Review exercise

Paper 1 style questions

1 The graph represents the temperature in °C in a certain city last Tuesday.

   a Write down the interval of time in which the temperature was below 0°C.

   b Write down the interval of time in which the temperature was above 11°C.

   c Write down the maximum temperature last Tuesday. Give your answer correct to the nearest unit.
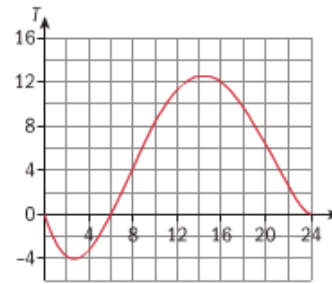
2 The cost c, in Singapore dollars (SGD), of renting an apartment for n months is a linear model

$$c = nr + s$$

where s is the security deposit and r is the amount of rent per month.

Wan Ning rented the apartment for 6 months and paid a total of 35 000 SGD.
Tanushree rented the same apartment for 2 years and paid a total of 116 000 SGD.
Find the value of

   a r, the rent per month   **b** s, the security deposit.

3 Given that $f(x) = x^2 + 5x$

   a factorize $x^2 + 5x$

   b sketch the graph of $y = f(x)$. Show on your sketch

      i the coordinates of the points of intersection with the axes

      ii the equation of the axis of symmetry

      iii the coordinates of the vertex of the parabola.

4 A signal rocket is fired vertically from ground level by a gun. The height, in metres, of the rocket above the ground is a function of the time t, in seconds, and is defined by:

$$h(t) = 30t - 5t^2, 0 \le t \le 6.$$

   a Find the height of the rocket above the ground after 4 seconds.

   b Find the maximum height of the rocket above the ground.

   c Use your GDC to find the length of time, in seconds, for which the rocket is at a height of 25 m or more above the ground.

5 The graph of the function $f(x) = \dfrac{2^x}{m}$ passes through the points $(3, 1.6)$ and $(0, n)$.

   a Calculate the value of m.

   **b** Calculate the value of n. Find $f(2)$.
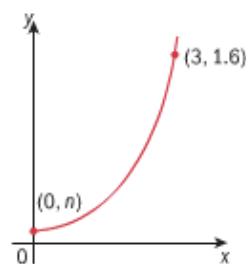
6 The diagram shows the graph of $y = x^2 - 2x - 15$.

The graph crosses the x-axis at the point A, and has a vertex at B.

a Factorize $x^2 - 2x - 15$.

b Find the coordinates of the point

   i A   **ii** B.
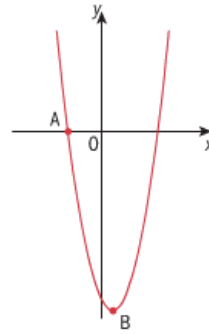
7 Consider the graphs of the following functions.

   i $y = 8x + x^2$

   ii $y = (x - 3)(x + 4)$

   iii $y = x^2 - 2x + 5$

   iv $y = 5 - 4x - 3x^2$

Which of these graphs

a has a y-intercept below the x-axis

b passes through the origin

c does not cross the x-axis

d could be represented by this diagram?
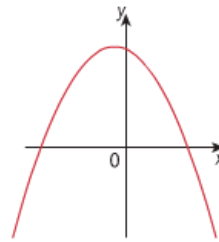
8 The figure shows the graphs of the functions

$$f(x) = (0.5)^x - 2 \quad \text{and} \quad g(x) = -x^2 + 4$$

for values of x between -3 and 3. The two graphs meet at the points A and B.

a Find the coordinates of

   i A   **ii** B.

b Write down the set of values of x for which $f(x) < g(x)$.

c Write down the equation of the horizontal asymptote to the graph of $f(x)$.

9 Gabriel is designing a rectangular window with a perimeter of 4.40 m. The length of the window is x m.

a Find an expression for the width of the window in terms of x.

b Find an expression for the area of the window, A, in terms of x.

Gabriel wants to make the amount of light passing through this window a maximum.

c Find the value of x that meets this condition.

