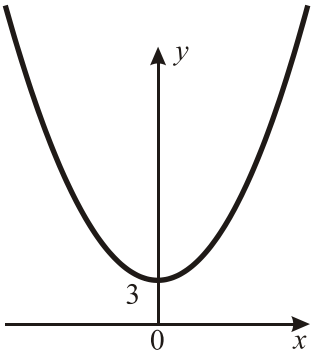


Diagram 2

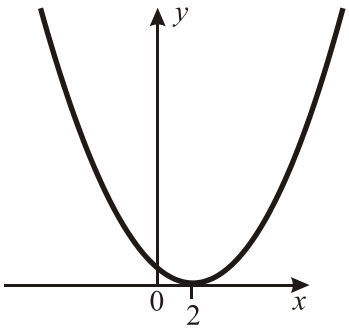


Diagram 3

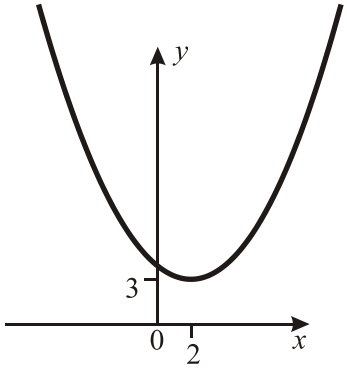


Diagram 4

Write down the equation of the graph shown in

(a) **Diagram 2**;

(b) **Diagram 3**;

(c) **Diagram 4**.

(Total 4 marks)

14. The perimeter of a rectangle is 24 metres.

(a) The table shows some of the possible dimensions of the rectangle.
Find the values of a, b, c, d and e.

Length (m)	Width (m)	Area (m^2)
1	11	11
a	10	b
3	c	27
4	d	e

(2)

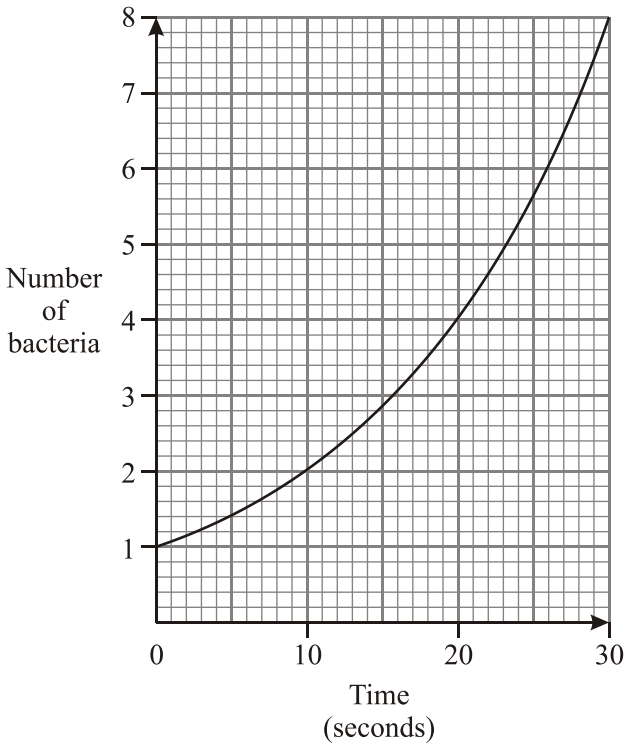
(b) If the length of the rectangle is x m, and the area is A m^2, express A in terms of x only.

(1)

(c) What are the length and width of the rectangle if the area is to be a maximum?

(3)
(Total 6 marks)

15. Under certain conditions the number of bacteria in a particular culture doubles every 10 seconds as shown by the graph below.



(a) Complete the table below.

Time (seconds)	0	10	20	30
Number of bacteria	1			

(b) Calculate the number of bacteria in the culture after 1 minute.

(Total 4 marks)

16. (a) Solve the equation $x^2 - 5x + 6 = 0$.

(b) Find the coordinates of the points where the graph of $y = x^2 - 5x + 6$ intersects the x-axis.

Working:

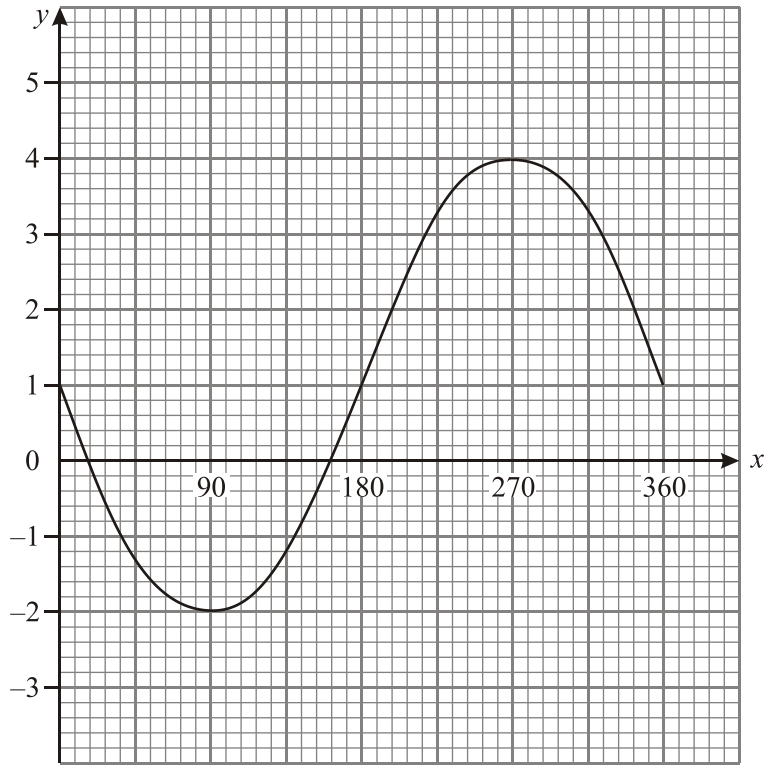
Answers:

(a) ……..

(b) .....

(Total 4 marks)

17. The diagram below shows the graph of $y = -a \sin x° + c$, $0 \leq x \leq 360$.



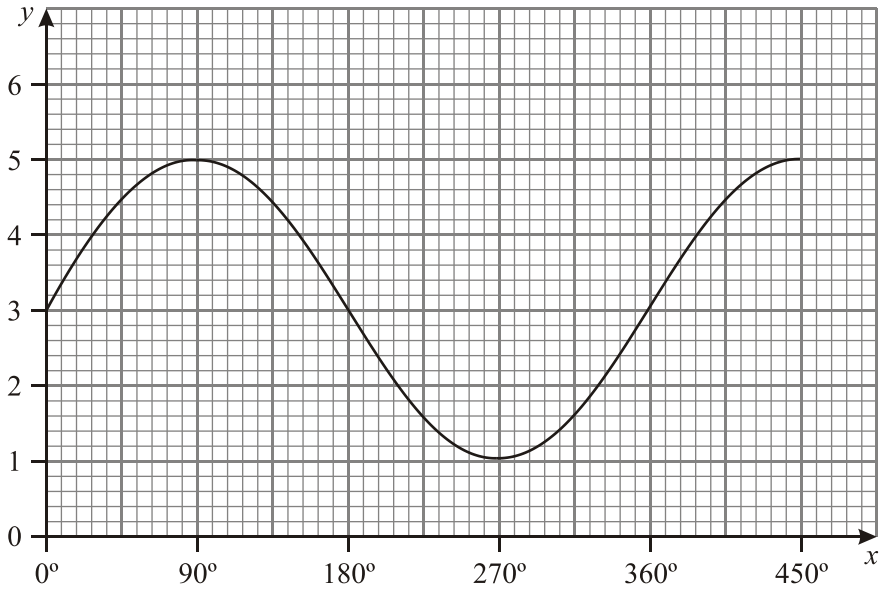
Use the graph to find the values of

(a) c;

(b) a.

(Total 4 marks)

18. The graph below shows part of the function $y = 2 \sin x + 3$.



(a) Write the domain of the part of the function shown on the graph.

(b) Write the range of the part of the function shown on the graph.

Working:

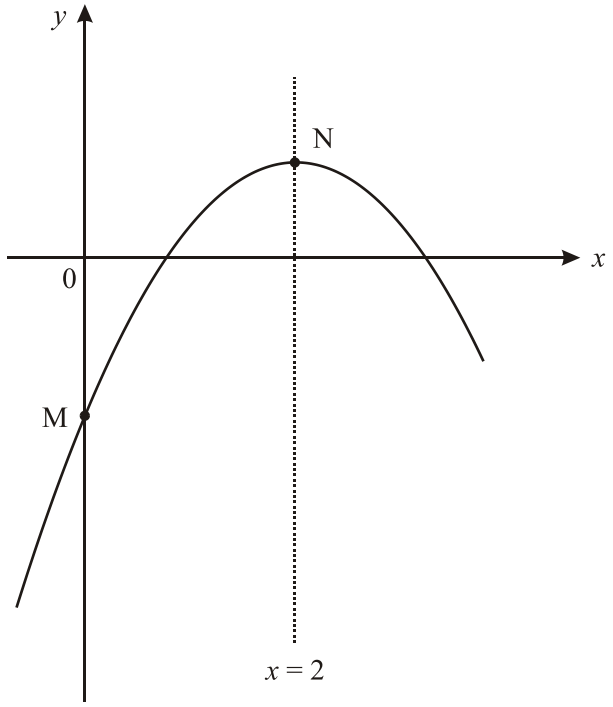
Answers:

(a) .....

