



# IGCSE Additional Mathematics Ch 9

## Functions Test

Student Name: \_\_\_\_\_

Time allowed: 55 minutes

### READ THESE INSTRUCTIONS FIRST

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate Answer Booklet/Paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

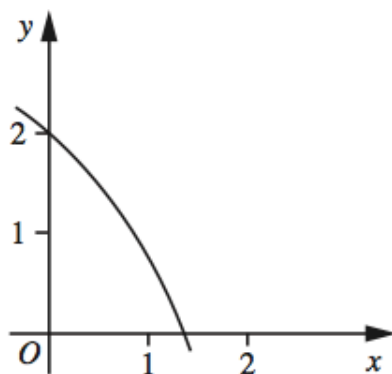
The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 36.

1.



The diagram shows part of the curve  $y = f(x)$ , where  $f(x) = p - e^x$  and  $p$  is a constant.  
The curve crosses the  $y$ -axis at  $(0, 2)$ .

- (i) Find the value of  $p$ . [2]
- (ii) Find the coordinates of the point where the curve crosses the  $x$ -axis. [2]
- (iii) Copy the diagram above and on it sketch the graph of  $y = f^{-1}(x)$ . [2]

2.

The function  $f$  is defined, for  $0^\circ \leq x \leq 180^\circ$ , by

$$f(x) = 3\cos 4x - 1.$$

- (i) Solve the equation  $f(x) = 0$ . [3]
- (ii) State the amplitude of  $f$ . [1]
- (iii) State the period of  $f$ . [1]
- (iv) State the maximum and minimum values of  $f$ . [2]
- (v) Sketch the graph of  $y = f(x)$ . [3]

3.

- (i) Express  $2x^2 - 8x + 3$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are integers. [2]

A function  $f$  is defined by  $f : x \mapsto 2x^2 - 8x + 3$ ,  $x \in \mathbb{R}$ .

- (ii) Find the coordinates of the stationary point on the graph of  $y = f(x)$ . [2]

- (iii) Find the value of  $f^2(0)$ . [2]

A function  $g$  is defined by  $g : x \mapsto 2x^2 - 8x + 3$ ,  $x \in \mathbb{R}$ , where  $x \leq N$ .

- (iv) State the greatest value of  $N$  for which  $g$  has an inverse. [1]

- (v) Using the result obtained in part (i), find an expression for  $g^{-1}$ . [3]

4.

The equation of a curve is  $y = 10 - x^2 + 6x$ .

- (i) Find the set of values of  $x$  for which  $y \geq 15$ . [3]

- (ii) Express  $y$  in the form  $a - (x + b)^2$ , where  $a$  and  $b$  are integers. [2]

- (iii) Hence, or otherwise, find the coordinates of the stationary point on the curve. [2]

Functions  $f$  and  $g$  are defined, for  $x \in \mathbb{R}$ , by

$$f : x \mapsto 10 - x^2 + 6x,$$

$$g : x \mapsto 2x - k, \text{ where } k \text{ is a constant.}$$

- (iv) Find the value of  $k$  for which the equation  $gf(x) = 0$  has two equal roots. [3]