10 a On the same graph sketch the curves $y = 3x^2$ and $y = \dfrac{1}{x}$ for values of x from -4 to 4 and values of y from -4 to 4.

b Write down the equations of the vertical and horizontal asymptotes of $y = \dfrac{1}{x}$.

c Solve the equation $3x^2 - \dfrac{1}{x} = 0$.
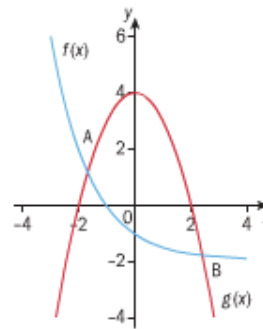
Paper 2 style questions

1 The number (n) of bacteria after t hours is given by the formula $n = 1500(1.32)^t$.

  a Copy and complete the table below for values of n and t.

Time (t hours)	0	1	2	3	4
Number of bacteria (n)	1500		2613	3450	

  b On graph paper, draw the graph of $n = 1500(1.32)^t$. Use a scale of 2 cm to represent 1 hour on the horizontal axis and 2 cm to represent 1000 bacteria on the vertical axis. Label the graph clearly.

  c Find

   i the number of bacteria after 2 hours 30 minutes. Give your answer to the nearest ten bacteria.

   ii the time it will take to form approximately 5000 bacteria. Give your answer to the nearest 10 minutes.

2 The functions f and g are defined by
$$f(x) = \frac{4}{x}, x \in \mathbb{R}, x \neq 0$$
$$g(x) = 2x, x \in \mathbb{R}$$

  a Sketch the graph of $f(x)$ for $-8 \leq x \leq 8$.

  b Write down the equations of the horizontal and vertical asymptotes of the function f.

  c Sketch the graph of g on the same axes.

  d Find the solutions of $\frac{4}{x} = 2x$.

  e Write down the range of function f.

3 A function is represented by the equation $f(x) = 2(1.5)^x + 3$. The table shows the values of $f(x)$ for $-3 \leq x \leq 2$.

x	−3	−2	−1	0	1	2
$f(x)$	3.59	3.89	a	5	6	b

  a Calculate the values for a and b.

  b On graph paper, draw the graph of $f(x)$ for $-3 \leq x \leq 2$, taking 1 cm to represent 1 unit on both axes.

The domain of the function $f(x)$ is the real numbers, $\mathbb{R}$.

  c Write down the range of $f(x)$.

  d Find the approximate value for x when $f(x) = 10$.

  e Write down the equation of the horizontal asymptote of $f(x) = 2(1.5)^x + 3$.

4 The graph shows the temperature, in degrees
  Celsius, of Leonie's cup of hot chocolate t minutes
  after pouring it. The equation of the graph is
  $f(t) = 21 + 77(0.8)^t$ where $f(t)$ is the temperature
  and t is the time in minutes after pouring the
  hot chocolate out.
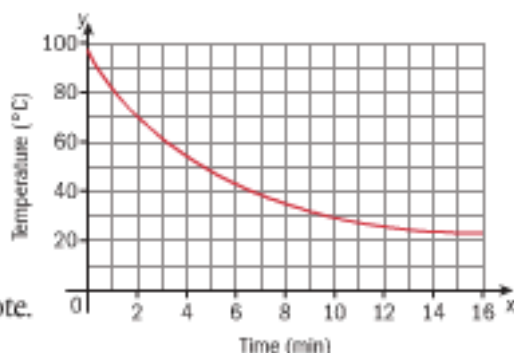
  a Find the initial temperature of the hot chocolate.
  b Write down the equation of the horizontal asymptote.
  c Write down the room temperature.
  d Find the temperature of the hot chocolate after 8 minutes.

5 Consider the functions
  $$f(x) = x^2 - x - 6 \quad \text{and} \quad g(x) = -2x + 1$$
  a On the same diagram draw the graphs of $f(x)$ and $g(x)$
     for $-10 \le x \le 10$.
  b Find the coordinates of the local minimum of the graph
     of $f(x)$.
  c Write down the gradient of the line $g(x)$.
  d Write down the coordinates of the point where the graph of
     $g(x)$ cuts the y-axis.
  e Find the coordinates of the points of intersection of the
     graphs of $f(x)$ and $g(x)$.
  f Hence, or otherwise, solve the equation $x^2 + x - 7 = 0$.