(b) .....

(Total 4 marks)

19. The diagram below shows part of the graph of $y = ax^2 + 4x - 3$. The line $x = 2$ is the axis of symmetry. M and N are points on the curve, as shown.



(a) Find the value of a.

(b) Find the coordinates of

(i) M;

(ii) N.

Working:

Answers:

(a) .....

(b) (i) ……...

(ii) ……...

(Total 4 marks)

20. The number (n) of bacteria in a colony after h hours is given by the formula $n = 1200(3^{0.25h})$. Initially, there are 1200 bacteria in the colony.

(a) Copy and complete the table below, which gives values of n and h.
Give your answers to the nearest hundred.

time in hours (h)	0	1	2	3	4
no. of bacteria (n)	1200		2100	2700	

(2)

(b) On graph paper, draw the graph of the above function. Use a scale of 3 cm to represent 1 hour on the horizontal axis and 4 cm to represent 1000 bacteria on the vertical axis. Label the graph clearly.

(5)

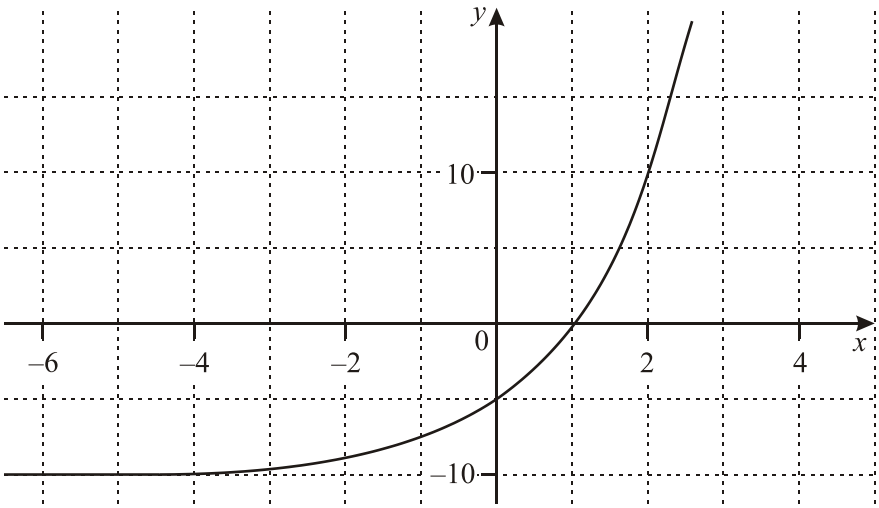
(c) Use your graph to answer each of the following, showing your method **clearly**.

(i) How many bacteria would there be after 2 hours and 40 minutes?
Give your answer to the nearest hundred bacteria.

(ii) After how long will there be approximately 3000 bacteria? Give your answer to the nearest 10 minutes.

(4)
(Total 11 marks)

21. The graph below shows the curve $y = k(2^x) + c$, where k and c are constants.



Find the values of c and k.

Working:

Answers:

.....

.....

(Total 4 marks)

22. A rectangle has dimensions $(5 + 2x)$ metres and $(7 - 2x)$ metres.

(a) Show that the area, A, of the rectangle can be written as $A = 35 + 4x - 4x^2$.

(1)

(b) The following is the table of values for the function $A = 35 + 4x - 4x^2$.

x	-3	-2	-1	0	1	2	3	4
A	-13	p	27	35	q	r	11	s

(i) Calculate the values of p, q, r and s.

(ii) On graph paper, using a scale of 1 cm for 1 unit on the x-axis and 1 cm for 5 units on the A-axis, plot the points from your table and join them up to form a smooth curve.

(6)

(c) Answer the following, using your graph or otherwise.

(i) Write down the equation of the axis of symmetry of the curve,

(ii) Find one value of x for a rectangle whose area is 27 m^2.

(iii) Using this value of x, write down the dimensions of the rectangle.

(4)

(d) (i) On the same graph, draw the line with equation $A = 5x + 30$.

(ii) Hence or otherwise, solve the equation $4x^2 + x - 5 = 0$.

(3)

(Total 14 marks)

23. Consider the graphs of the following functions.

(i) $y = 7x + x^2$;

(ii) $y = (x - 2)(x + 3)$;

(iii) $y = 3x^2 - 2x + 5$;

(iv) $y = 5 - 3x - 2x^2$.

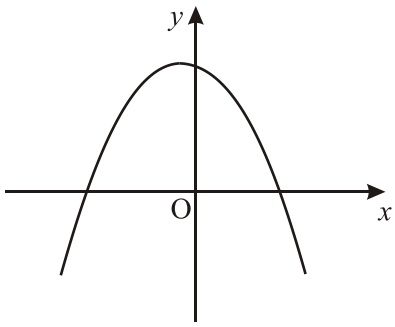
Which of these graphs

(a) has a y-intercept below the x-axis?

(b) passes through the origin?

(c) does not cross the x-axis?

(d) could be represented by the following diagram?



Working:

Answers:

(a) .....

(b) .....

(c) .....

(d) .....

(Total 8 marks)

24. The cost c, in Australian dollars (AUD), of renting a bungalow for n weeks is given by the linear relationship $c = nr + s$, where s is the security deposit and r is the amount of rent per week.

Ana rented the bungalow for 12 weeks and paid a total of 2925 AUD.

Raquel rented the same bungalow for 20 weeks and paid a total of 4525 AUD.

Find the value of

(a) r, the rent per week;

(b) s, the security deposit.

Working:

Answers:

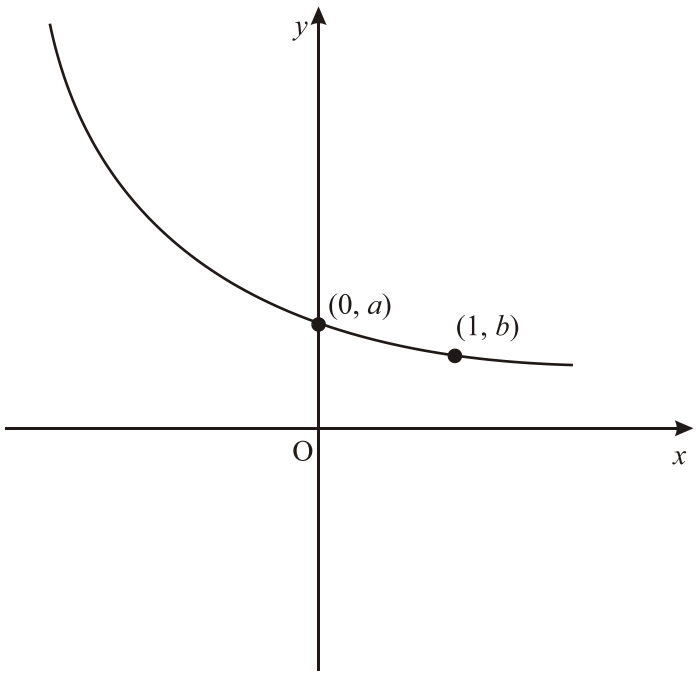
(a) .....

(b) .....

(Total 8 marks)

25. The following diagram shows the graph of $y = 3^{-x} + 2$. The curve passes through the points $(0, a)$ and $(1, b)$.

Diagram not to scale



(a) Find the value of

(i) a;

(ii) b.

(b) Write down the equation of the asymptote to this curve.

Working:

Answers:

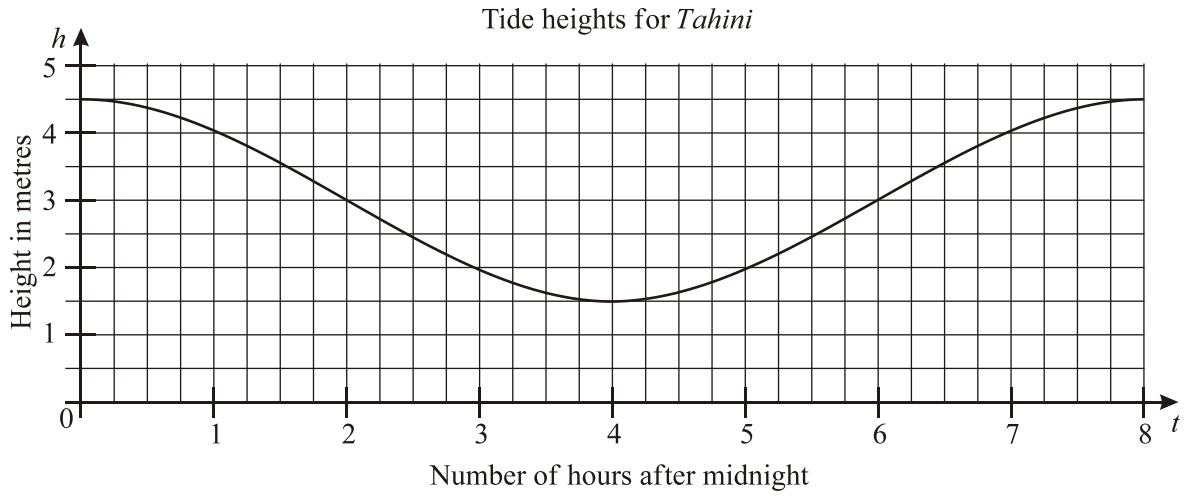
(a) (i) .....

