IGCSE Additional Mathematics Ch 9

Functions Test 2

Student Name:

Time allowed: 40 minutes

READ THESE INSTRUCTIONS FIRST

Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 29.

5.

The function f is defined for the domain $-3 \leqslant x \leqslant 3$ by

$$\mathrm{f}(x) = 9\left(x - \tfrac{1}{3}\right)^2 - 11.$$

(i) Find the range of f. [3]

(ii) State the coordinates and nature of the turning point of

(a) the curve $y = \mathrm{f}(x)$,

(b) the curve $y = |\mathrm{f}(x)|$.

[4]

6.

The functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f}: x \mapsto x^3,$$

$$\mathrm{g}: x \mapsto x + 2.$$

Express each of the following as a composite function, using only f, g, f^{-1} and/or g^{-1} :

(i) $x \mapsto x^3 + 2$, [1]

(ii) $x \mapsto x^3 - 2$, [1]

(iii) $x \mapsto (x+2)^{\frac{1}{3}}$. [1]

7.

The functions f and g are defined, for $x \in \mathbb{R}$, by

$$f : x \mapsto 3x - 2,$$

$$g : x \mapsto \frac{7x - a}{x + 1}, \text{ where } x \neq -1 \text{ and } a \text{ is a positive constant.}$$

(i) Obtain expressions for f^{-1} and g^{-1}. [3]

(ii) Determine the value of a for which $f^{-1}g(4) = 2$. [3]

(iii) If $a = 9$, show that there is only one value of x for which $g(x) = g^{-1}(x)$. [3]

8.

The function f is defined, for $x > 0$, by $f : x \mapsto \ln x$.

(i) State the range of f. [1]

(ii) State the range of f^{-1}. [1]

(iii) On the same diagram, sketch and label the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [2]

The function g is defined, for $x > 0$, by $g : x \mapsto 3x + 2$.

(iv) Solve the equation $fg(x) = 3$. [2]

(v) Solve the equation $f^{-1}g^{-1}(x) = 7$. [4]