6 a Sketch the graph of $f(x) = x^2 - \dfrac{3}{x}$, for $-4 \le x \le 4$.
  b Write down the equation of the vertical asymptote of $f(x)$.
  c On the same diagram draw the graph of $g(x) = -3(2)^x + 9$,
     for $-4 \le x \le 4$.
  d Write down the equation of the horizontal asymptote
     of $g(x)$.
  e Find the coordinates of the points of intersection of
     $f(x)$ and $g(x)$.

7 The profit (P) in euros made by selling homemade lemonade is modeled by the function

$$P = -\frac{x^2}{10} + 10x - 60$$

where x is the number of glasses of lemonade sold.

a Copy and complete the table.

x	0	10	20	30	40	50	60	70	80	90
P		30			180			150	100	

b On graph paper draw axes for x and $P(x)$, placing x on the horizontal axis and $P(x)$ on the vertical axis. Draw the graph of $P(x)$ against x by plotting the points.

c **Use your graph** to find

  i the maximum possible profit

  ii the number of glasses that need to be sold to make the maximum profit

  iii the number of glasses that need to be sold to make a profit of 160 euros

  iv the amount of money initially invested.

8 a Sketch the graph of the function $f(x) = x^2 - 7$, $x \in \mathbb{R}$, $-4 \le x \le 4$. Write down the coordinates of the points where the graph of $y = f(x)$ intersects the axes.

b On the same diagram sketch the graph of the function
$$g(x) = 7 - x^2, x \in \mathbb{R}, -4 \le x \le 4.$$

c Solve the equation $f(x) = g(x)$ in the given domain.

d The graph of the function $h(x) = x + c$, $x \in \mathbb{R}$, $-4 \le x \le 4$, where c is a positive integer, intersects twice with both $f(x)$ and $g(x)$ in the given domain.
Find the possible values for c.

Answers

Review exercise
Paper 1 style questions

1 a 00:00–06:00

  b 11:30–17:00
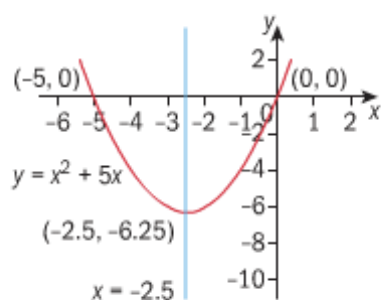
  c 13 °C

2 a 4500 SGD

  b 8000 SGD

3 a $x(x+5)$

  b

4 a 40 m

  b 45 m

  c 4 s

5 a $m = 5$

  b $n = \dfrac{1}{5}$ $f(2) = \dfrac{4}{5}$

6 a $(x-5)(x+3)$

  b i (−3, 0)   ii (1, −16)

7 a ii   b i   c iii   d iv

8 a i (−1.68, 1.19)