(ii) .....

(b) .....

(Total 8 marks)

26. The graph below shows the tide heights, h metres, at time t hours after midnight, for *Tahini* island.



(a) Use the graph to find

(i) the height of the tide at 03:15;

(ii) the times when the height of the tide is 3.5 metres.

(3)

(b) The best time to catch fish is when the tide is **below** 3 metres. Find this best time, giving your answer as an inequality in t.

(3)

Due to the location of *Tahini* island, there is very little variation in the pattern of tidal heights. The maximum tide height is 4.5 metres and the minimum tide height is 1.5 metres. The height h can be modelled by the function

$$h(t) = a\cos(bt°) + 3.$$

(c) Use the graph above to find the values of the variables a and b.

(4)

(d) Hence **calculate** the height of the tide at 13:00.

(3)

(e) At what time would the tide be at its lowest point in the **second** 8 hour period?

(2)

(Total 15 marks)

27. The diagram below shows a part of the graph of $y = a^x$. The graph crosses the y-axis at the point P. The point Q (4, 16) is on the graph.

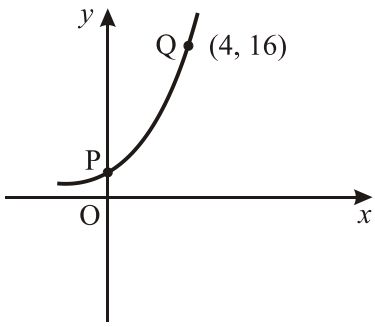


Diagram not to scale

Find

(a) the coordinates of the point P;

(b) the value of a.

Working:

Answers:

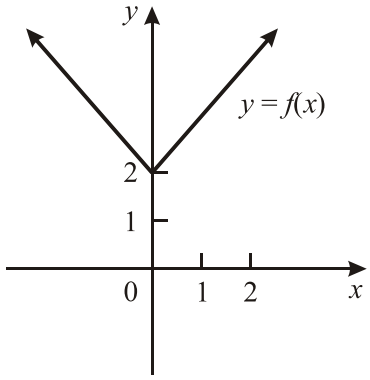
(a) .....

(b) …….....

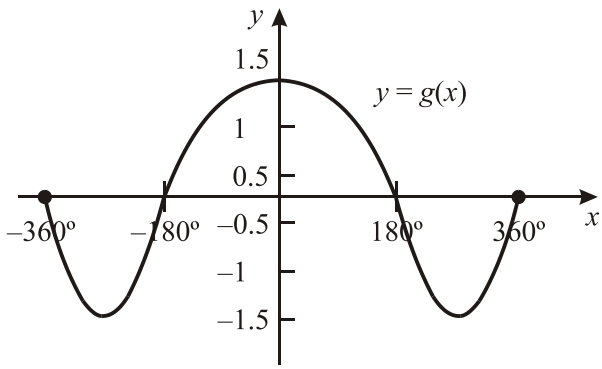
(Total 8 marks)

28. The diagrams below show the graphs of two functions, $y = f(x)$, and $y = g(x)$.

(a)



(b)



State the domain and range of

(a) the function f;

(b) the function g.

Answers:

(a) Domain of f .....

Range of f .....

(b) Domain of g .....

Range of g .....

(Total 8 marks)

29. The conversion formula for temperature from the Fahrenheit (F) to the Celsius (C) scale is given by

$$C = \frac{5(F-32)}{9}.$$

(a) What is the temperature in degrees Celsius when it is 50° Fahrenheit?

There is another temperature scale called the Kelvin (K) scale.
The temperature in degrees Kelvin is given by $K = C + 273$.

(b) What is the temperature in **Fahrenheit** when it is zero degrees on the Kelvin scale?

Working:

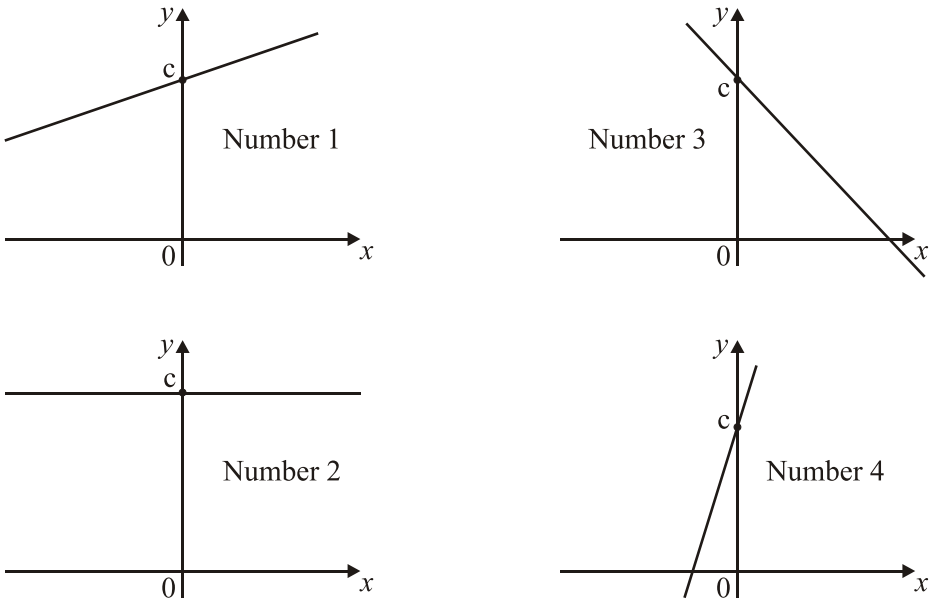
Answers:

(a) .....

(b) .....

(Total 8 marks)

30. The four diagrams below show the graphs of four different straight lines, all drawn to the same scale. Each diagram is numbered and c is a positive constant.

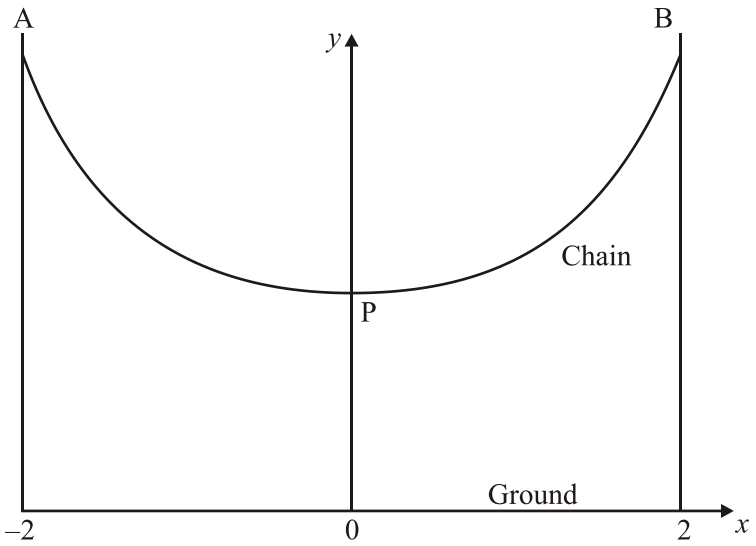


In the table below, write the number of the diagram whose straight line corresponds to the equation in the table.

Equation	Diagram number
$y = c$	
$y = -x + c$	
$y = 3x + c$	
$y = \frac{1}{3}x + c$	

(Total 8 marks)

31. The diagram shows a chain hanging between two hooks A and B.
The points A and B are at equal heights above the ground. P is the lowest point on the chain. The ground is represented by the x-axis. The x-coordinate of A is –2 and the x-coordinate of B is 2. Point P is on the y-axis. The shape of the chain is given by $y = 2^x + 2^{-x}$ where $-2 \leq x \leq 2$.



(a) Calculate the height of the point P.

(b) Find the range of y. Write your answer as an interval or using inequality symbols.

Working:

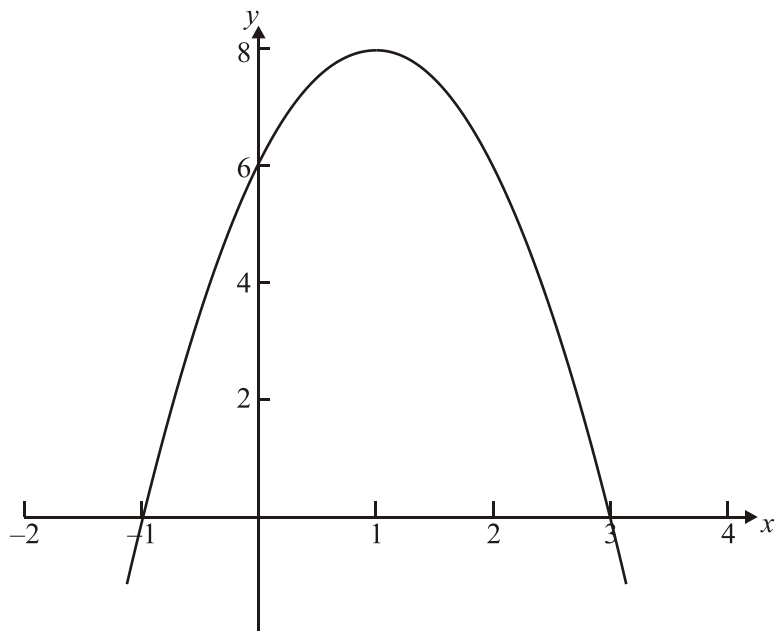
Answers:

(a) .....

(b) .....

(Total 8 marks)

32. The figure below shows part of the graph of a quadratic function $y = ax^2 + 4x + c$.



(a) Write down the value of c.

(b) Find the value of a.

(c) Write the quadratic function in its factorized form.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 8 marks)

33. The number of bacteria (y) present at any time is given by the formula:

$y = 15\,000e^{-025t}$, where t is the time in seconds and e = 2.72 correct to 3 s.f.

(a) Calculate the values of a, b and c to the nearest hundred in the table below:

Time in seconds (t)	0	1	2	3	4	5	6	7	8
Amount of bacteria (y) (nearest hundred)	a	11700	9100	7100	b	4300	3300	2600	c

(3)

(b) On graph paper using 1 cm for each second on the horizontal axis and 1 cm for each thousand on the vertical axis, draw and label the graph representing this information.

(5)

(c) Using your graph, answer the following questions:

(i) After how many seconds will there be 5000 bacteria? Give your answer correct to the nearest tenth of a second.

(ii) How many bacteria will there be after 6.8 seconds? Give your answer correct to the nearest hundred bacteria.

(iii) Will there be a time when there are no bacteria left? Explain your answer.

(6)
(Total 14 marks)

34. Consider the function $f(x) = x^3 - 4x^2 - 3x + 18$

(b) Find the values of $f(x)$ for a and b in the table below:

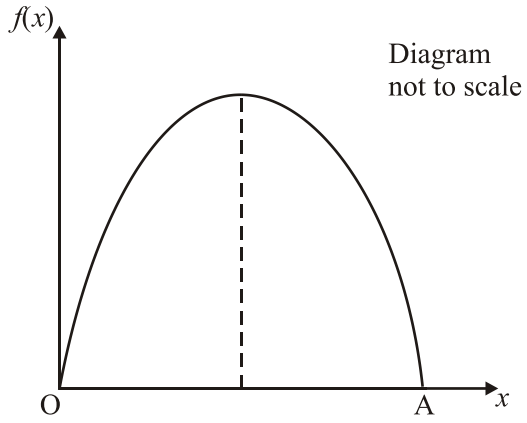
x	–3	–2	–1	0	1	2	3	4	5
$f(x)$	–36	a	16	b	12	4	0	6	28

(2)

(c) Using a scale of 1 cm for each unit on the x-axis and 1 cm for each 5 units on the y-axis, draw the graph of $f(x)$ for $-3 \leq x \leq 5$. Label clearly.

(5)

35. The graph of the function $f : x \mapsto 30x - 5x^2$ is given in the diagram below.



(a) Factorize fully $30x - 5x^2$.

(b) Find the coordinates of the point A.

(c) Write down the equation of the axis of symmetry.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 8 marks)

36. Consider the function $f(x) = 2 \sin x - 1$ where $0 \leq x \leq 720°$.

(a) Write down the period of the function.

(b) Find the minimum value of the function.

(c) Solve $f(x) = 1$.

Working:

Answers:

(a) .....

(b) .....

(c) .....

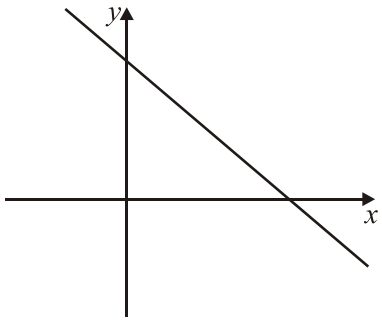
(Total 8 marks)

37. The diagrams below are sketches of some of the following functions.

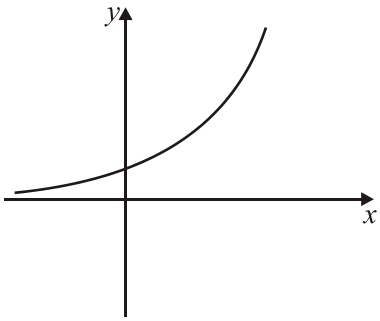
(i) $y = a^x$ (ii) $y = x^2 - a$ (iii) $y = a - x^2$

(iv) $y = a - x$ (v) $y = x - a$

(a)

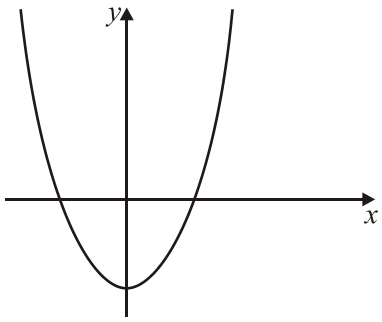


(b)

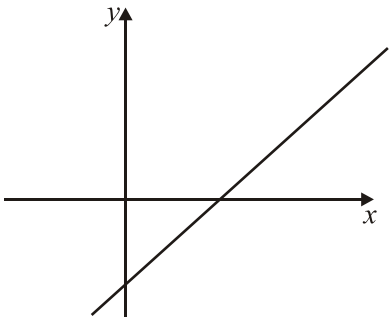


DIAGRAMS NOT TO SCALE

(c)



(d)



Complete the table to match each sketch to the correct function.

Sketch	Function
(a)	
(b)	
(c)	
(d)	

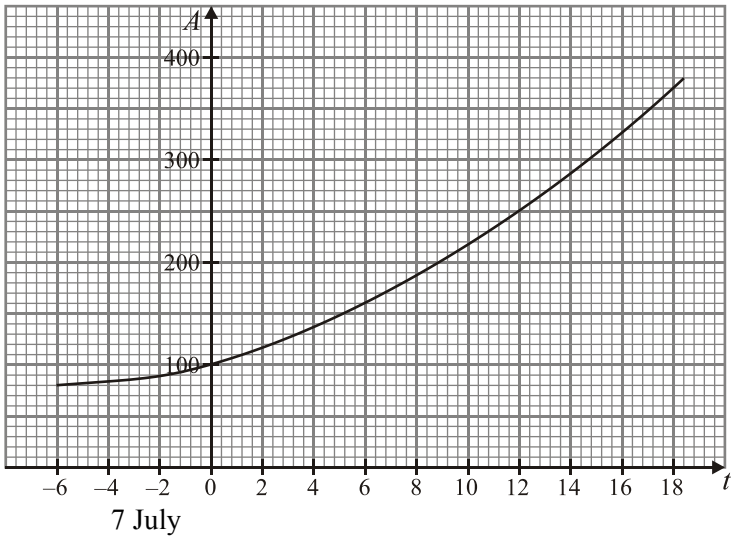
Working:

(Total 8 marks)

38. The area, A m^2, of a fast growing plant is measured at noon (12:00) each day. On 7 July the area was 100 m^2. Every day the plant grew by 7.5%. The formula for A is given by

$$A = 100\,(1.075)^t$$

where t is the number of days after 7 July. (on 7 July, $t = 0$)
The graph of $A = 100(1.075)^t$ is shown below.



(a) What does the graph represent when t is negative?

(2)

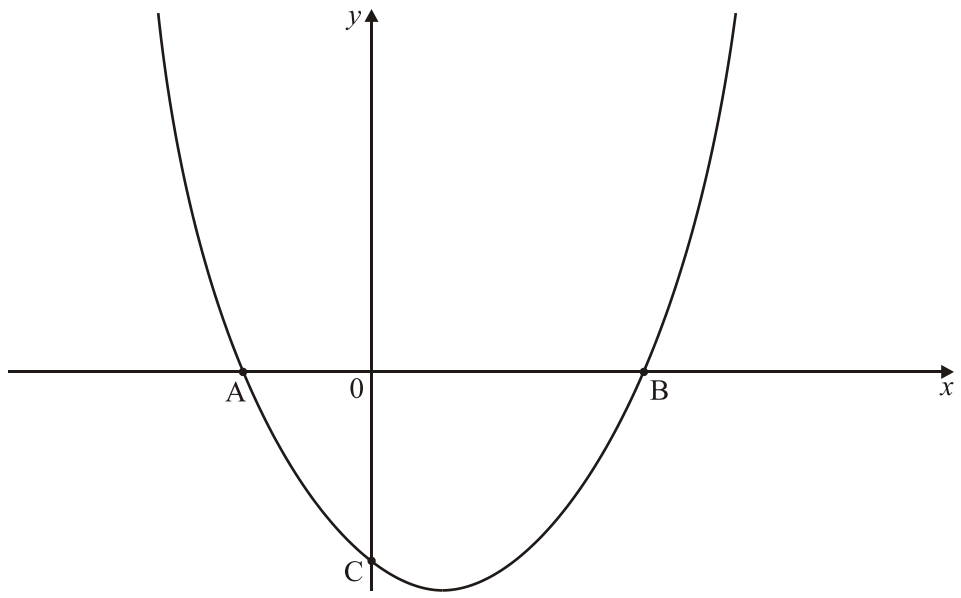
(b) Use the graph to find the value of t when $A = 178$.