    ii (2.41, − 1.81)
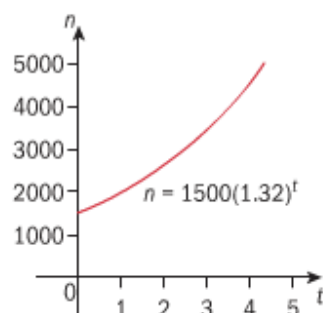
  b − 1.68 < x < 2.41

  c $y = -2$

9 a $2.2 - x$

  b $A = x(2.2 - x)$

  c $x = 1.1$ m

Paper 2 style questions

1 a 1980, 4554

  b
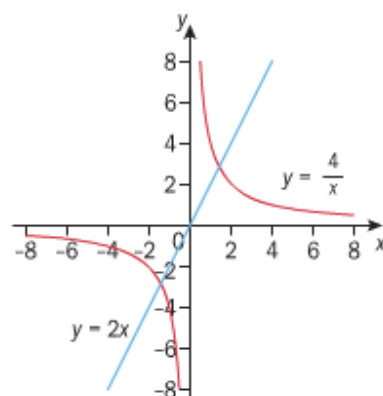

  c i 3000

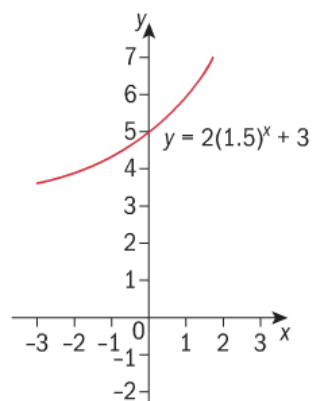   ii 4 hours 20 minutes

2 a, c

  b $y = 0, x = 0$ d $x = \pm 1.41$

  e $\{y \mid y \in \mathbb{R}, y \neq 0\}$
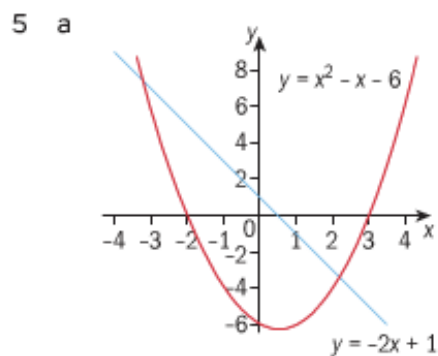
3 a $a = 4.33, b = 7.5$

  b

  c $f(x) > 3$
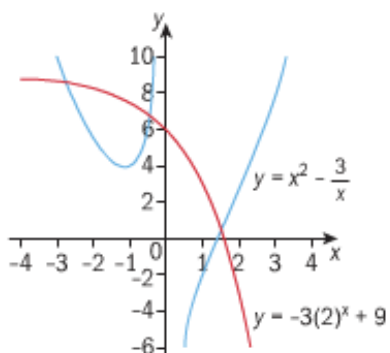
  d $x = 3$ (approximately)

  e $y = 3$

4 a 98 °C   b $y = 21$
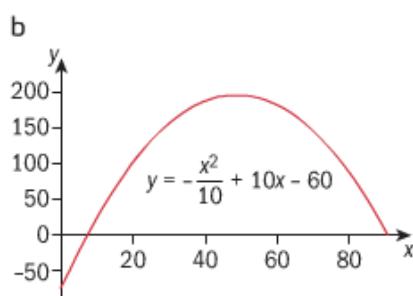
  c 21 °C   d 33.9 °C

5 **a**

$y = x^2 - x - 6$

$y = -2x + 1$

b $(0.5, -6.25)$

c -2 **d** $(0, 1)$

e $(2.19, -3.39), (-3.19, 7.39)$

f $x = 2.19, -3.19$

6 **a, c**

$y = x^2 - \dfrac{3}{x}$

$y = -3(2)^x + 9$

b $x = 0$ **d** $y = 9$

e $(-2.73, 8.55), (-0.454, 6.81),$
$(1.53, 0.362)$

b

$y = -\dfrac{x^2}{10} + 10x - 60$

c **i** 190 euros
ii 50
iii 33 or 67
iv 60 euros

8 **a, b**

$(0, 7)$ $y = 7 - x^2$

$(-2.65, 0)$ $(2.65, 0)$

$(0, -7)$ $y = x^2 - 7$
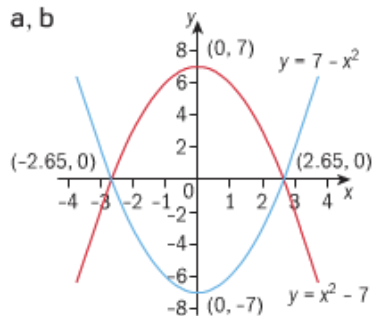
c $x = \pm 2.65$

d $c = 1, 2, 3, 4, 5$

7 **a**

x	0	10	20	30	40	50	60	70	80	90
P	−60	30	100	150	180	190	180	150	100	30