(1)

(c) Calculate the area covered by the plant at noon on 28 July.

(3)
(Total 6 marks)

39. The graph of the function $y = x^2 - x - 2$ is drawn below.



(a) Write down the coordinates of the point C.

(b) Calculate the coordinates of the points A and B.

Working:

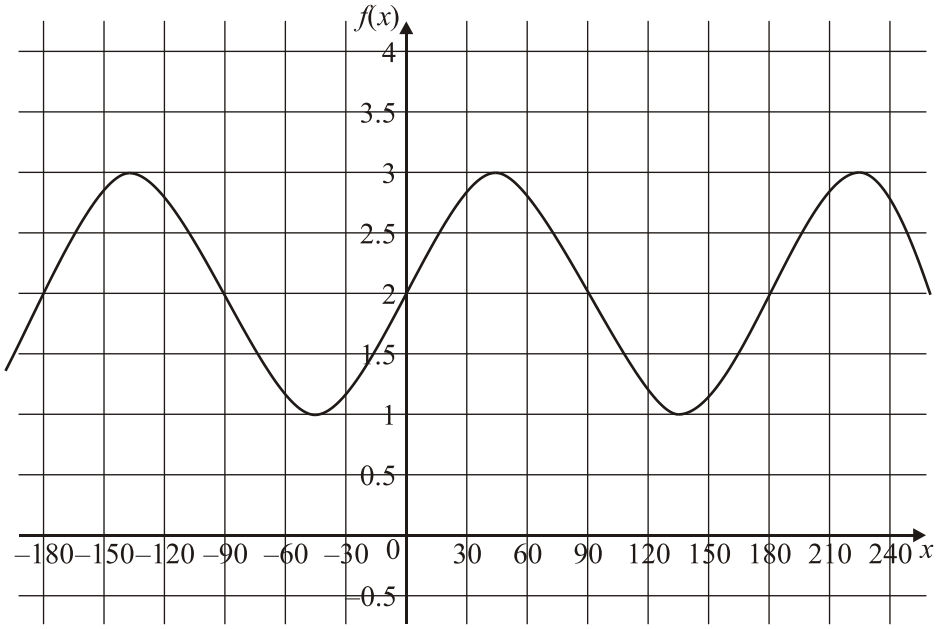
Answers:

(a) .....

(b) .....

(Total 8 marks)

40. The curve shown in the figure below is part of the graph of the function, $f(x) = 2 + \sin(2x)$, where x is measured in degrees.



(a) Find the range of $f(x)$.

(b) Find the amplitude of $f(x)$.

(c) Find the period of $f(x)$.

(d) If the function is changed to $f(x) = 2 + \sin(4x)$ what is the effect on the period, compared to the period of the original function?

Working:

Answers:

(a) .....

(b) .....

(c) .....

(d) .....

(Total 8 marks)

41. The following table gives the postage rates for sending letters from the Netherlands. All prices are given in Euros (€).

Destination	Weight not more than 20 g	Each additional 20 g or part of 20 g
Within the Netherlands (zone 1)	€0.40	€0.35
Other destinations within Europe (zone 2)	€0.55	€0.50
Outside Europe (zone 3)	€0.80	€0.70

(a) Write down the cost of sending a letter weighing 15 g from the Netherlands to a destination within the Netherlands (zone 1).

(b) Find the cost of sending a letter weighing 35 g from the Netherlands to a destination in France (zone 2).

(c) Find the cost of sending a letter weighing 50 g from the Netherlands to a destination in the USA (zone 3).

Working:

Answers:

(a) .....

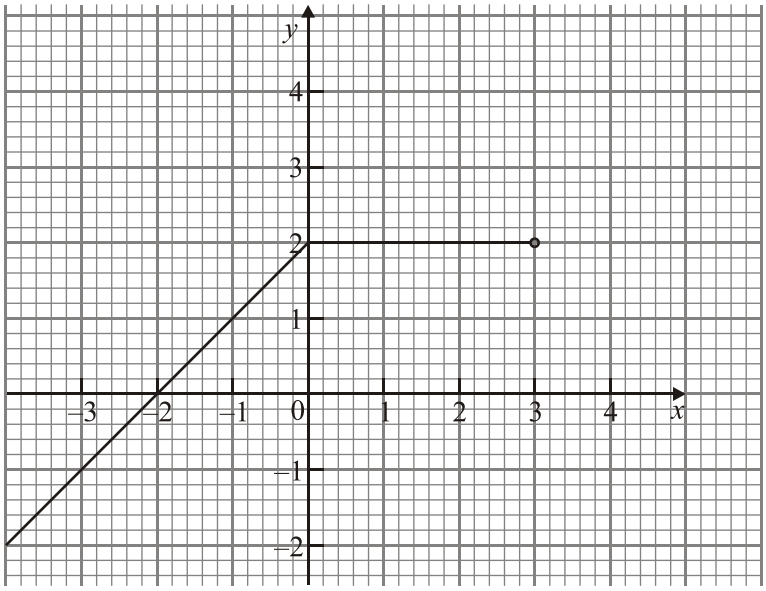
(b) .....

(c) .....

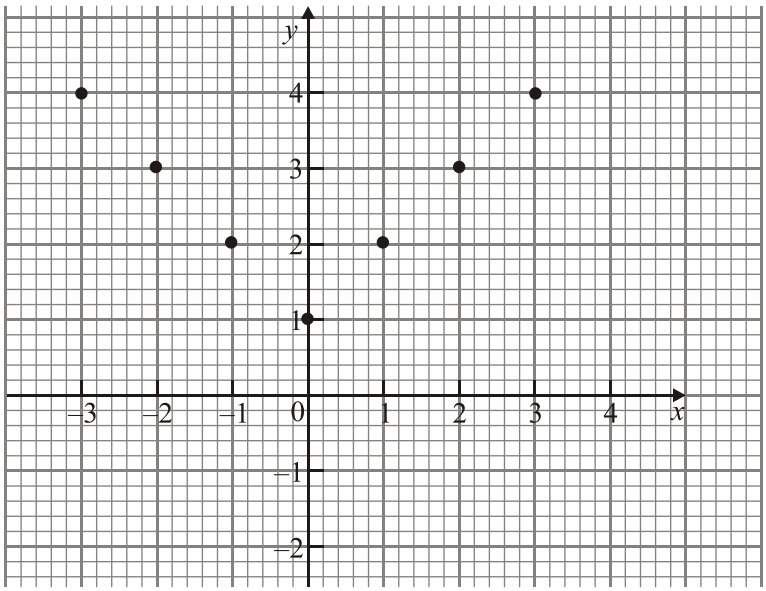
(Total 8 marks)

42. Write down the domain and range of the following functions.

(a)



(b)



Working:

Answers:

(a) .....

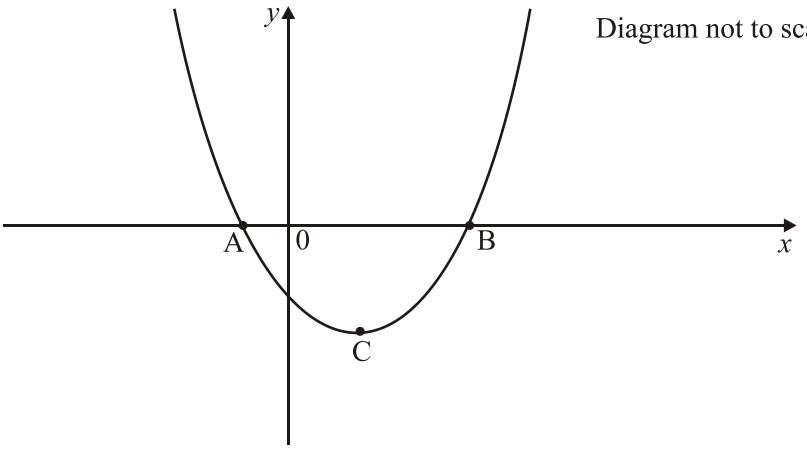
.....

(b) .....

.....

(Total 8 marks)

43. The graph of the function $f(x) = x^2 - 2x - 3$ is shown in the diagram below.



(a) Factorize the expression $x^2 - 2x - 3$.

(b) Write down the coordinates of the points A and B.

(c) Write down the equation of the axis of symmetry.

(d) Write down the coordinates of the point C, the vertex of the parabola.

Working:

Answers:

(a) .....

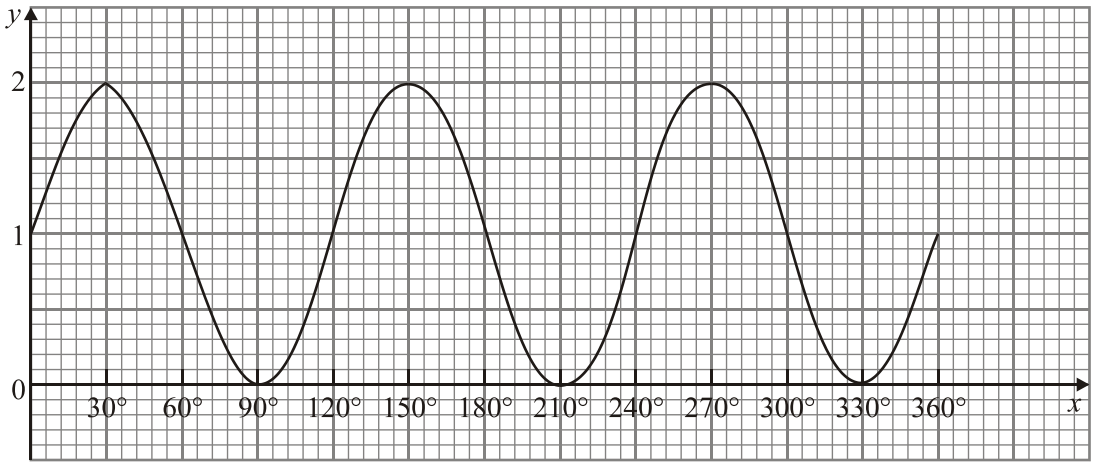
(b) .....

(c) .....

(d) .....

(Total 8 marks)

44. The diagram shows the graph of $y = \sin ax + b$.



(a) Using the graph, write down the following values

(i) the period;

(ii) the amplitude;

(iii) b.

(b) Calculate the value of a.

Working:

Answers:

(a) (i) .....

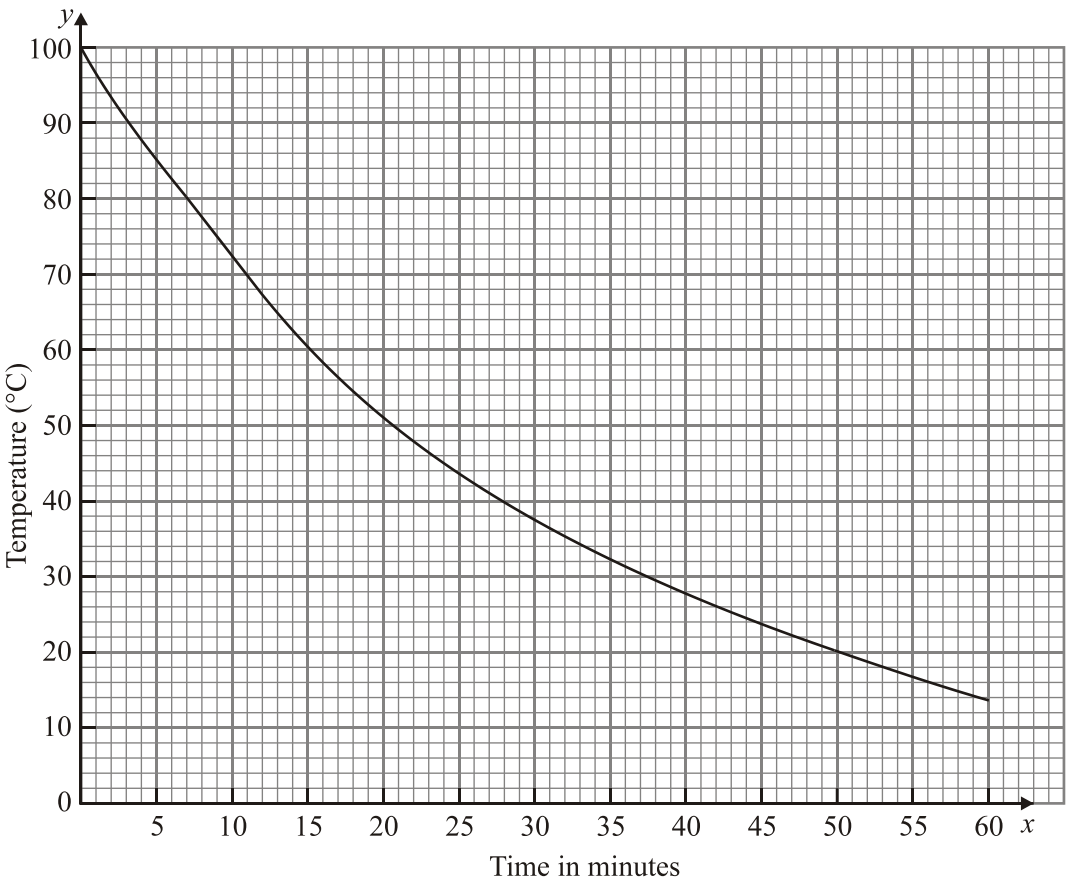
(ii) .....

(iii) .....

(b) .....

(Total 8 marks)

45. The graph below shows the temperature of a liquid as it is cooling.



(a) Write down the temperature after 5 minutes.

(b) After how many minutes is the temperature 50°C?

The equation of the graph for all positive x can be written in the form $y = 100(5^{-0.02x})$.

(c) Calculate the temperature after 80 minutes.

(d) Write down the equation of the asymptote to the curve.

Working:

Answers:

(a) .....

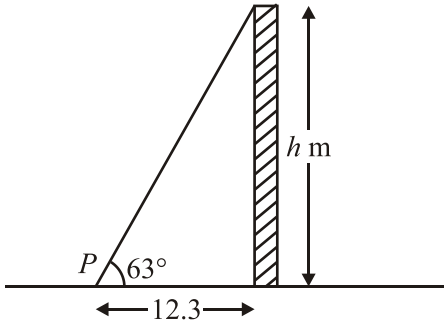
(b) .....

(c) .....

(d) .....

(Total 8 marks)

47. The diagram shows a point P, 12.3 m from the base of a building of height h m. The angle measured to the top of the building from point P is 63°.



(a) Calculate the height h of the building.

Consider the formula $h = 4.9t^2$, where h is the height of the building and t is the time in seconds to fall to the ground from the top of the building.

(b) Calculate how long it would take for a stone to fall from the top of the building to the ground.

Working:

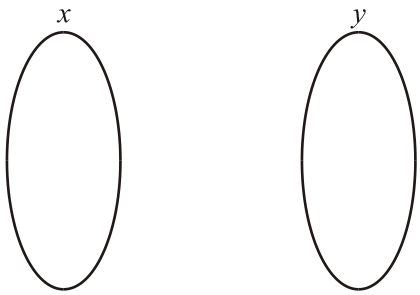
Answers:

(a) .....

(b) .....

(Total 6 marks)

48. (a) Represent the function $y = 2x^2 - 5$, where $x \in \{-2, -1, 0, 1, 2, 3\}$ by a mapping diagram.



(b) List the elements of the domain of this function.

(c) List the elements of the range of this function.

(Total 6 marks)

49. It is thought that a joke would spread in a school according to an exponential model $N = 4 \times (1.356)^{0.4t}$, $t \geq 0$; where N is the number of people who have heard the joke, and t is the time in minutes after the joke is first told.

(a) How many people heard the joke initially?

(b) How many people had heard the joke after 16 minutes?

There are 1200 people in the school.

(c) Estimate how long it would take for everybody in the school to hear this joke.

Working:

Answers:

(a) .....

(b) .....

(c) .....

(Total 6 marks)

50. (a) Sketch the graph of the function $y = 2x^2 - 6x + 5$.

(b) Write down the coordinates of the local maximum or minimum of the function.

(c) Find the equation of the axis of symmetry of the function.

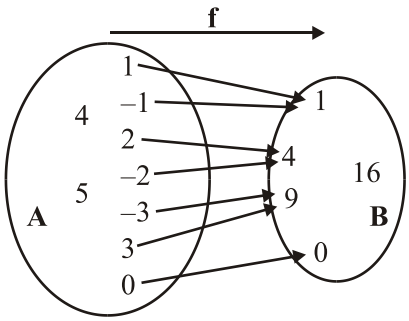
(Total 6 marks)

51. (a) Sketch a graph of $y = \dfrac{x}{2+x}$ for $-10 \leq x \leq 10$.

(b) Hence write down the equations of the horizontal and vertical asymptotes.

(Total 6 marks)

52.



The diagram shows a function *f*, mapping members of set A to members of set B.

(a) (i) Using set notation, write down all members of the domain of *f*.

(ii) Using set notation, write down all members of the range of *f*.

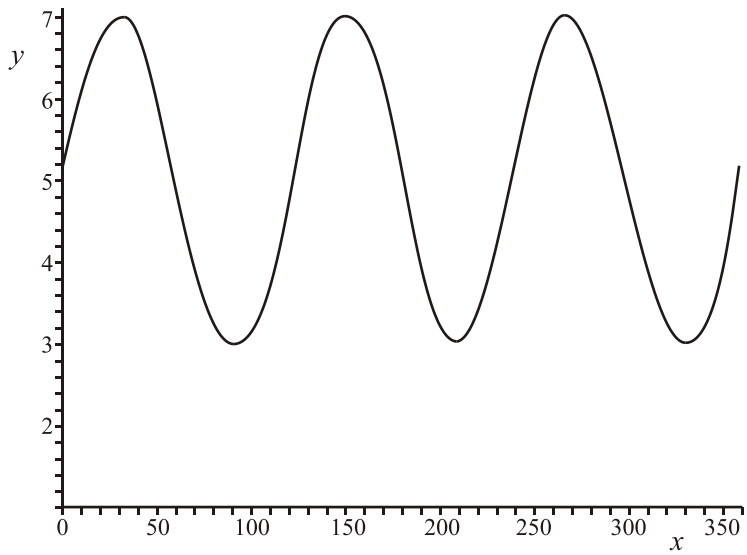
(iii) Write down the equation of the function *f*.

The equation of a function *g* is $g(x) = x^2 + 1$. The domain of *g* is $\mathbb{R}$.

(b) Write down the range of *g*.

(Total 6 marks)

53. Below is a graph of the function $y = a + b \sin(cx)$ where *a*, *b* and *c* are positive integers and *x* is measured in degrees.



Find the values of *a*, *b* and *c*.

(Total 6 marks)

54. (a) Sketch the graph of the function $f : x \mapsto 1 + 2 \sin x$, where $x \in \mathbb{R}$, $-360° \leq x \leq 360°$.

(4)

(b) Write down the range of this function for the given domain.

(2)

(c) Write down the amplitude of this function.

(1)

(d) On the same diagram sketch the graph of the function $g : x \mapsto \sin 2x$, where $x \in \mathbb{R}$, $-360° \leq x \leq 360°$.

(4)

(e) Write down the period of this function.

(1)

(f) Use the sketch graphs drawn to find the number of solutions to the equation $f(x) = g(x)$ in the given domain.

(1)

(g) Hence solve the equation $1 + 2 \sin x = \sin 2x$ for $\mathbf{0° \leq x \leq 360°}$.

(4)

(Total 17 marks)

55. The functions f and g are defined by

$$f : x \mapsto \frac{x+4}{x}, x \in \mathbb{R}, x \neq 0$$

$$g : x \mapsto x, x \in \mathbb{R}$$

(a) Sketch the graph of f for $-10 \leq x \leq 10$.

(4)

(b) Write down the equations of the horizontal and vertical asymptotes of the function f.

(4)

(c) Sketch the graph of g on the same axes.

(2)

(d) Hence, or otherwise, find the solutions of $\frac{x+4}{x} = x$.

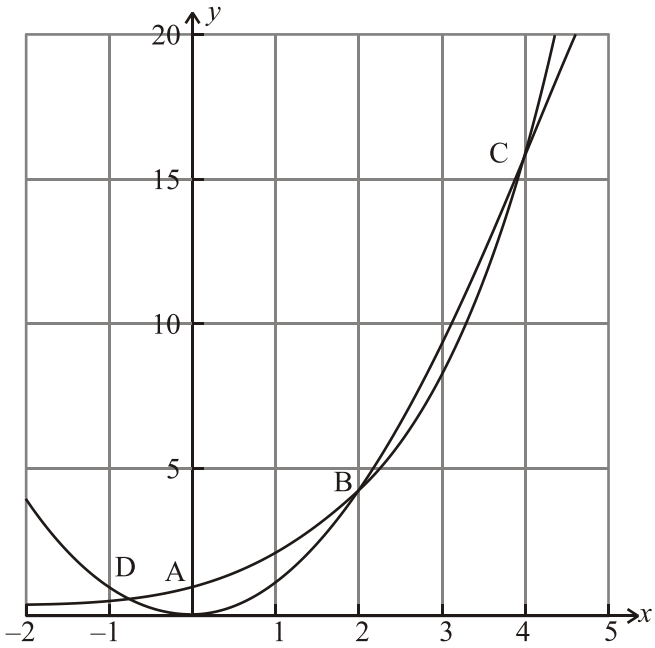
(4)

(e) Write down the range of function f.

(2)
(Total 16 marks)

56. The figure below shows the graphs of the functions $y = x^2$ and $y = 2^x$ for values of x between –2 and 5.

The points of intersection of the two curves are labelled as B, C and D.



(a) Write down the coordinates of the point A.

(2)

(b) Write down the coordinates of the points B and C.

(2)

(c) Find the x-coordinate of the point D.

(1)

(d) Write down, using interval notation, all values of x for which $2^x \leq x^2$.

(3)

(Total 8 marks)

57. The equation $M = 90 \times 2^{-t/20}$ gives the amount, in grams, of radioactive material held in a laboratory over t years.

(a) What was the original mass of the radioactive material?

The table below lists some values for M.

t	60	80	100
M	11.25	v	2.8125

(b) Find the value of v.

(c) Calculate the number of years it would take for the radioactive material to have a mass of 45 grams.

Working:

Answers:

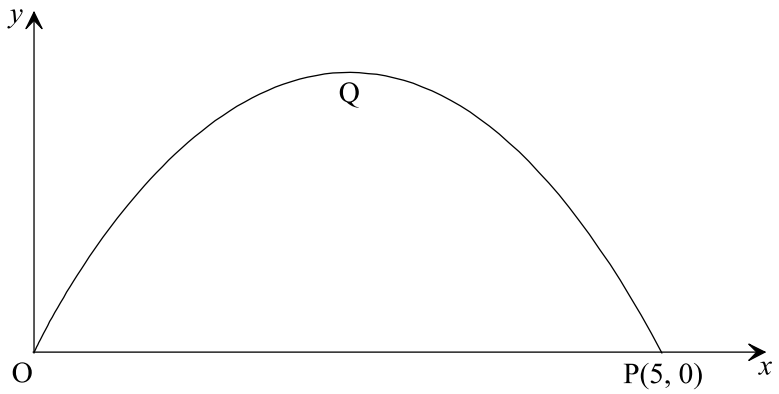
(a) .....

(b) .....

(c) .....

(Total 8 marks)

58. The diagram below shows the graph of $y = c + kx - x^2$, where k and c are constants.



(a) Find the values of k and c.

(b) Find the coordinates of Q, the highest point on the graph.

Working:

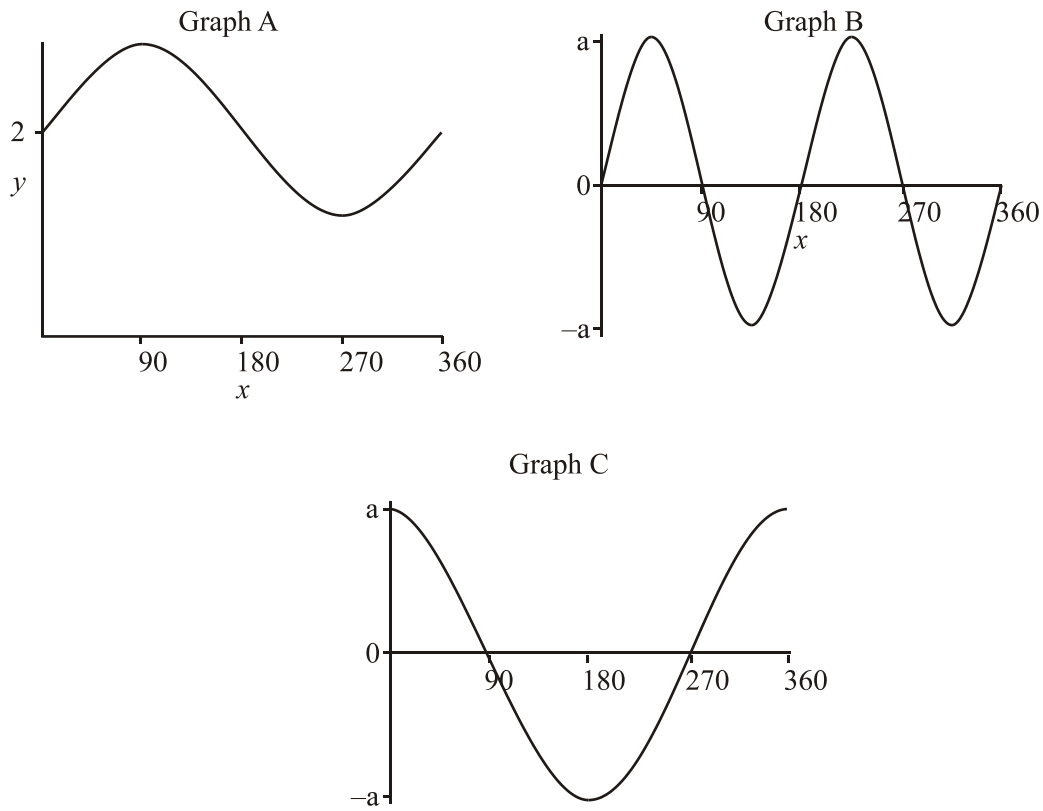
Answers:

(a) .....

(b) .....

(Total 8 marks)

59. The graphs of three trigonometric functions are drawn below. The x variable is measured in degrees, with $0 \leq x \leq 360°$. The amplitude 'a' is a positive constant with $0 < a \leq 1$.



(a) Write the letter of the graph next to the function representing that graph in the box below.

FUNCTION	GRAPH
$y = a \cos(x)$	
$y = a \sin(2x)$	
$y = 2 + a \sin(x)$	

(b) State the period of the function shown in graph B.

(c) State the range of the function $2 + a \sin(x)$ in terms of the constant a.

Working:

Answers:

(b) .....

(c) .....

(Total 8 marks)

60. A small manufacturing company makes and sells x machines each month. The monthly cost C, in dollars, of making x machines is given by

$$C(x) = 2600 + 0.4x^2.$$

The monthly income I, in dollars, obtained by selling x machines is given by

$$I(x) = 150x - 0.6x^2.$$

(a) Show that the company's monthly profit can be calculated using the quadratic function

$$P(x) = -x^2 + 150x - 2600.$$

(2)

(b) The maximum profit occurs at the vertex of the function $P(x)$. How many machines should be made and sold each month for a maximum profit?

(2)

(c) If the company does maximize profit, what is the selling price of each machine?

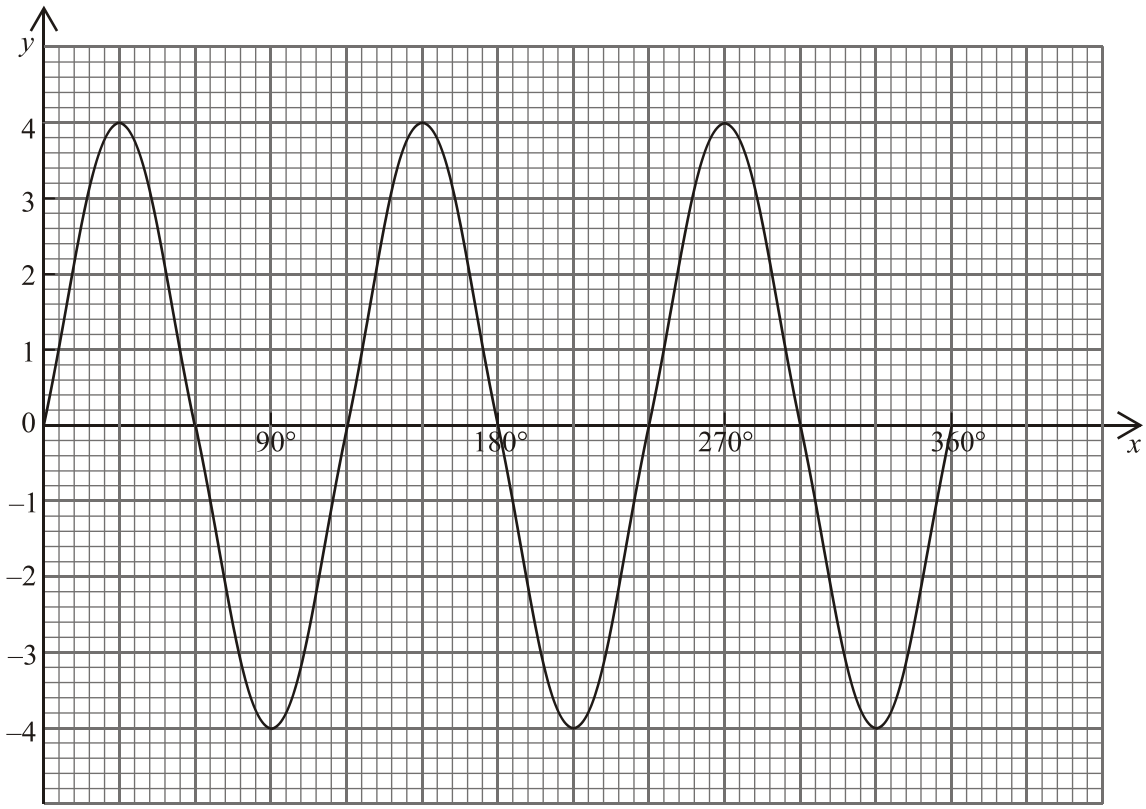
(4)

(d) Given that $P(x) = (x - 20)(130 - x)$, find the smallest number of machines the company must make and sell each month in order to make **positive** profit.

(4)

(Total 12 marks)

61.



The graph represents the function $y = 4 \sin (3x)$.

(a) (i) Write down the period of the function.

(ii) Write down the amplitude of the function.

(b) Draw the line $y = 2$ on the diagram.

(c) Using the graph, or otherwise, solve the equation $4 \sin (3x) = 2$ for $0° \leq x \leq 90°$.

Working:

Answers:

(a) (i).....

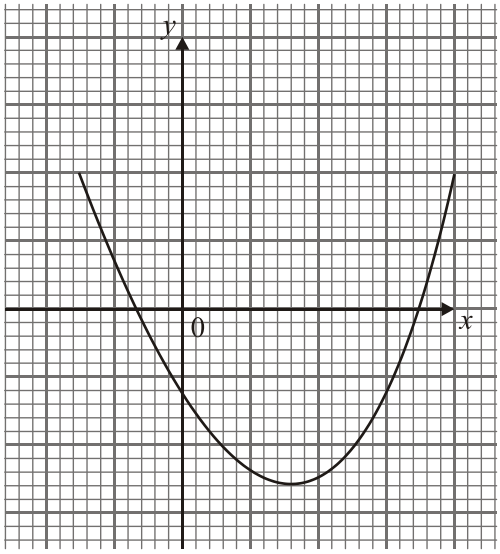
(ii).....

(c) .....

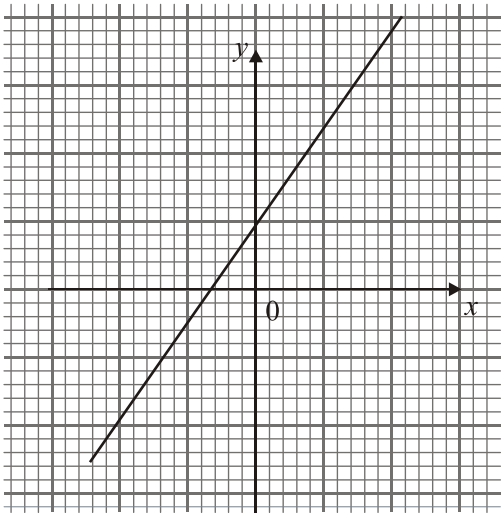
(Total 8 marks)

62. The diagrams below include sketches of the graphs of the following equations where a and b are **positive** integers.

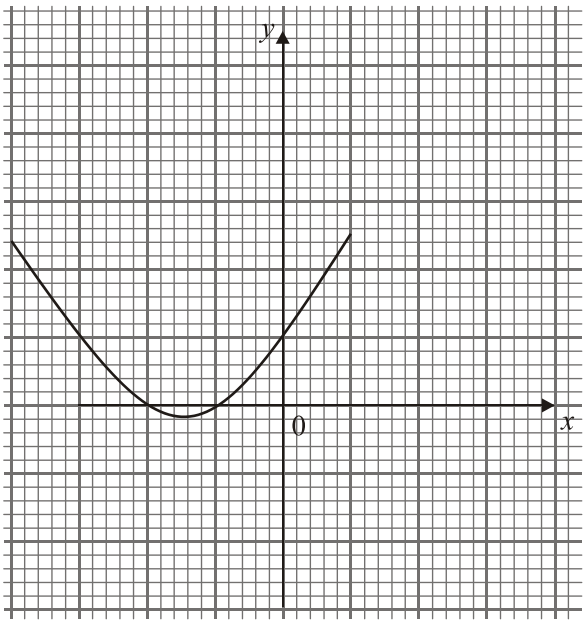
1.



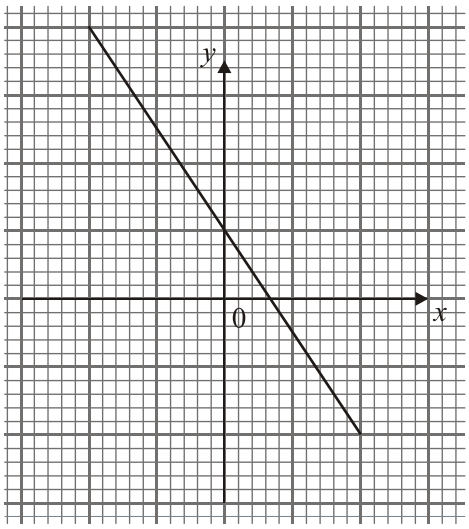
2.



3.



4.



Complete the table to match each **equation** to the correct **sketch**.

	Equation	Sketch
(i)	$y = ax + b$	
(ii)	$y = -ax + b$	
(iii)	$y = x^2 + ax + b$	
(iv)	$y = x^2 - ax - b$	

(Total 8 marks)

63. A function is represented by the equation $f(x) = 3(2)^x + 1$.

The table of values of $f(x)$ – $3 \leq x \leq 2$ is given below.

x	–3	–2	–1	0	1	2
$f(x)$	1.375	1.75	a	4	7	b

(a) Calculate the values for a and b.

(2)

(b) On graph paper, draw the graph of $f(x)$, for $-3 \leq x \leq 2$, taking 1 cm to represent 1 unit on both axes.

(4)

The domain of the function $f(x)$ is the real numbers, $\mathbb{R}$.

(c) Write down the range of $f(x)$.

(2)

(d) Using your graph, or otherwise, find the approximate value for x when $f(x) = 10$.

(2)

(Total 10 marks)